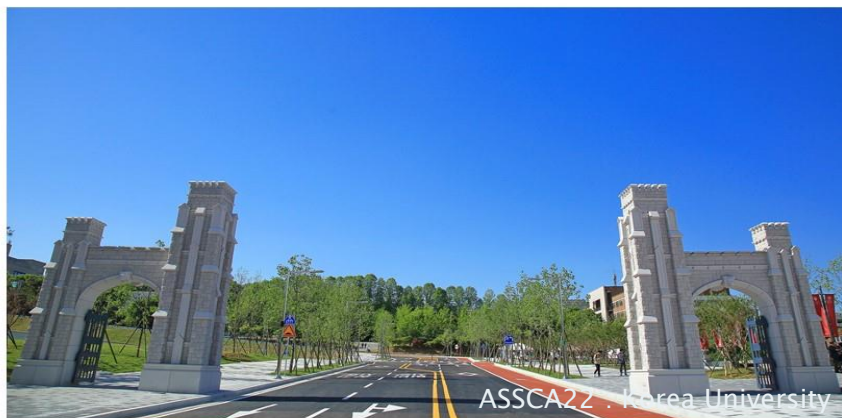




Cryogenics : Basic Thermodynamics and Liquefaction Cycles

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Superconductivity and Cryogenics for Accelerators
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ASSCA22 : Korea University : Feb.17,2023 : T S Datta

Outline of my Lectures

- 1. Introduction on Cryogenics**
- 2. How to Produce Cryogen**
 - **Thermodynamic Process (Carnot)**
 - **Liquefaction Cycle**
 - **Performance of Cycle**
- 3. Practical Helium Liquefier/ Refrigerator**
 - **Components**
- 4. Properties of Matter at Cryogenic Temperature
(If Time Permits)**





Temperature Scale

6000K	Sun
373K	Water Boils
300K	Room Temp
273K	Ice
263K	SO ₂ -liquid
240K	NH ₃ - liquid

Lowest Temp:
500 pico K

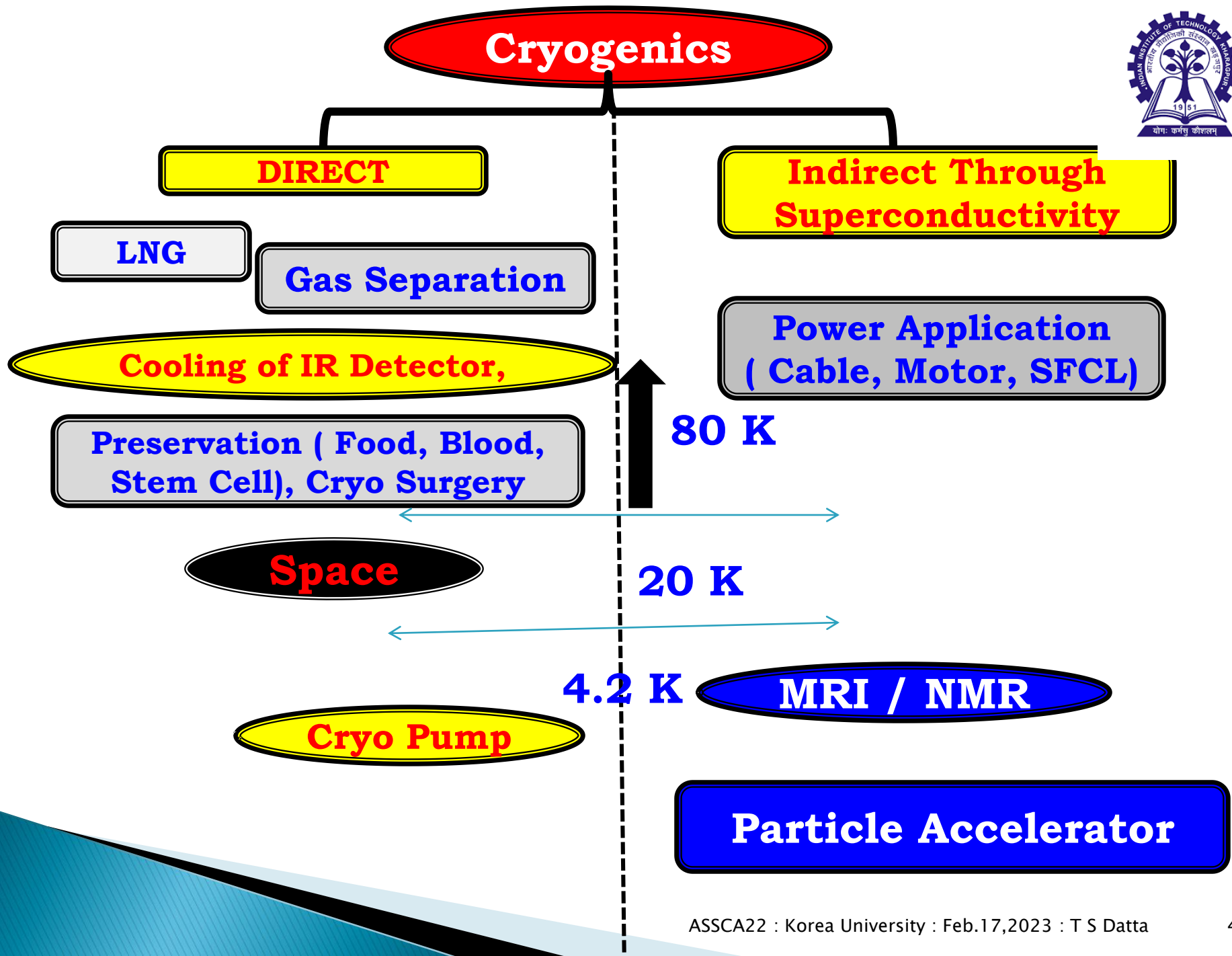
Why?

120K

Cryogenic
Temperature
range

		Cryogenics boundary
111K	CH ₄ -liquid (LNG)	
90K	LOX -liquid Oxygen	
77K	LN ₂ -liquid Nitrogen	
20K	LH ₂ -liquid Hydrogen	
4.2K	LHe -liquid Helium	
2.1K	Superfluid Helium	

0 K - Absolute Zero



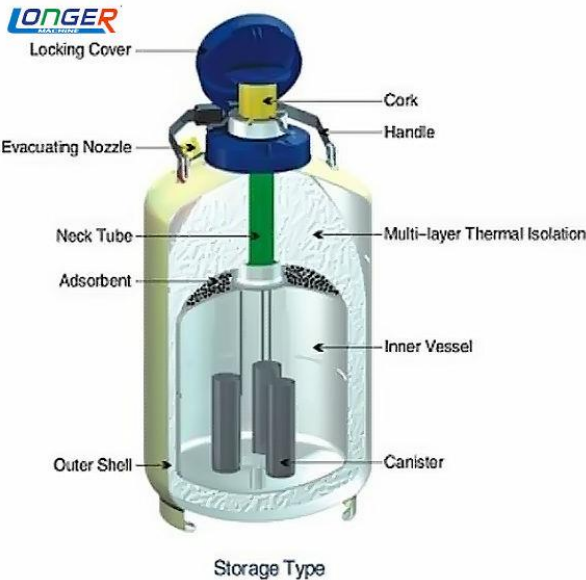
CRYOGEN STORAGE VESSEL



100- 500L Pressurized



Small container 10- 60 L



20,000 L Storage Vessel at IUAC

Transport Vessel



WHY STORAGE IS SO CRITICAL ?



A. LOW LATENT HEAT (L)

B. LARGE TEMPERATURE DIFFERENCE ($T_r - T_b$)

Property	He	N ₂	Water
Boiling Point	4.2K	78K	373K
Density	0.12kg/liter	0.81kg/liter	1kg/litre
<i>Heat of Vaporisation</i>	<i>20KJ/ kg</i>	<i>198</i>	<i>2250</i>
Liquid evaporated on 100 W Heat input	140 L/ hr	2.2 L/hr	0.16 L/hr

1 W HEAT LOAD EVAPORATES 34 LITRES LHe in ONE DAY

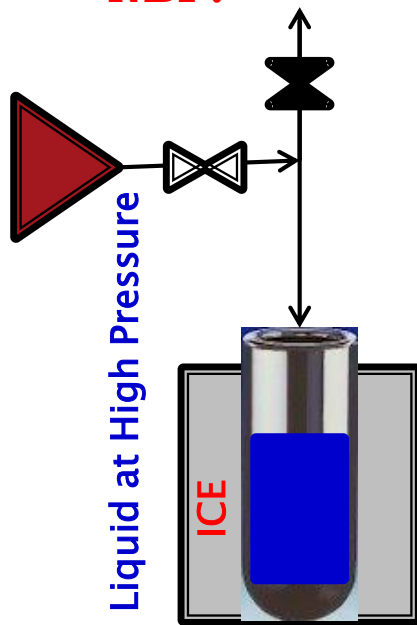
THANKS TO MLI (MULTILAYER INSULATION TECHNOGY)

EVAPORATION RATE ONLY 1 LITRES/d in 100 Litres Vessel

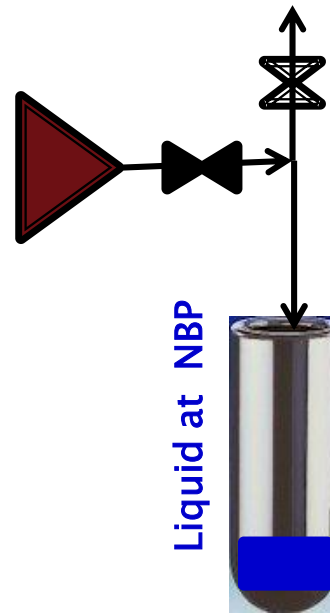
PERMANENT GASES

Laboratory techniques for reducing temperatures and Liquefaction

- Liquefaction of gas at high pressure in a thick-walled glass tube surrounded by ice (273 K),**
- A rapid expansion of the vapor phase to atmospheric pressure through a valve.**
- The temperature of the remaining liquid phase then dropped to its NBP.**



a) Isothermal Compression



b) Rapid Expansion

Ethylene, could be liquefied with a critical temperature of **282 K** and a normal boiling point temperature of **169 K**,

Methane, Nitrogen, hydrogen, Helium that could not be liquefied by this technique, even with pressures up to 40 MPa, were called **“permanent” gases.**

CRYOGENIC RANGE



	T(boil)	T(critical)	P(critical)
SO ₂	263K	432K	79 Bar
NH ₃	240K	405K	115 Bar
C ₂ H ₄	169	282	50
CH ₄	112	191K	46 Bar
O ₂ (LOX)	90K	155K	50 Bar
N ₂ (LN2)	78K	126K	34Bar
H ₂ (LH2)	20K	33K	13Bar
He(LHe)	4.2K	5.2K	2.2 Bar

T_c [SO₂/NH₃] > 273K (ICE temp)

gas

Pressure

liquid

T_c [N₂/He] < 273 K (ICE Temp)

gas

Whatever
Pressure ?

No liquid

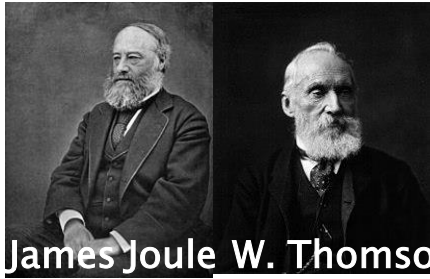
Permanent Gases

Hence they are called

Ar, N₂, O₂, Air, Ne, H₂ and He

LNG
&
LPG ??

HISTORY ON LIQUEFACTION OF PERMANENT GASES



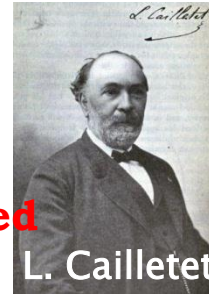
James Joule W. Thomson

1852

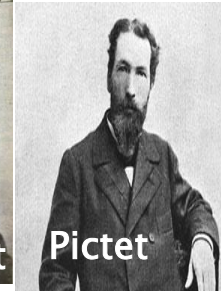
JT Effect Discovered

1877- 1883

Oxygen / Nitrogen Liquefied



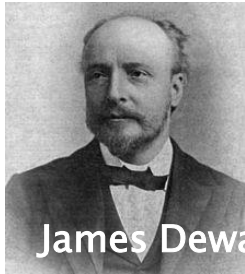
L. Cailletet



Pictet



Wroblewski and Olszewski



James Dewar

1898

Hydrogen Liquefied

1902

Expansion Engine for Air



George Claude



Carl von Linde



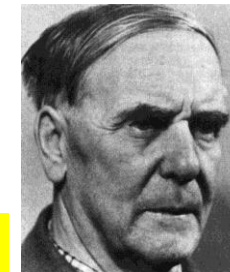
Heike
Kamerlingh Onnes

July 10, 2008

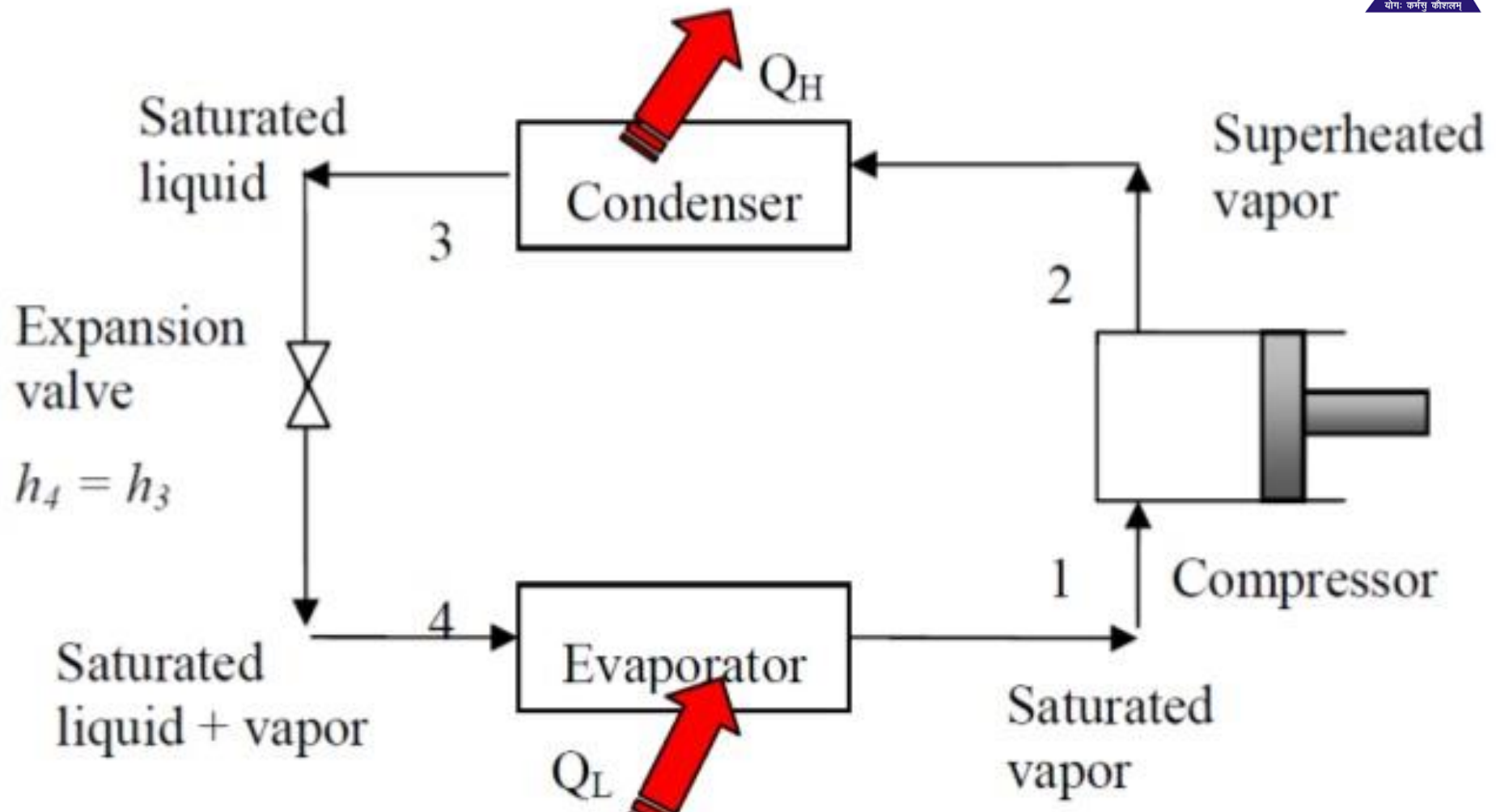
Helium Liquefaction

1934

Helium Liquefaction with Claude Cycle



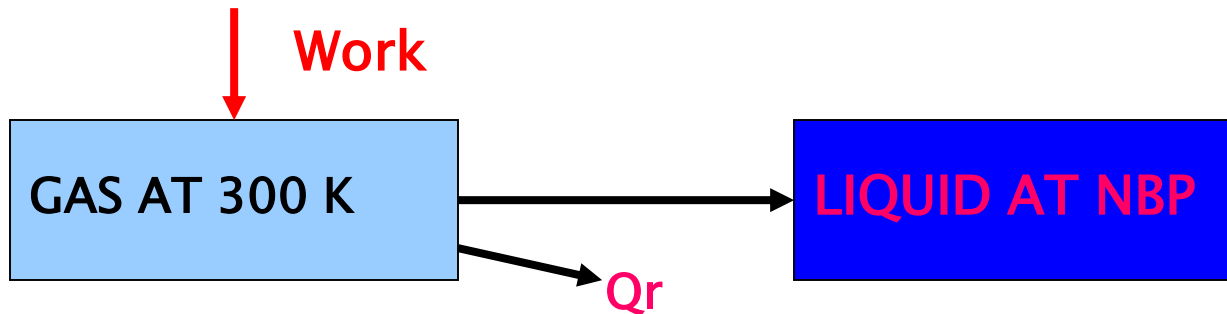
P. Kapitsa



We can not use similar principle for Cryogenic Refrigerator, because Critical Temperature of Cryogenic gases are much below Room Temperature

LIQUEFACTION OF PERMANENT GASES

$$Q_r = \text{Sensible Heat} + \text{Heat of Vaporisation}$$



$$(Q_r) = \text{Nitrogen } 234 \text{ J/ gm (300K to 78 K)} + 199 \text{ J/gm}$$

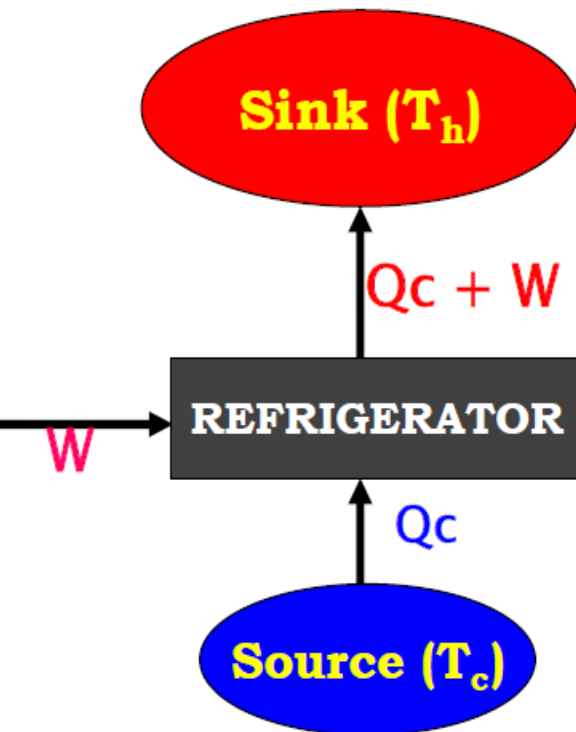
$$\text{Helium : } 1542 \text{ J/ gm (300 K to 4.2 K)} + 20 \text{ J/ gm}$$

Sensible Heat

Latent Heat

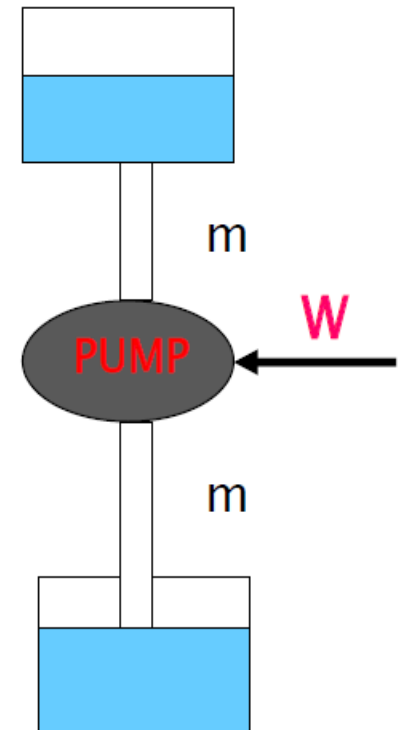
Refrigerator

To Transfer Heat from Source to Sink if Source Temp is less than Sink



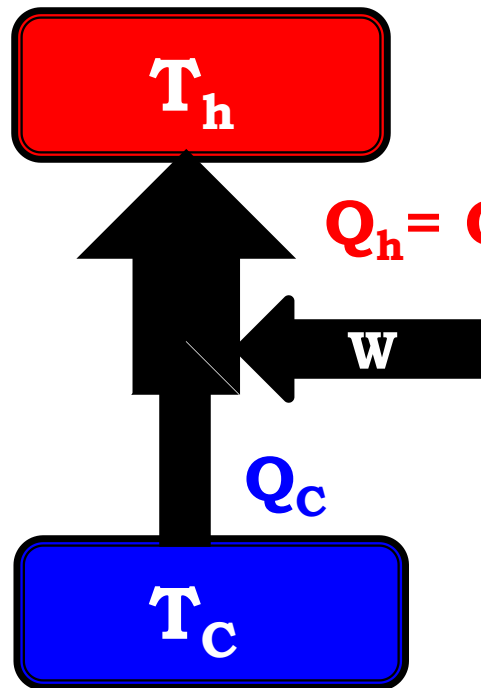
Refrigerator is analogous to Water Pump to transfer Heat (Water) from Lower Temp (Lower level) to Higher Temp (Higher Level)

Power required or pump size depends on water capacity (Ref. Load in Watt) and the difference of level (Diff on Temp)



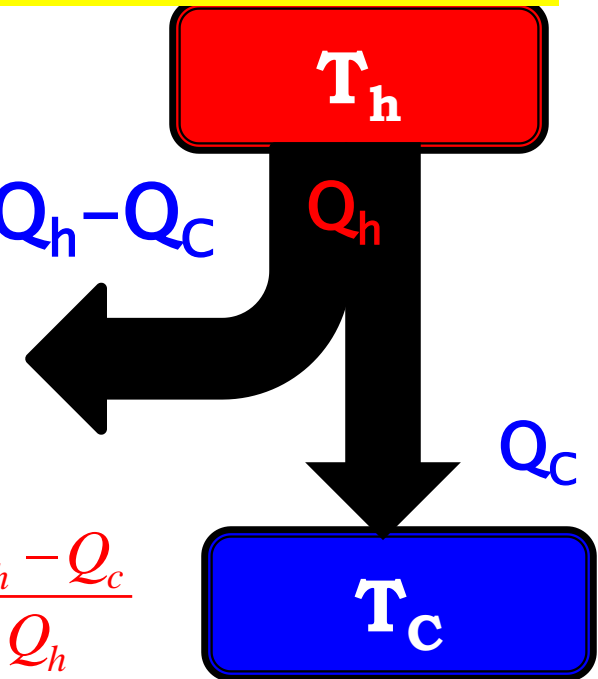
Transfer Amount of Heat energy is different between Source and Sink unlike pump. That embodies the concept the " Quality " of Thermal energy

Second Law of Thermodynamics: It is impossible to extract an amount of heat Q_H from a hot reservoir and use it all to do work W .



$$W = Q_h - Q_c$$

$$\eta = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h}$$



$$\begin{aligned} \text{COP}_{R,\text{Carnot}} &= Q_c / W \\ &= T_c / (T_h - T_c) \end{aligned}$$

2nd Law of Thermodynamics: It is not possible for heat to flow from a colder body to a warmer body without any input of Work

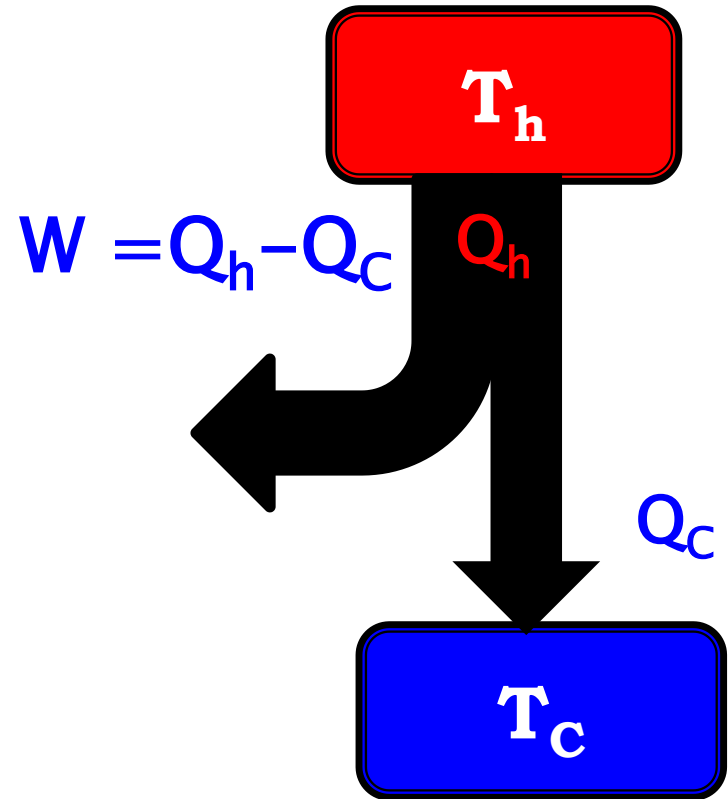
CARNOT CYCLE is the Most Efficient Cycle to have Maximum Work



$$\eta = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = \frac{T_h - T_c}{T_h}$$

Here T_c Is fixed (300 K) and T_h Can be varied . Higher T_h Higher Efficiency.

$$\begin{aligned} T_h &= 600, \eta = 50 \% \\ T_h &= 1000\text{K}, \eta = 70\% \end{aligned}$$



VCR used for Cooling

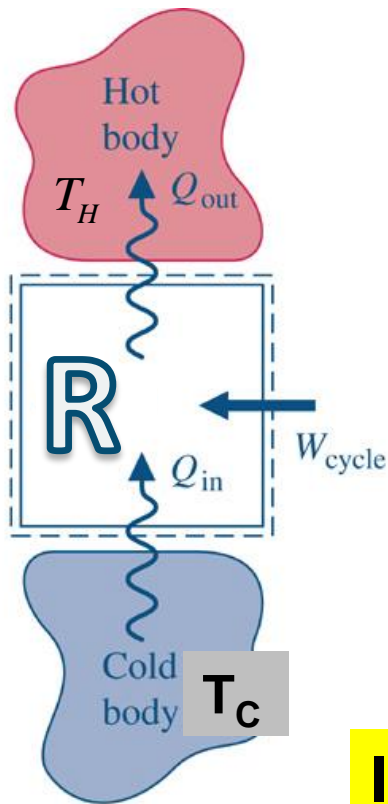


$$\eta_E = \frac{\text{energy sought}}{\text{energy that costs}} = \frac{Q_{in}}{W_{cycle}} = \text{COP}_C$$

$$\text{COP}_{R,\text{Carnot}} = T_C / (T_h - T_C)$$

Observation: η_E may be >1 ($\eta_E > 100\%$)

The concept of an efficiency being greater than 100% makes people uneasy. Therefore, the conversion efficiency for a refrigerator is called the **Cooling Coefficient of Performance (COP_C)**. A refrigeration system that is used for cooling is called a **refrigerator**.



Inverse COP : Power required to have refrigeration of 1 W : $W / Q_c = (T_h - T_C) / T_C$

Power (W) required to extract 1 W refrigeration at T_c is : $1 / (\text{COP}) =$

$$= \frac{W}{Q_c} = \frac{Q_h - Q_c}{Q_c} = \frac{T_h - T_c}{T_c}$$

$T_h = 300 \text{ K}$ T_c vary from 200 K to $.000001 \text{ K}$

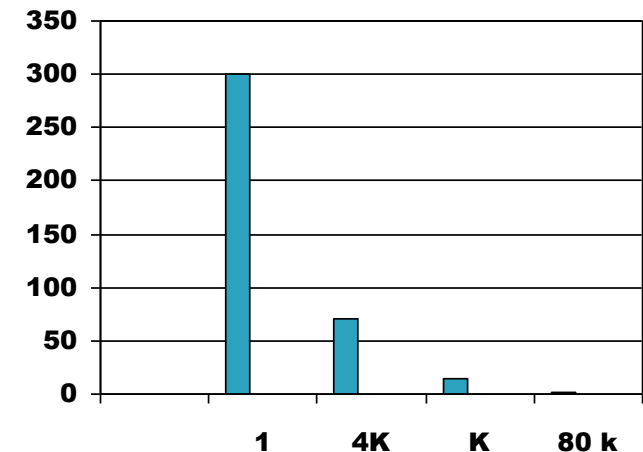
$T_c = 200 \text{ K}$, $W = 0.5 \text{ W}$

$\text{LN}_2 : T_c = 78 \text{ K}$, $W = 1.68 \text{ W}$

$\text{LH}_2 : T_c = 4.2 \text{ K}$, $W = 70 \text{ W}$

$T_c = 2 \text{ K}$, $W = 150 \text{ W}$

$T_c = 0.01$ $W = 30 \text{ k W}$



These are Theoretical minimum Power. We have to multiply first with efficiency of the Cycle and then multiply with mechanical efficiency of all Components (Compressor, Heat Exchanger, Expander of refrigerator)

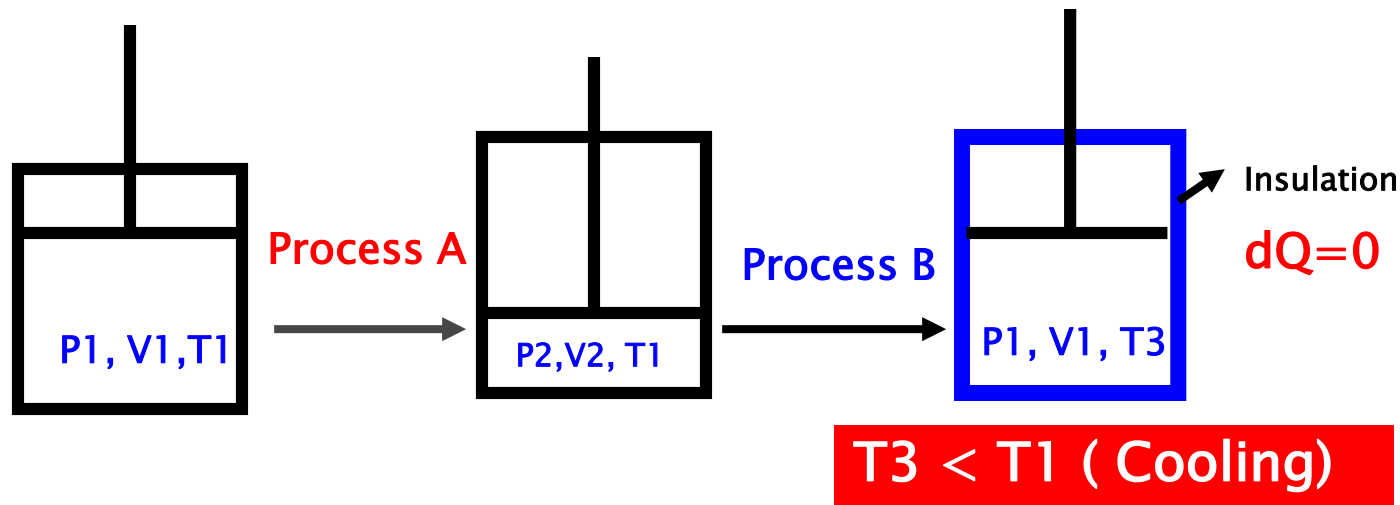
Actual work = $W_c / (\eta_{\text{Cycle}} * \eta_{\text{Comp}})$ Total efficiency may be 10 to 30 % at 4.2 K

Required Plug Power for 1 W refrigeration at 4.2 K = 500- 225 W

Required Plug Power for 1 W refrigeration at 2K = 1000- 1500 W

BASIC THERMODYNAMIC PROCESS FOR COOLING

- ▶ **A. ISOTHERMAL COMPRESSION (Compressor)**
- ▶ **B. ADIABATIC EXPANSION (Turbine)**
- ▶ **C. ISENTHALPIC EXPANSION (JT VALVE)**
- ▶ **D. ISOBARIC COOLING (Heat Exchanger, Precooler)**



Isothermal compression is achieved with water/ air cooling System. $W = m \cdot T \left(\frac{R}{M} \right) \ln \left(\frac{P_2}{P_1} \right)$.

Example : 1 gm/s gas $T = 300$ K, $P_2 / P_1 = 15$

Helium : 1600 W (20 NM³), N₂ = 200 W (2. 8 NM³)

Fundamental :

Pressure (P)

Temperature (T)

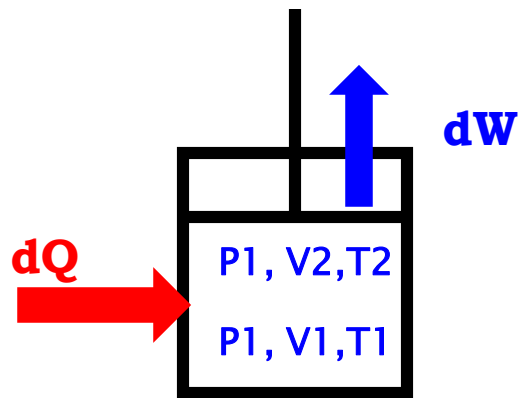
Volume (V)

Gas Constant (R)

Work (W)

Heat (Q)

Internal Energy (U)



$$dQ = dU + dW = C_p dT + p dV$$

Other Important Parameters

1. Entropy (S) :

- $dS = \frac{dQ}{T}$
- Entropy is a measure of Disorder.
 - Second Law of Thermodynamics: In any cyclic process the entropy will either increase or remain the same.

$$\oint ds = \oint \frac{dq}{T} \geq 0$$

2. Enthalpy (h)

- equivalent to the total heat content of a system. It is equal to the internal energy of the system plus the product of pressure and volume.

$$dh = du + p dv$$

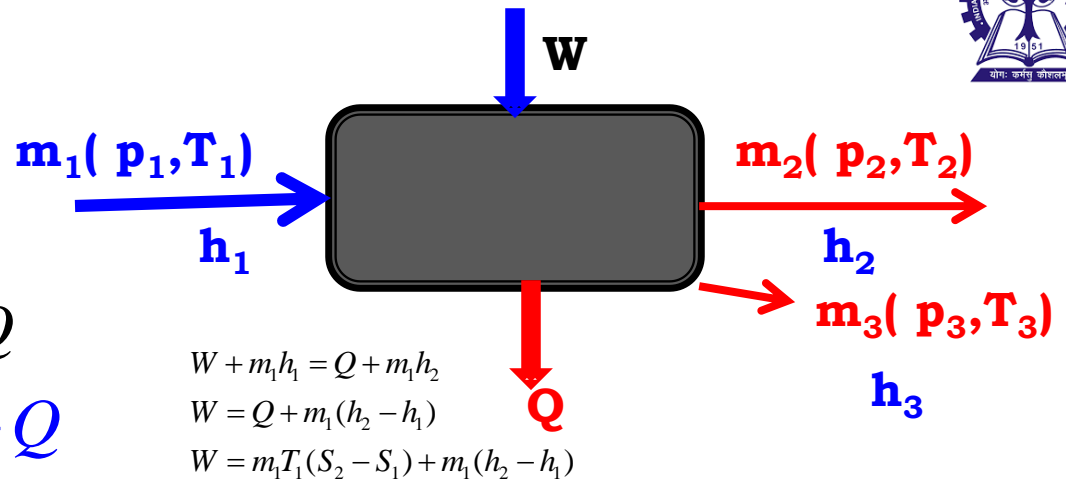
Energy & Mass Balance



$$m_1 = m_2 + m_3$$

$$m_1 h_1 + W = m_2 h_2 + m_3 h_3 + Q$$

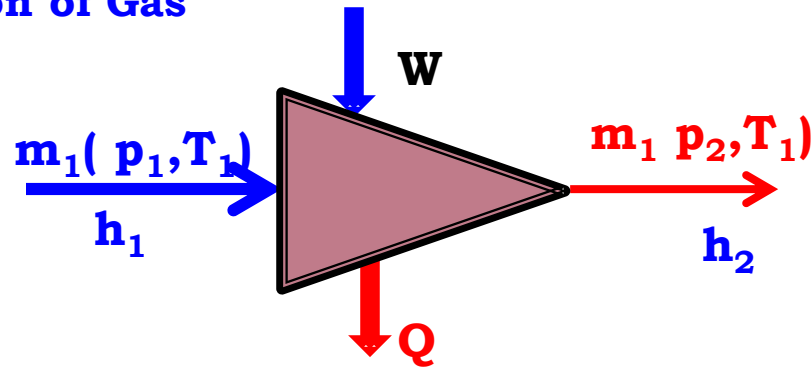
$$W = m_2 h_2 + m_3 h_3 - m_1 h_1 + Q$$



A. Example : Isothermal Compression of Gas

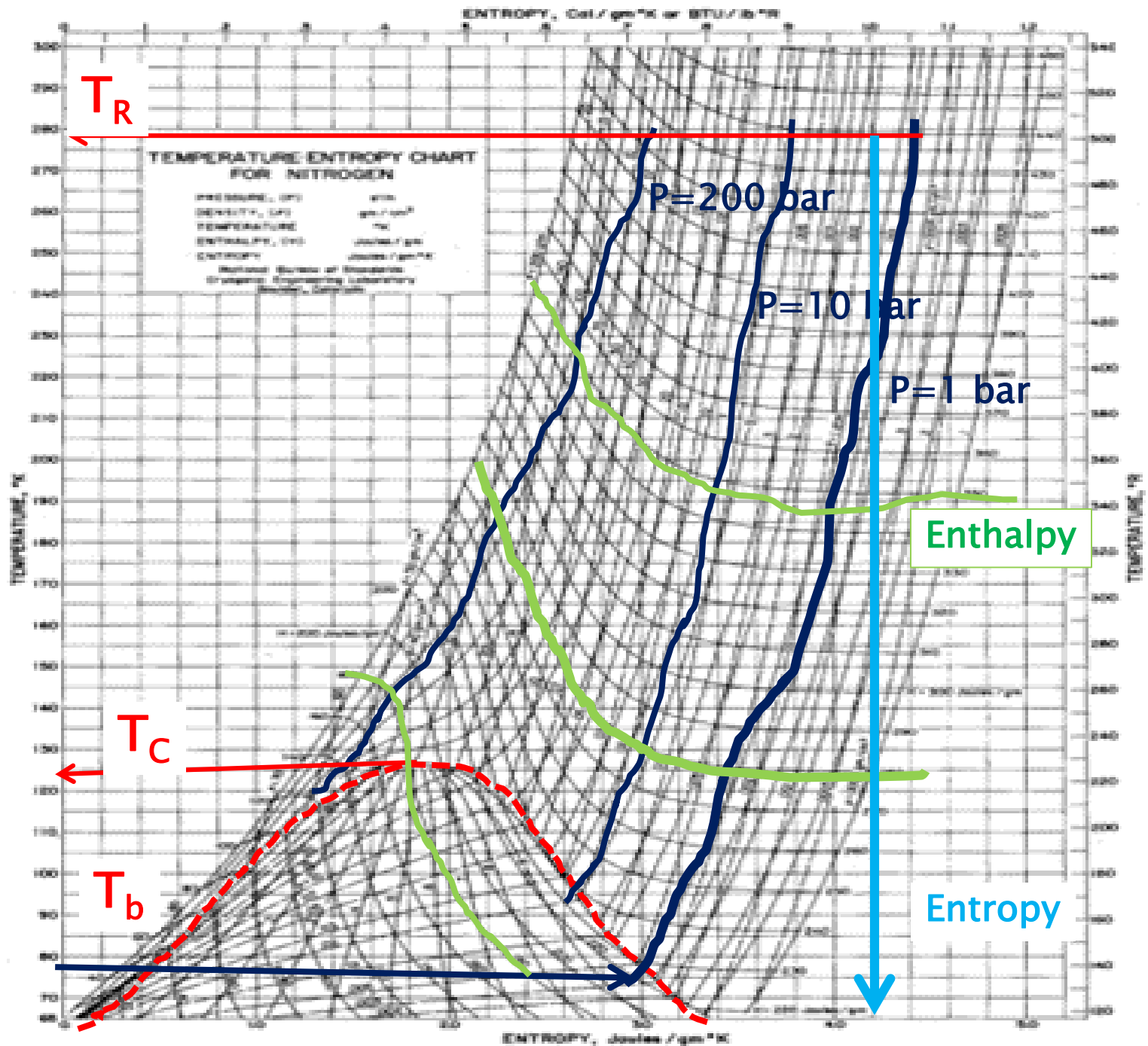
$$W + m_1 h_1 = Q + m_1 h_2$$

$$W = Q + m_1 (h_2 - h_1)$$



$$W = m_1 T_1 (S_2 - S_1) + m_1 (h_2 - h_1)$$

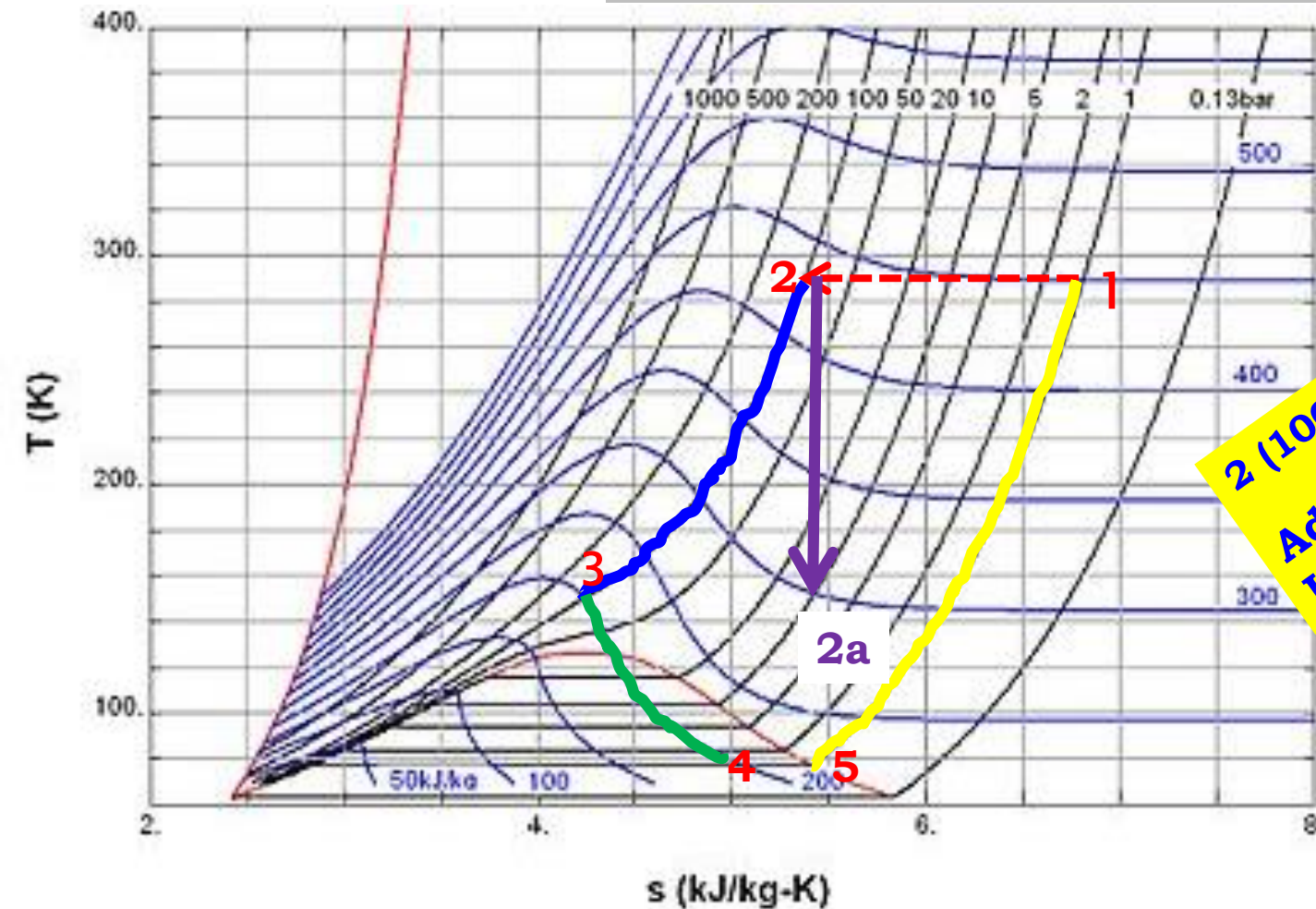
(Temperature, Entropy Chart with enthalpy)



SAMPLE TEMPERATURE- ENTROPY CHART FOR NITROGEN

- Constant Enthalpy Curve
- Constant Pressure Line
- Liquid Vapour Inter-phase

- 1 (1bar, 280 K)–2 (100, 280) : **Isothermal Compression**
- 2(100,280)-3(100,150) : **Isobaric Cooling (HX)**
- 3(100,150) -4(1,78) : **Isenthalpic Cooling (JT)**
- 5(1,78) -1(1,280) : **Isobaric Heating (HX)**



2 (100, 280)-2a (10-155):
Adiabatic Expansion/
Isentropic Cooling



To Liquefy Permanent Gases

Cooling

**Isenthalpic
(JT)**

Isentropic

**Indirectly By Other
Cold Liquid/ Gas
(Heat Exchanger**

By JT Valve

(No External Work)

By Expansion

Engine/Turbine

(Do External Work)

High Pressure to Low pressure

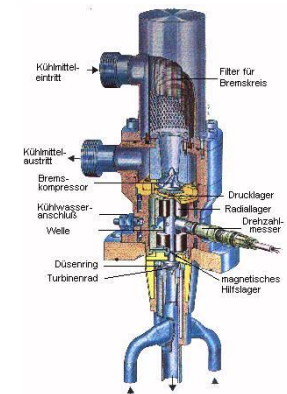
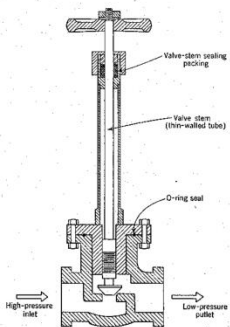
Cooling co- efficient

$$\mu_{JT} = \left(\frac{\partial T}{\partial p} \right)_h$$

$$\mu = \left(\frac{dT}{dp} \right)_x$$

$$\mu_s = \left(\frac{\partial T}{\partial p} \right)_s$$

**μ should be positive
Higher is Better**



$$\mu_{JT} = \left(\frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_p - v \right]$$

For an Ideal Gas $pv = RT$, $\left(\frac{\partial v}{\partial T} \right)_p = R / p = v / T$

$$\mu_{JT} = 0$$

Fortunately Gas does not behave ideally

Real Gas : Vander Wall $\left(p + \frac{a}{v^2} \right) (v - b) = RT$

$$\mu_{JT} = \frac{(2a / RT) \left(1 - \frac{b}{v} \right)^2 - b}{C_p \left[1 - (2a / vRT) \left(1 - \frac{b}{v} \right)^2 \right]}$$

Isenthalpic /JT Cooling



$$\mu_{JT} = \frac{(2a/RT)\left(1 - \frac{b}{v}\right)^2 - b}{C_p \left[1 - (2a/vRT)\left(1 - \frac{b}{v}\right)^2 \right]}$$

At large specific volumes

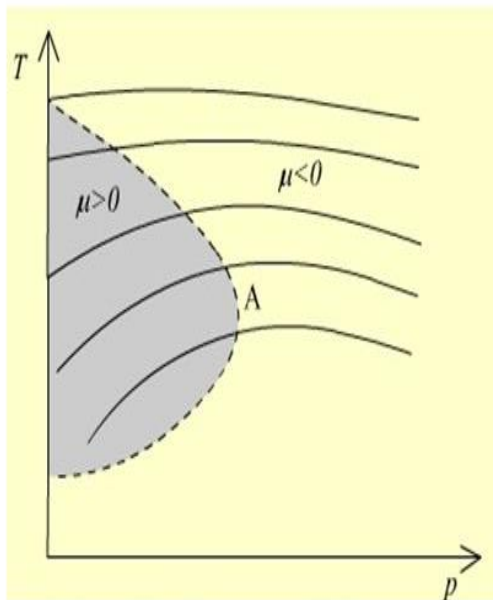
$$\mu_{JT} = \frac{1}{C_p} \left(\frac{2a}{RT} - b \right)$$

**When , $2a/RT > b$,
Or $T < 2a/bR$**

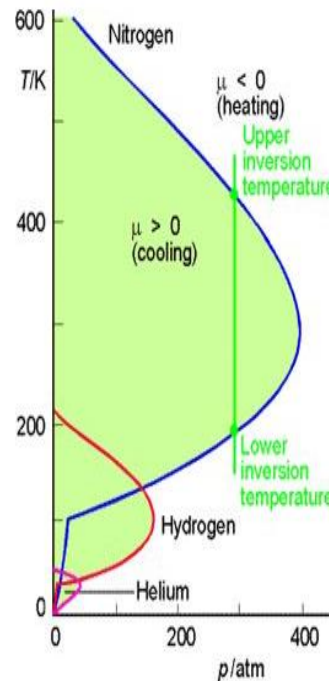
μ_{JT} is Positive , Hence Cooling

When , $2a/RT < b$,

μ_{JT} is Negative , Hence Heating



Inversion curve



Inversion curve is represented by all points , where $\mu_{JT} = 0$

$$T_i = \frac{2a}{bR} \left(1 - \frac{b}{v} \right)^2$$

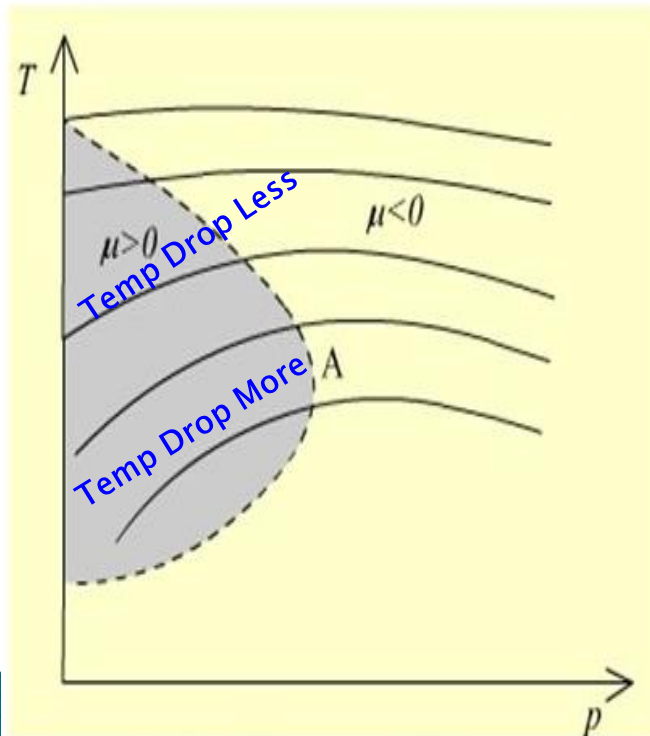
**Maximum Inversion Temperature
 $T_{imax} = 2a/bR$
(at $p=0$ or $b/v=0$)**

Above Max inversion temperature (T_{imax}) we will not be able to cool the gas for any set of pressure combination.



Gas	He	H2	Ne	N2	Ar	O2
T_{imax} (K)	45	205	250	621	794	761

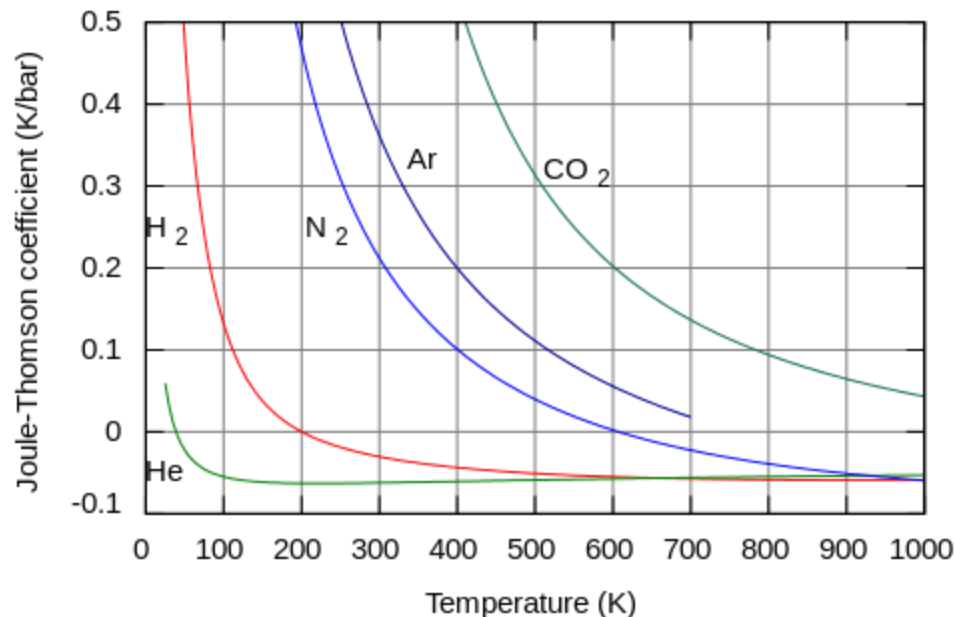
Above RT



Just below their max inversion temperature drop in temperature is not significant and temperature drop increases as we lower the inlet temperature and max above their critical temperature.

That's the reason JT is always incorporated in the last stage of liquefaction cycle. It can also handles liquid gas mixture unlike turbine

Temperature is the measure of thermal kinetic energy (energy associated with molecular motion); so a change in temperature indicates a change in thermal kinetic energy. **The internal Energy is the sum of thermal kinetic energy and thermal potential energy. Thus, even if the internal energy does not change, the temperature can change due to conversion between kinetic and potential energy;** this is what happens in a free expansion and typically produces a decrease in temperature as the fluid expands



$$dh = du + pdv$$

$$Pdv=0, dh=0 \text{ and hence } du=0$$

$$d(\text{KE} + \text{PE}) = 0$$

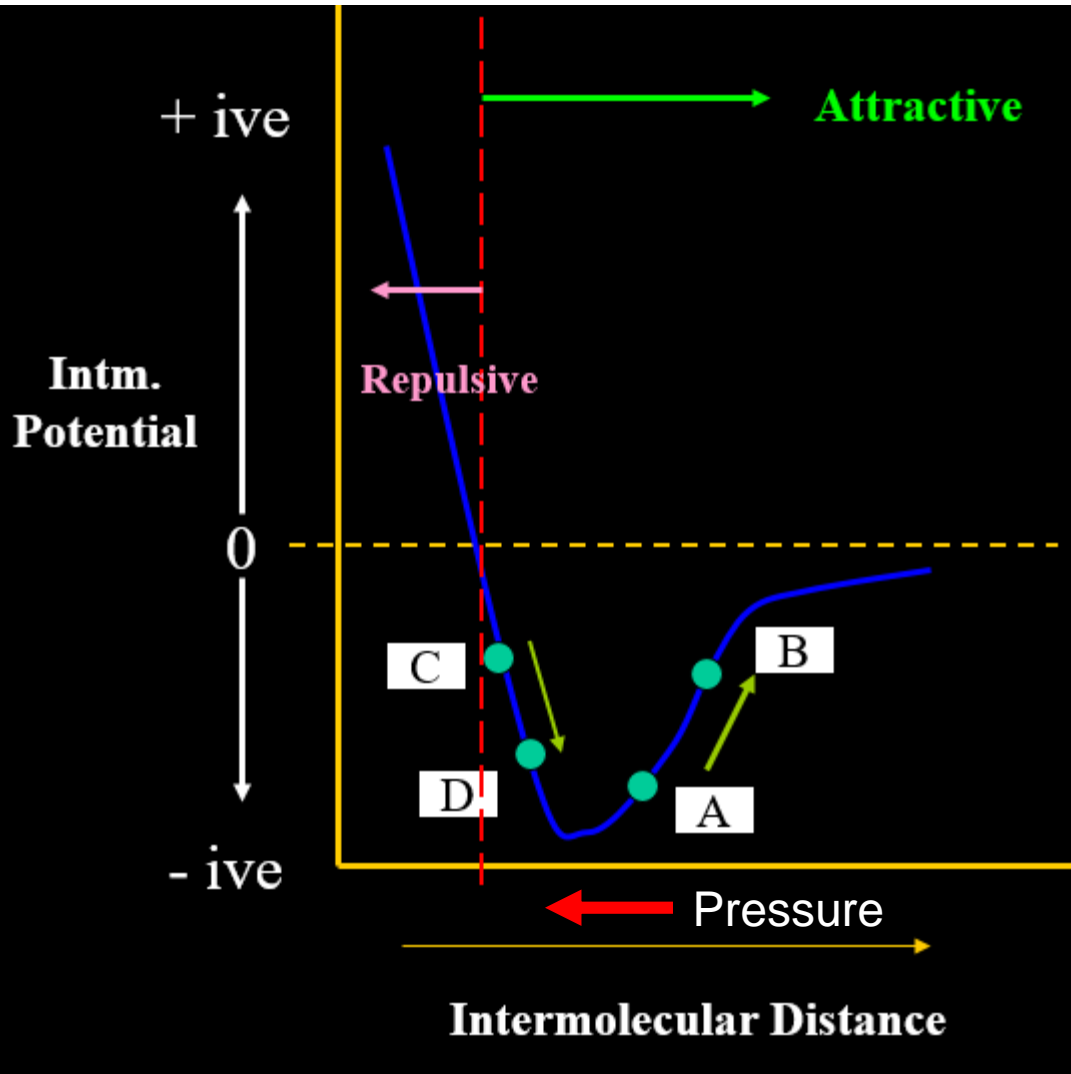


JT COOLING

$$dh = du + pdv$$

$Pdv=0$, $dh=0$ and
Hence $du=0$

Difference (KE+ PE) = 0



Expansion from A to B :
Potential Energy increases
and hence KE decreases (Temp Drop)

Expansion from C to D :
Potential Energy decreases
and KE increases , Temp increases

Adiabatic Expansion (Isentropic Cooling)

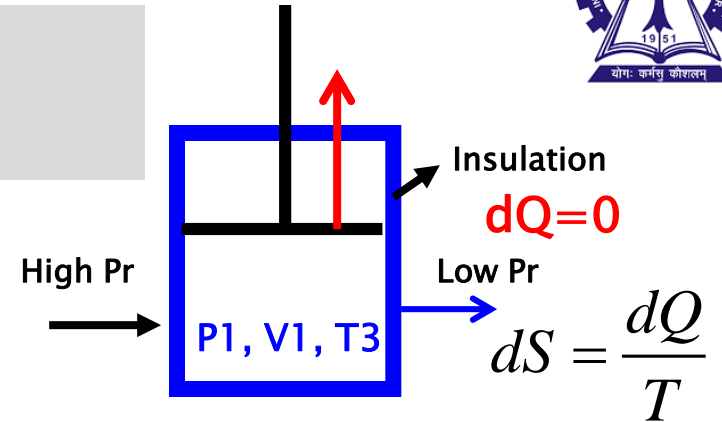


High Pressure Gas does an External work (Lifting Piston) and comes out at low Pressure and low temperature

$$\mu_s = \left(\frac{\partial T}{\partial p} \right)_s = \frac{T}{C_p} \left(\frac{\partial v}{\partial T} \right)_p$$

$$\left(\frac{\partial v}{\partial T} \right)_p = R / p = v / T \quad \mu_s = v / C_p$$

$$\mu_s > \mu_{JT}$$



$$dS=0 \text{ (Isentropic)}$$

$$dQ = dU + dW = C_p dT + p dV$$

- **Unlike JT Expansion, There will be always cooling effect on adiabatic expansion at any temperature and Pressure**
- **Temperature drop is much higher compared to JT Expansion**

Comparison of Cooling Effect by JT & Isentropic Expansion Through T- S Chart

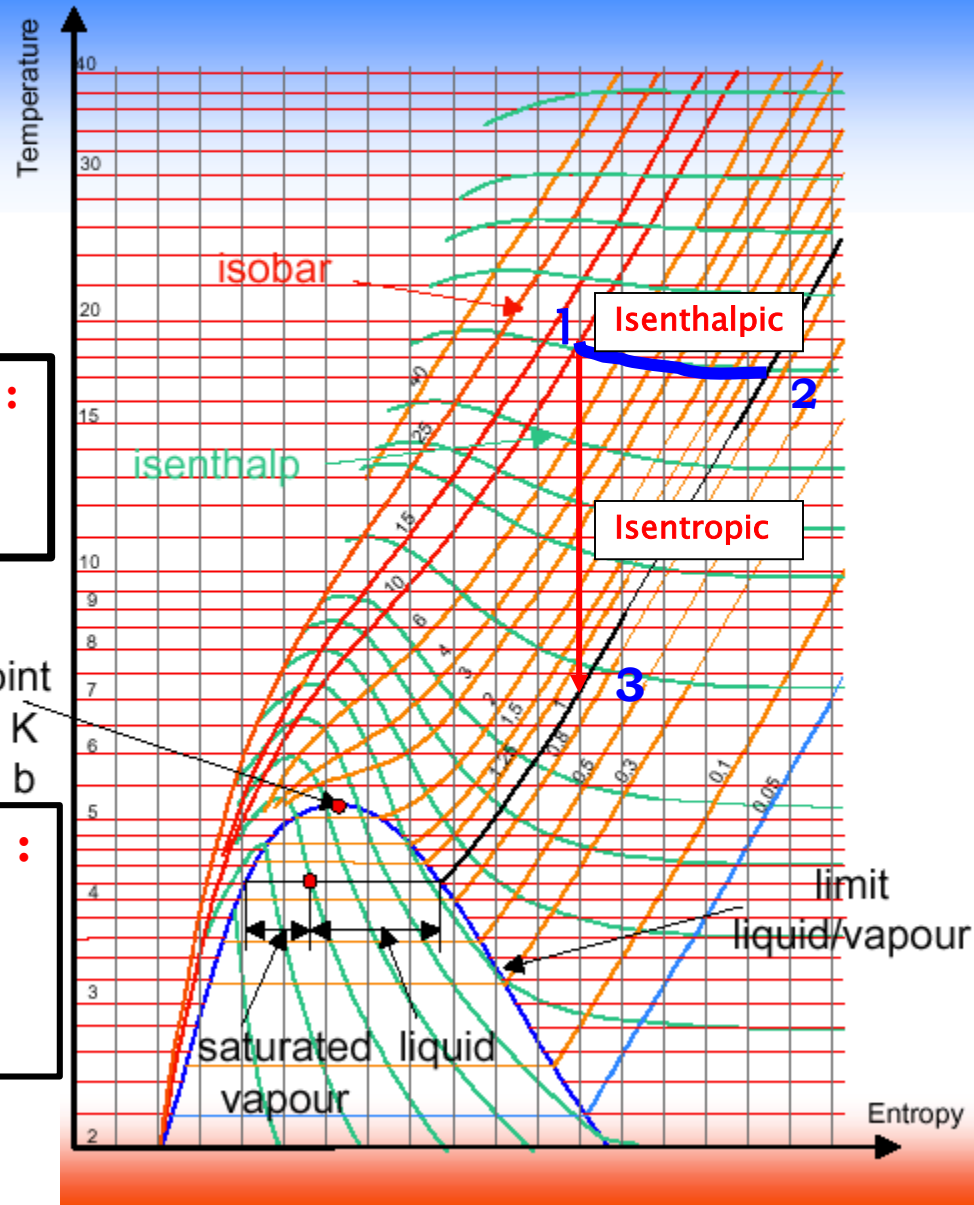
$$(dT)_s \gg (dT)_h$$

HELIUM

1 (19k, 10 bar) - 2 (17 K, 1 Bar) :
JT Cooling
Drop only 2 K

1 (19k, 10 bar) - 3 (7 K, 1 Bar) :
Isentropic Cooling
Drop 12 K

Critical point
T = 5,195 K
P = 2,274 b



Adiabatic (Isentropic) Expansion

$$PV^\gamma = \text{Constant} \quad \gamma = \frac{C_p}{C_v} \quad \begin{aligned} \gamma &= 1.66 \text{ (He, Monatomic Gas)} \\ &= 1.4 \text{ (N}_2\text{, Diatomic Gas)} \end{aligned}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

Example : If Helium Gas is Expanded from 220 Psi (p_1) to 18 Psi (p_2) at Inlet temperature (T_1) = 60 K

$T_2 = 23.4 \text{ K}$, Actual Case it is 30K

Efficiency = 80 %

LIQUEFACTION SYSTEM PARAMETERS



- **Work Required per unit mass liquefied = W/m_f**
- **Work Required per unit Mass Gas Compressed = W/m**
- **Yield (What fraction of Compressed Gas is liquefied) , $y= m_f/m$**
- **Ideal work required per unit mass liquefied= W_i/m_f**
- **Figure of Merit : Ideal work/ Actual work for the cycle**

$$\text{FOM} = (W_i/m_f) / (W/m_f) : 0 \text{ to } 1$$

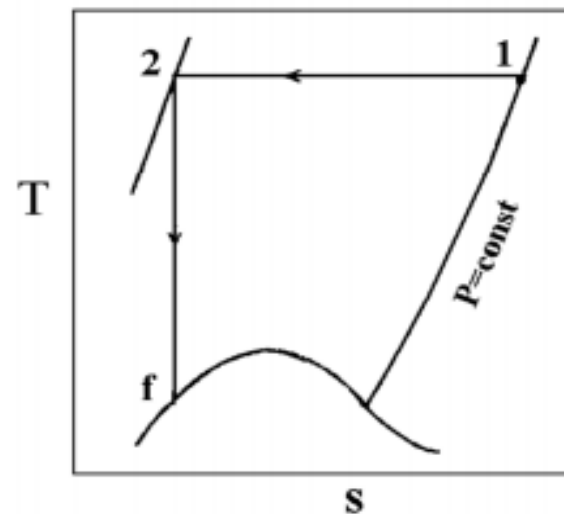
Assumptions

- ❑ Compressor and Expander Efficiency = 100%
- ❑ Heat Exchanger Effectiveness = 1
- ❑ Pressure drop is neglected
- ❑ Heat Transfer between the system and the Surrounding is neglected

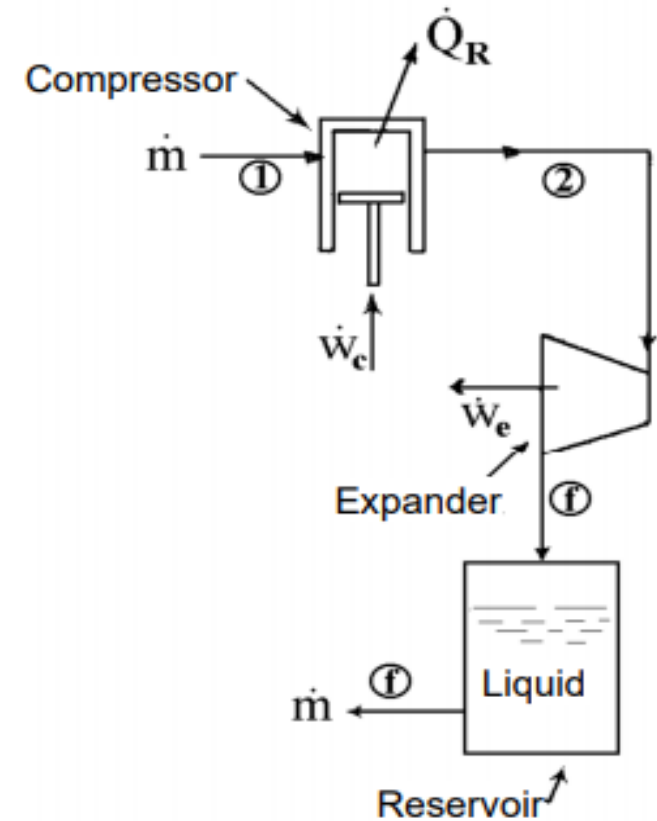
We will consider the effect of efficiency on cycle performance in later stage

5.3 Thermodynamic cycle

Ideal Thermodynamic Liquefaction Cycle



(a)



(b)

Ideal Thermodynamic Liquefaction System
(a) T-S diagram (b) System diagram

5.3 Thermodynamic cycle

Ideal Thermodynamic Liquefaction Cycle

Table 3.1. Ideal-work requirements for liquefaction of gases beginning at 300 K (80°F) and 101.3 kPa (14.7 psia)

Gas	Normal Boiling Point		Ideal Work of Liquefaction, $-\dot{W}_i/\dot{m}_f$	
	K	°R	kJ/kg	Btu/lb _m
Helium-3	3.19	5.74	8.178	3.516
Helium-4	4.21	7.58	6.819	2.931
Hydrogen, H ₂	20.27	36.5	12.019	5.167
Neon, Ne	27.09	48.8	1.335	0.574
Nitrogen, N ₂	77.36	139.2	768.1	330.2
Air	78.8	142	738.9	317.7

➤ Theoretical minimum work for liquefaction of gas

$$-\frac{\dot{W}_i}{\dot{m}} = T_1 \cdot (s_1 - s_f) - (h_1 - h_f) = -\frac{\dot{W}_i}{\dot{m}_f}$$

Ideal Thermodynamic Liquefaction Cycle



1----2 : Isothermal Compression

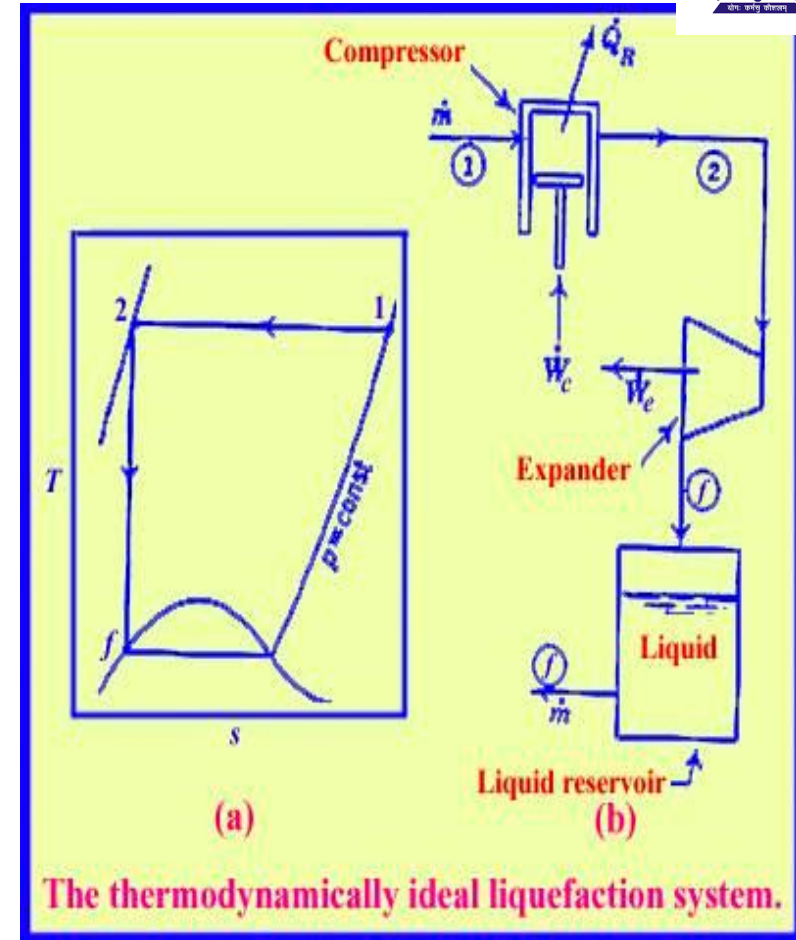
2----f : Isentropic Expansion

Not Possible , Why ???

For N_2 , if P_1 is 1 bar, then P_2 must be **700000 Bar** (very high pressure) to liquefy all the gases

Not Practical

Liquid Yield (Y) = $m_f/m = 1$



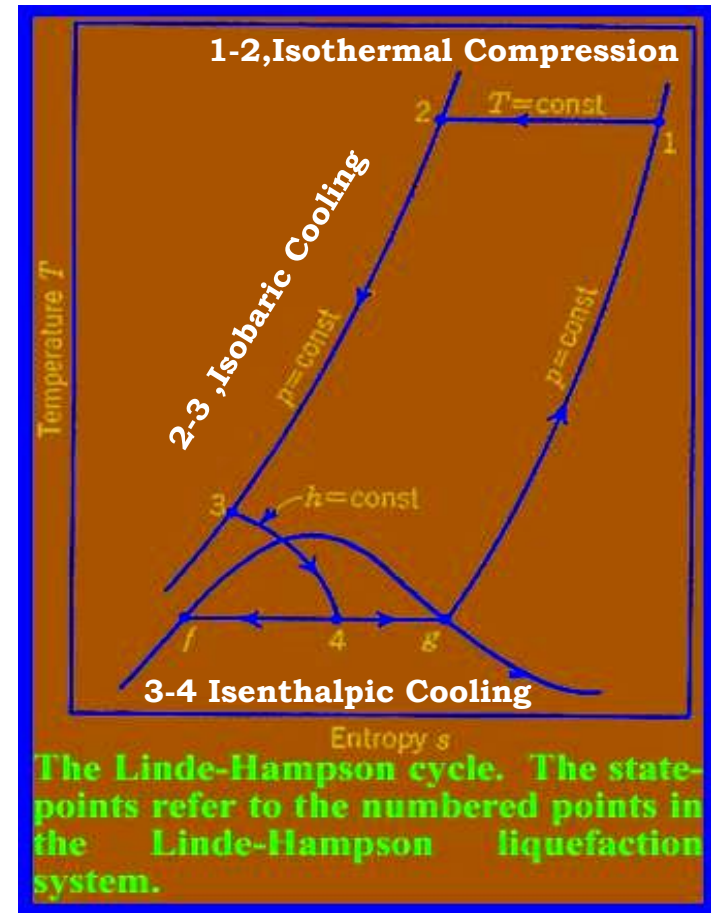
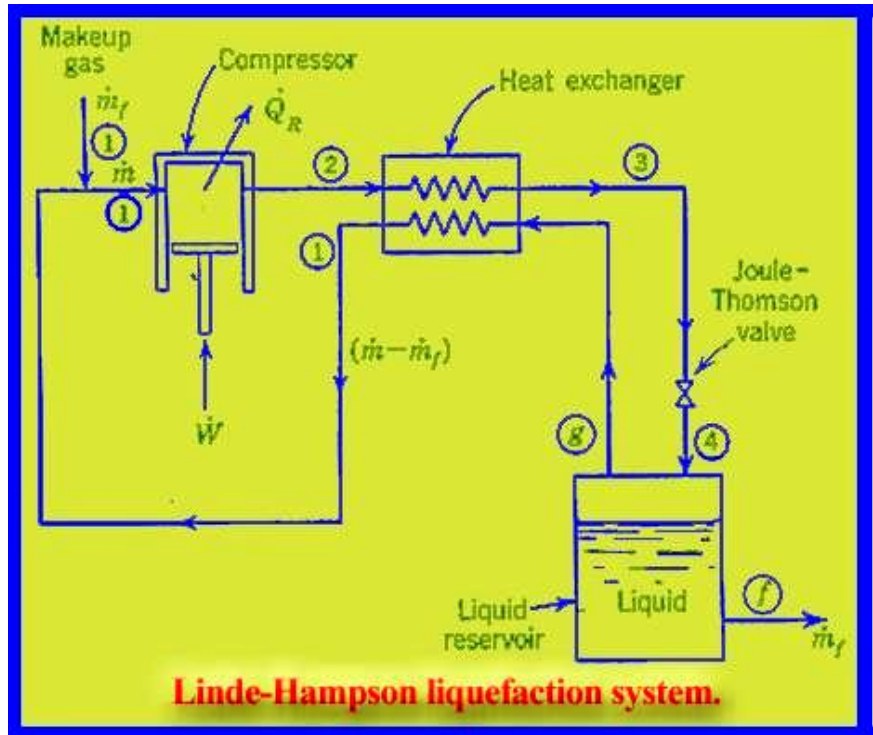
Ideal Work Requirement for 1 Kg Liquid Production

$N_2 = 768 \text{ KJ}$, $He = 6800 \text{ KJ}$,

Value will be used for Comparison

Primary Practical Nitrogen Liquefier (Linde- Hampson Cucle)

It is only for those gases whose T_{imax} is above RT



$$\frac{\dot{m}_f}{\dot{m}} = y = \frac{h_1 - h_2}{h_1 - h_f} \ll 1$$

Yield can be increased by Increasing Pressure (lower h_2), Other parameters are fixed **but at what cost ??**

A simple table on liquid yield and work required per unit production of liquid nitrogen for various compressor discharge pressure is presented here

($T_1 = 300$ K, $P_1 = 1$ bar absolute pressure, $h_f = 30$ J /gm. $h_1 = 462$ J/ gm)

$$\frac{\dot{m}_f}{\dot{m}} = y = \frac{h_1 - h_2}{h_1 - h_f}$$

$$\frac{W}{\dot{m}_f} = y \left[T_1 (S_1 - S_2) - (h_1 - h_2) \right]$$

Pressure (P_2)	h_2	$Y = \dot{m}_f / \dot{m}$	W / \dot{m}_f	FOM
20 bar	454	0.02	12888	0.06
50	448	0.03	9937	0.08
100	438	0.06	7200	0.11
200	425	0.09	5564	0.13

FOM = Ideal Work Required/ Actual Work

We Need Very High Pressure to have Significant Percentage of Liquid Production on this L- H Cycle

Linde-Hampson Performance



- Optimum theoretical performance realized by minimizing h_2 (P_2 such that h is on the inversion curve)
- P_2 is typically ~ 100 atm.
- Theoretical performance with $P_2 = 20$ atm.(from Barron):

Table 3.3. Performance of the Linde-Hampson system using different fluids. $p_1 = 101.3$ kPa (14.7 psia); $p_2 = 20.265$ MPa (200 atm); $T_1 = T_2 = 300$ K (80°F); heat-exchanger effectiveness = 100 percent; compressor overall efficiency = 100 percent

Fluid	Normal Boiling Point		Liquid Yield $y = \dot{m}_f/\dot{m}$	Work per Unit Mass Compressed		Work per Unit Mass Liquefied		Figure of Merit FOM = \dot{W}_f/\dot{W}
	K	°R		kJ/kg	Btu/lb _m	kJ/kg	Btu/lb _m	
N ₂	77.36	139.3	0.0708	472.5	203.2	6673	2869	0.1151
Air	78.8	142	0.0808	454.1	195.2	5621	2416	0.1313
CO	81.6	146.9	0.0871	468.9	201.6	5381	2313	0.1428
A	87.28	157.1	0.1183	325.3	139.8	2750	1182	0.1741
O ₂	90.18	162.3	0.1065	405.0	174.1	3804	1636	0.1671
CH ₄	111.7	201.1	0.1977	782.4	336.4	3957	1701	0.2758
C ₂ H ₆	184.5	332.1	0.5257	320.9	138.0	611	262	0.5882
C ₃ H ₈	231.1	416.0	0.6769	159.0	68.4	235.0	101.0	0.5976
NH ₃	239.8	431.6	0.8079	363.1	156.1	449.4	193.2	0.7991

No Helium??

Analysis on Linde- Hampson Cycle

Pressure	Yield mf/m	W/mf (J/ Kg)
100 Bar	0.06	7200

For 10 litre/ hr (8 kg/hr) liquid nitrogen production : Compressor capacity required at 100 bar discharge pressure

$$m = m_f \times y = 8 \text{ kg/hr} / 0.06 = 133 \text{ kg/hr} = 106 \text{ M}^3/\text{hr}$$

$$\text{Theoretical Power : } 7200 \text{ kJ/kg} \times 8 / 3600 = 16 \text{ kW} \quad ?$$

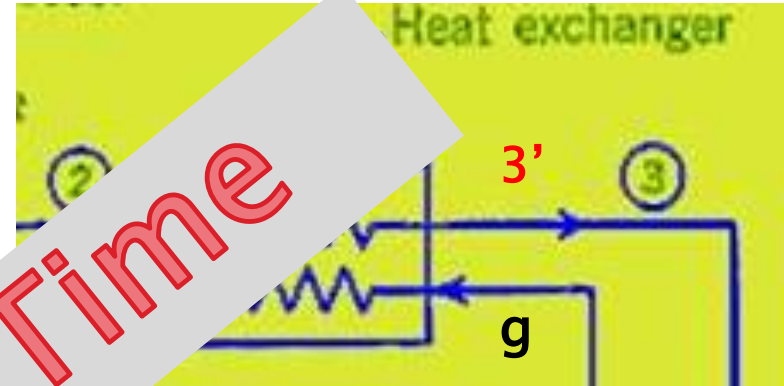
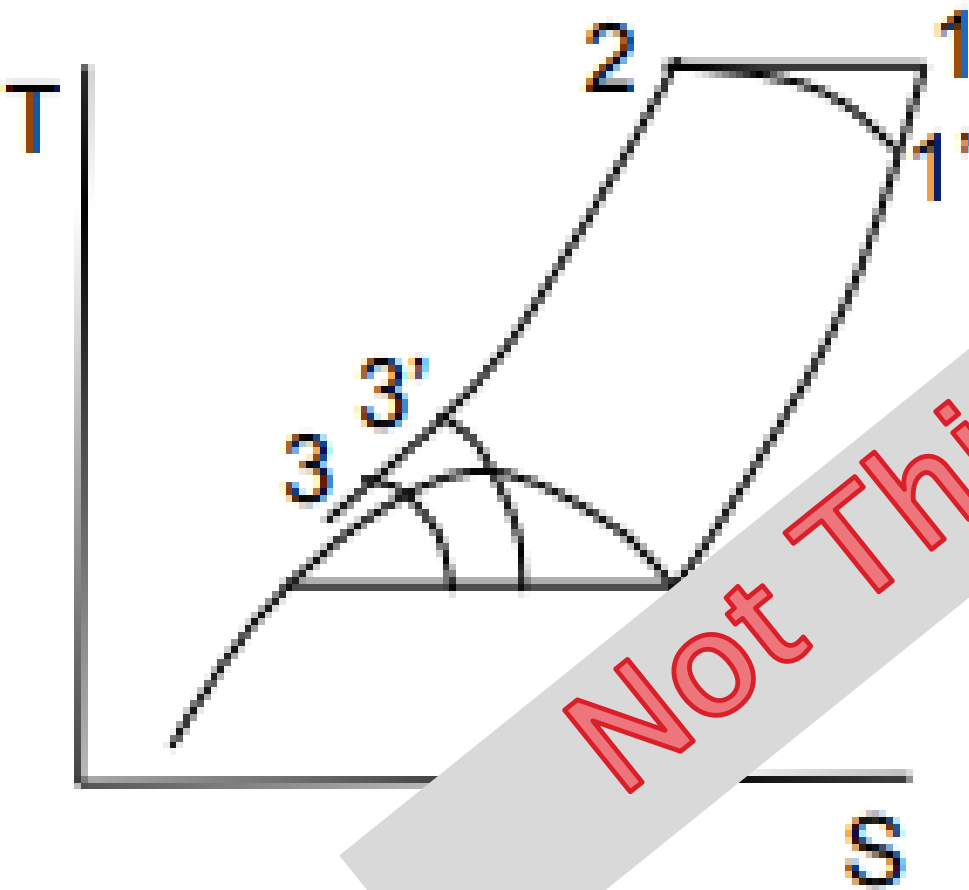
Considering the efficiency of Compressor and Heat Exchanger, the Actual power Requirement will be more than Double 40 kW
Power Cost of Liquid nitrogen will be Rs 20/ Litre

Actual Power cost is only Rs 5/ litre
This Cycle is Simple but not Cost Effective

Can we use the same Cycle for Helium Liquefaction ??



EFFECT : HEAT Exchanger EFFECTIVENESS



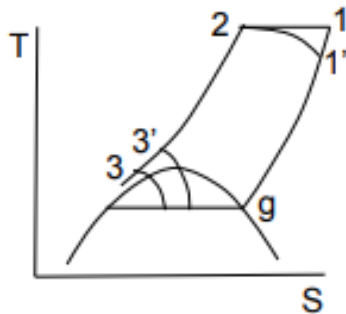
$$y = \frac{h'_1 - h_2}{h'_1 - h_f}$$

$$\varepsilon = \frac{h_1 - h_g}{h_1 - h_g}$$

We will replace h_1' by h_1 with effectiveness

Influence of Non-Ideal Components

- A non-ideal heat exchanger will have an effectiveness less than 1.
- A non-isothermal compressor will require more work than an isothermal compressor.



$$\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$$

$$\eta_c = \frac{(\dot{W}/\dot{m})_{\text{isothermal}}}{(\dot{W}/\dot{m})_{\text{actual}}}$$

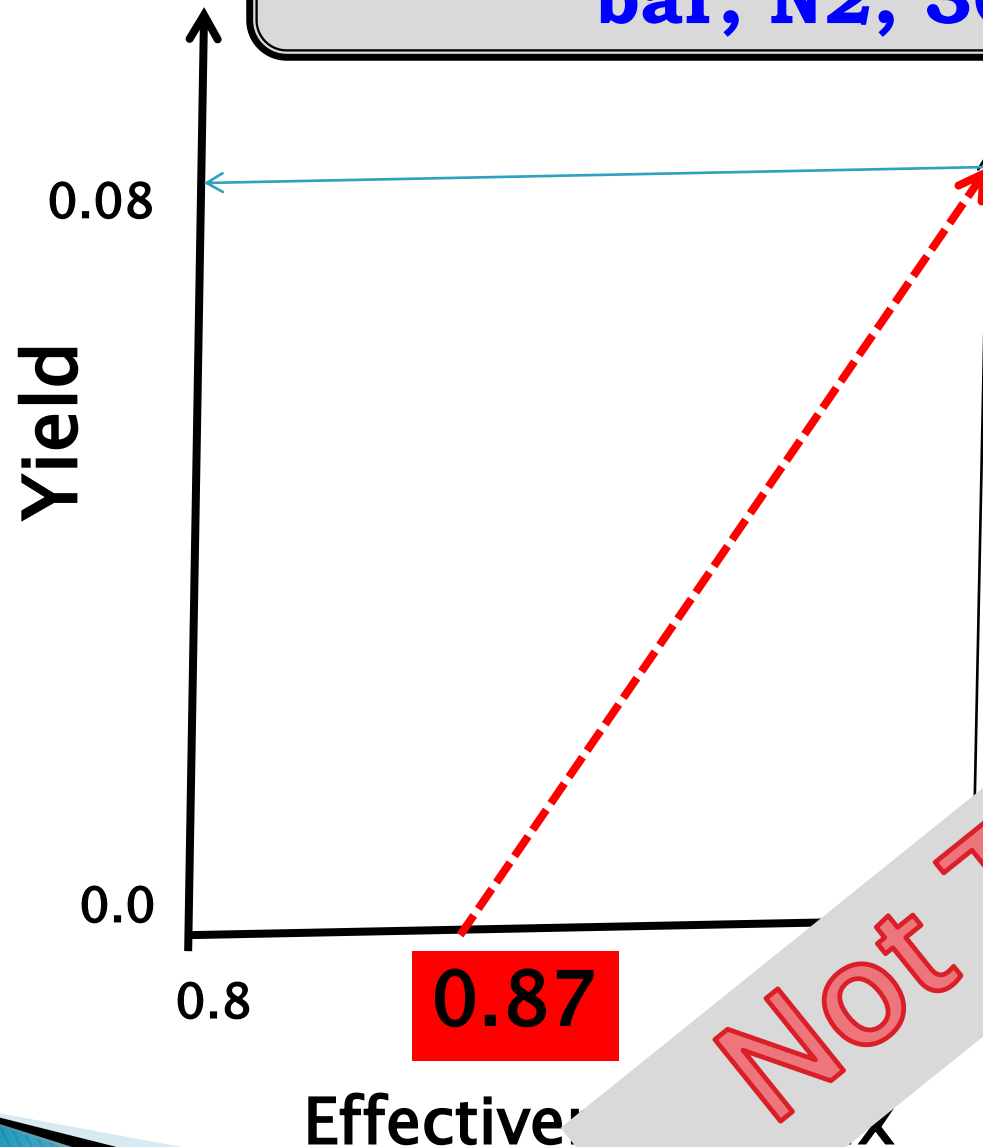
- The influence of these parameters on the cooling capacity (refrigerator), liquid refrigerant, and compression work for a simple Linde-Hampson system.

$$(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)$$

$$y = \frac{h_{1'} - h_2}{h_{1'} - h_f} = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

$$\frac{\dot{W}}{\dot{m}} = \frac{1}{\eta_c} \left[T_1 (s_{1'} - s_2) - (h_1 - h_2) + (1 - \varepsilon)(h_1 - h_g) \right]$$

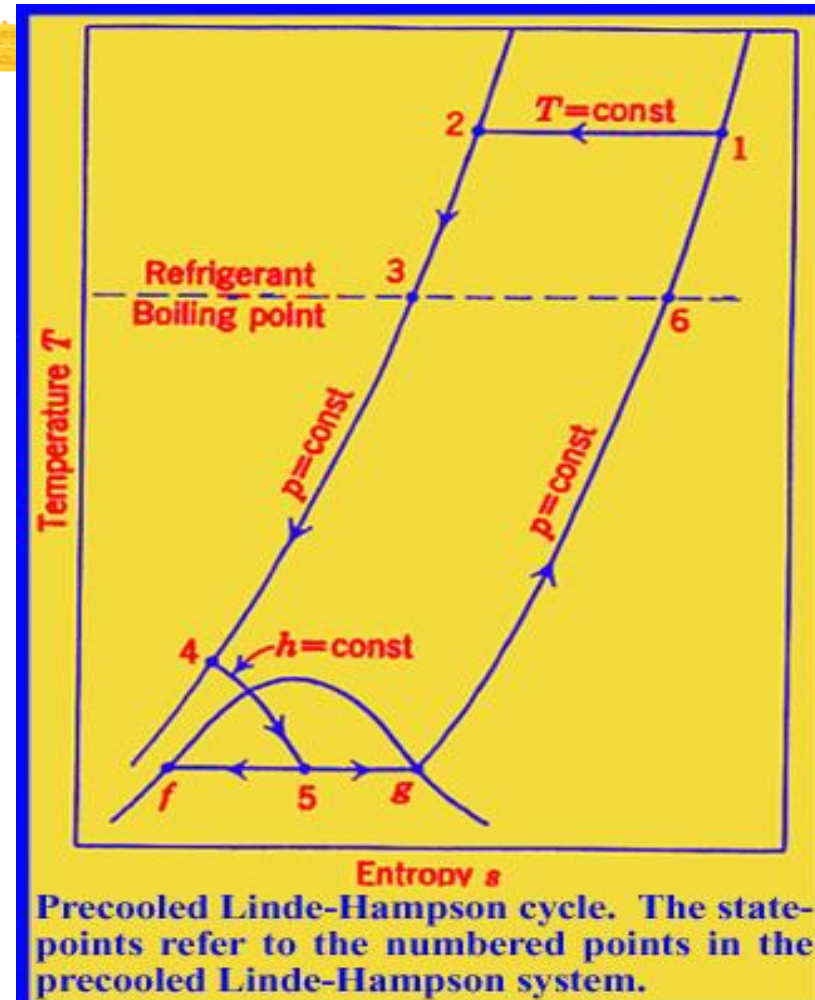
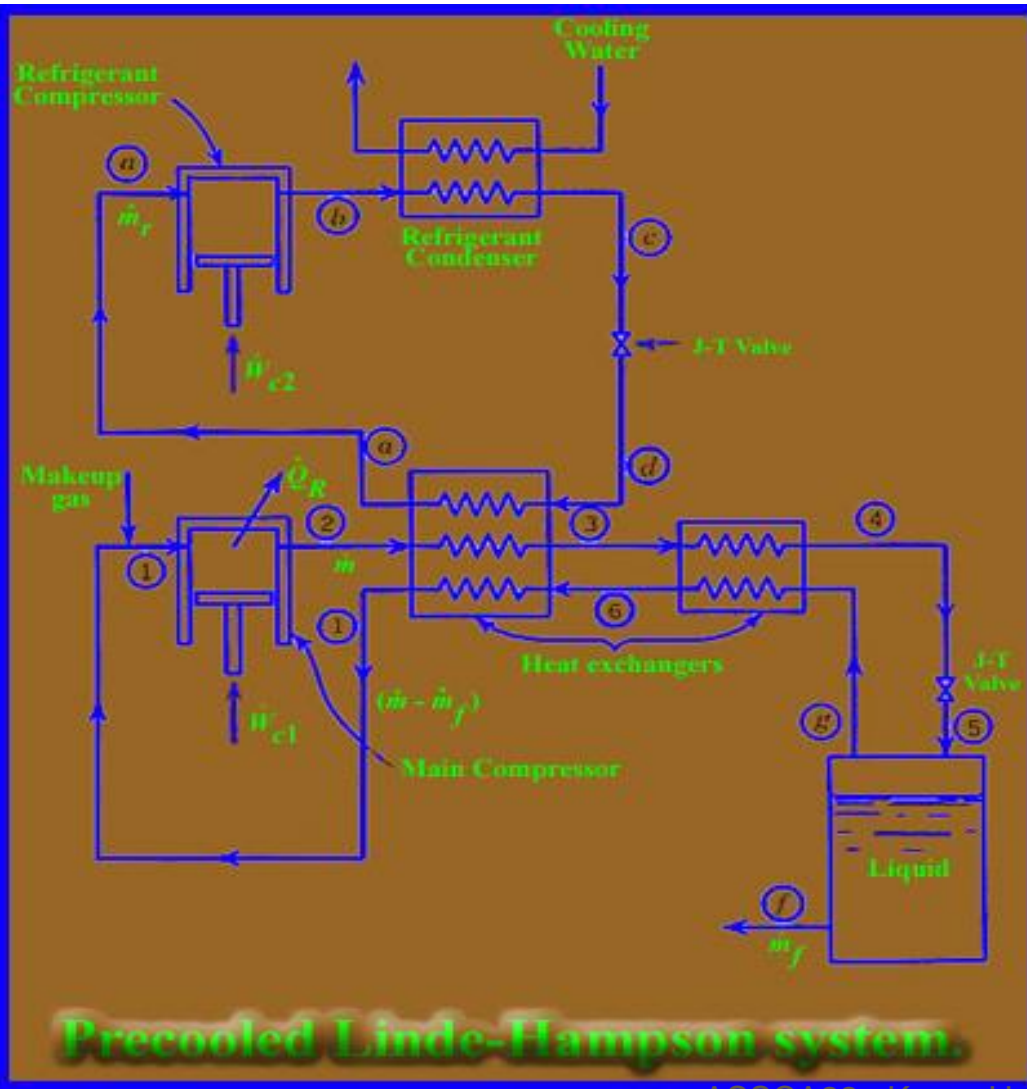
Example : $p_1 = 1$ bar , $p_2 = 200$ bar, N_2 , 300 K

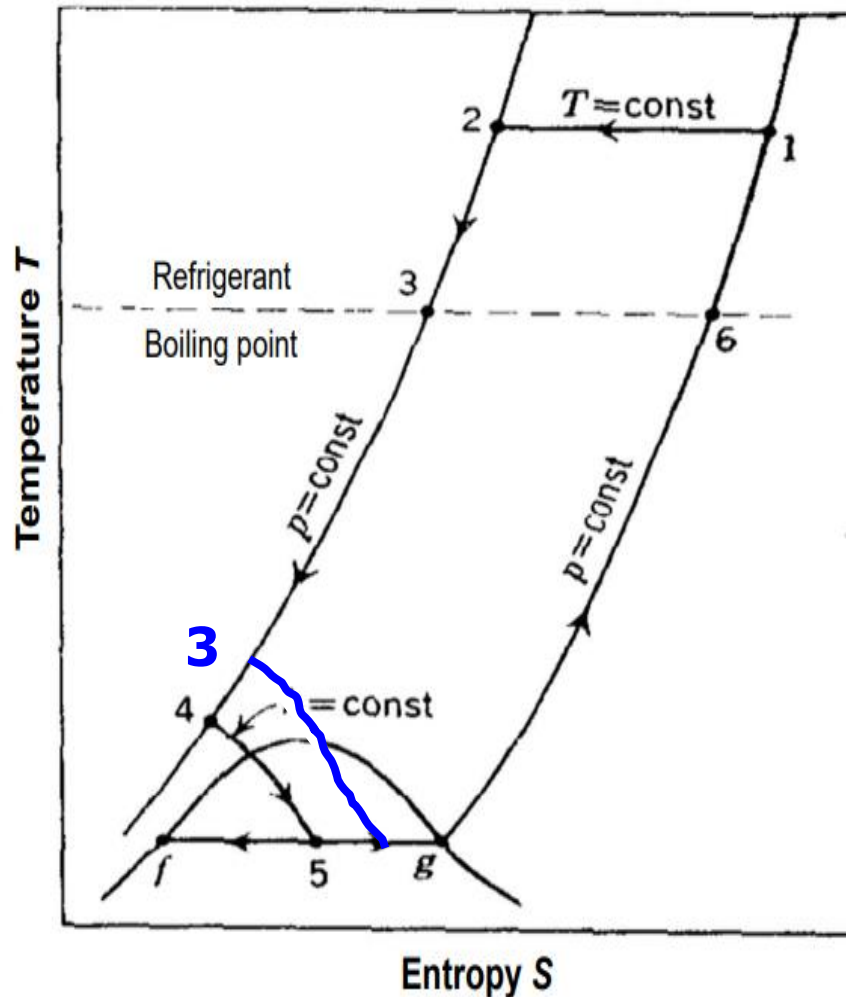


Next Class
talking
Helmholtz free energy
reaction
equilibrium

Not This Time

Precooled Linde-Hampson Cycle





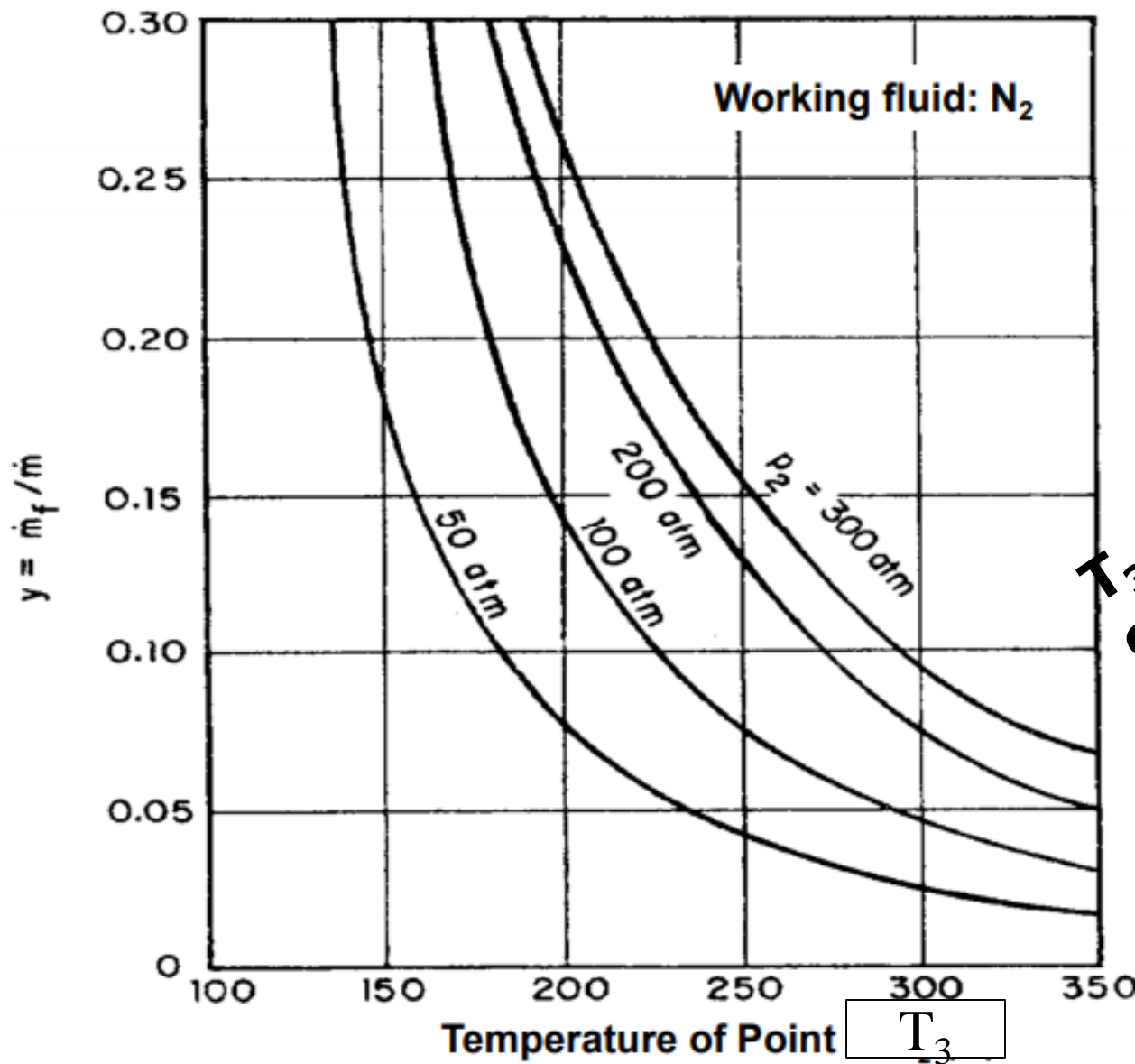
T-s diagram of Single throttling Linde cycle with precooling

$$\frac{m_f}{m} = y = \frac{h_1 - h_2}{h_1 - h_f}$$

$$y = (h_6 - h_3) / (h_6 - h_f)$$

$$\frac{W}{m_f} = [T(S_1 - S_2) - (h_1 - h_2)] / y$$

Yield is higher, Hence W / m_f will be less



T_3 depends on cooling medium like Ammonia / LNG

Relationship between liquefaction rate and inlet T_3

LIQUID YIELD CAN BE INCREASED BY PRECOOLING

(by lowering the value h_1)

Liquid yield for Linde precooled system with $P_2 = 100$ bar

Precool temp.	300	250	200	150
$Y = m_f/m$	0.06	0.08	0.14	0.57

Power remains same , Add Refrigeration power for precoolant



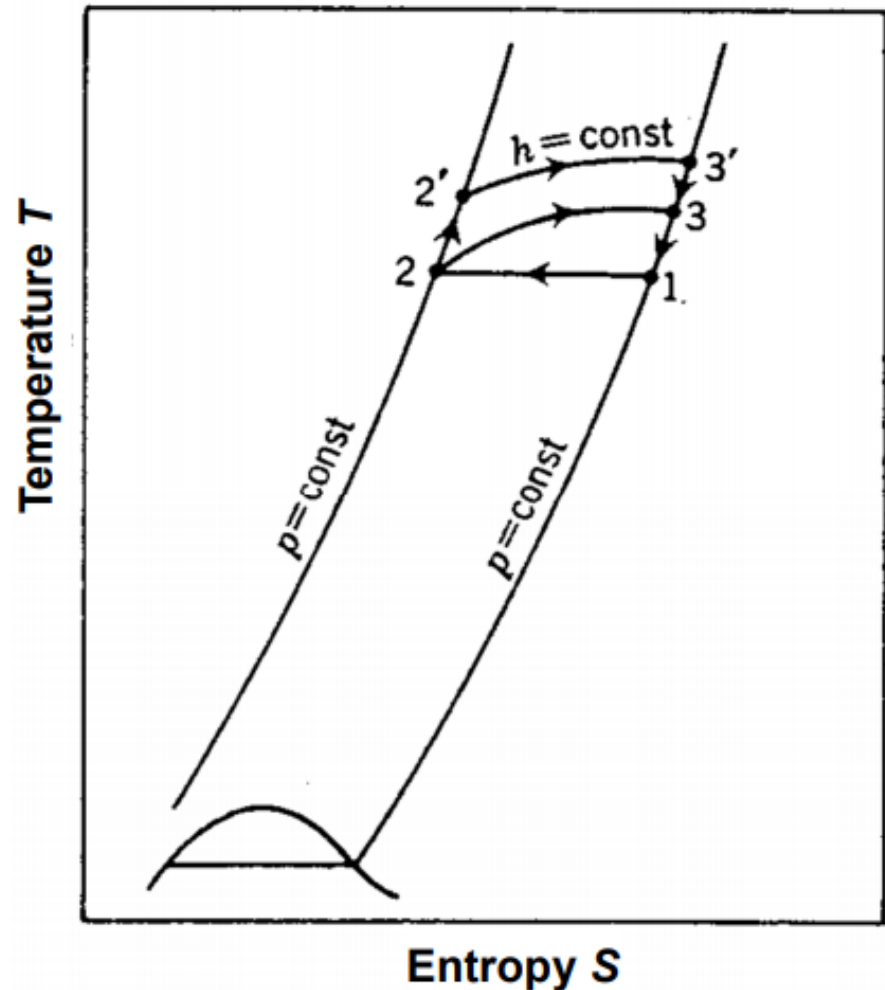
Simple L-H System for H₂ and He ???



Do you Remember Max Inversion Temperature For He/ H₂

A Linde cycle without pre-cooling can not be used for the liquefaction of neon, hydrogen and helium.

1. The inversion temperature of these gases is lower than the ambient temperature and can not be cooled down to start the process
2. The inversion temperature of helium is 46 K, so if the Linde cycle with single throttling is used to realize the liquefaction of helium, it is necessary to use LH₂ to precool the temperature of the helium to below 46 K.



T-s diagram of Linde cycle with single throttling for helium or hydrogen

LIQUEFACTION OF HELIUM

**Max Inversion Temperature
for Helium : 45 K (Below RT)**

**BASIC LINDE- HAMPSON SYSTEM
WILL HAVE HEATING EFFECT.**

**We have to Precool with
Liquid Nitrogen (78 k) and
Liquid Hydrogen (20 K) prior
to JT .**

**(First Liquefaction of Helium
in 1908 : By Precooling L- H)**

**ALTERNATIVELY : ADDING
ONE ADIABATIC EXPANSION
PROCESS BY USING A TURBINE :**

**First Helium Liquefier
By Kamerlingh Onnes**

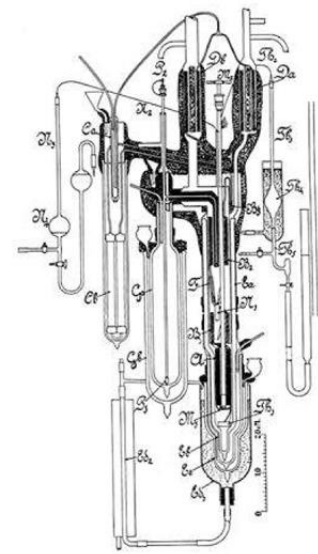
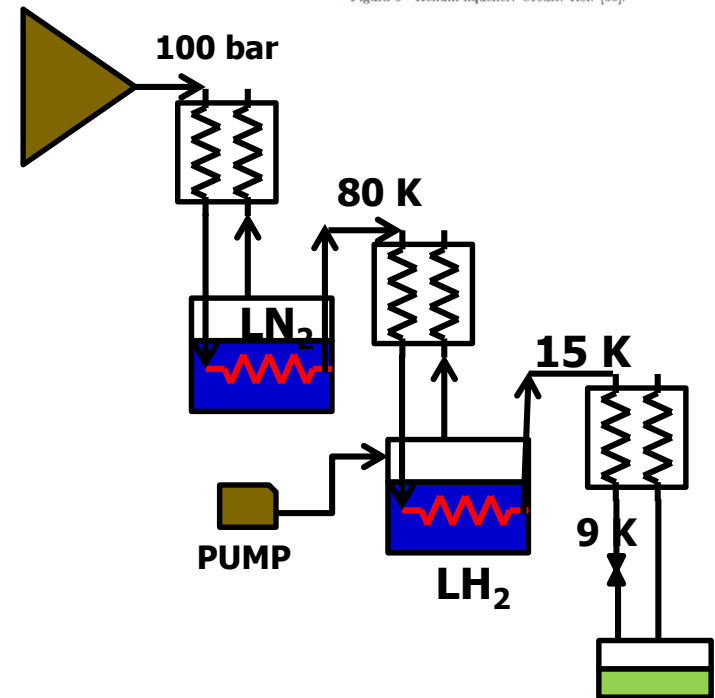


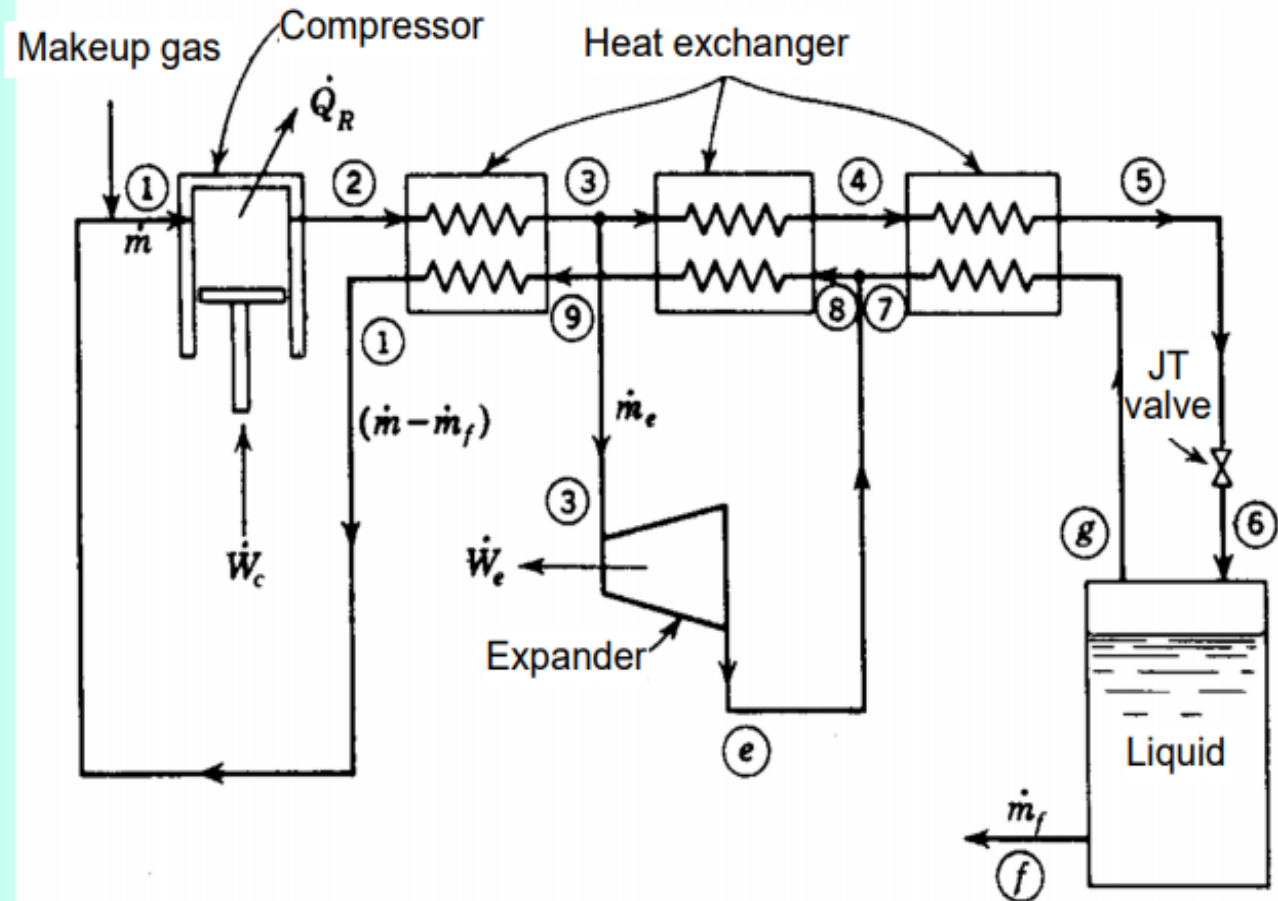
Figura 6 - Helium liquefier. Credit: Ref. [35].



CLAUDE CYCLE



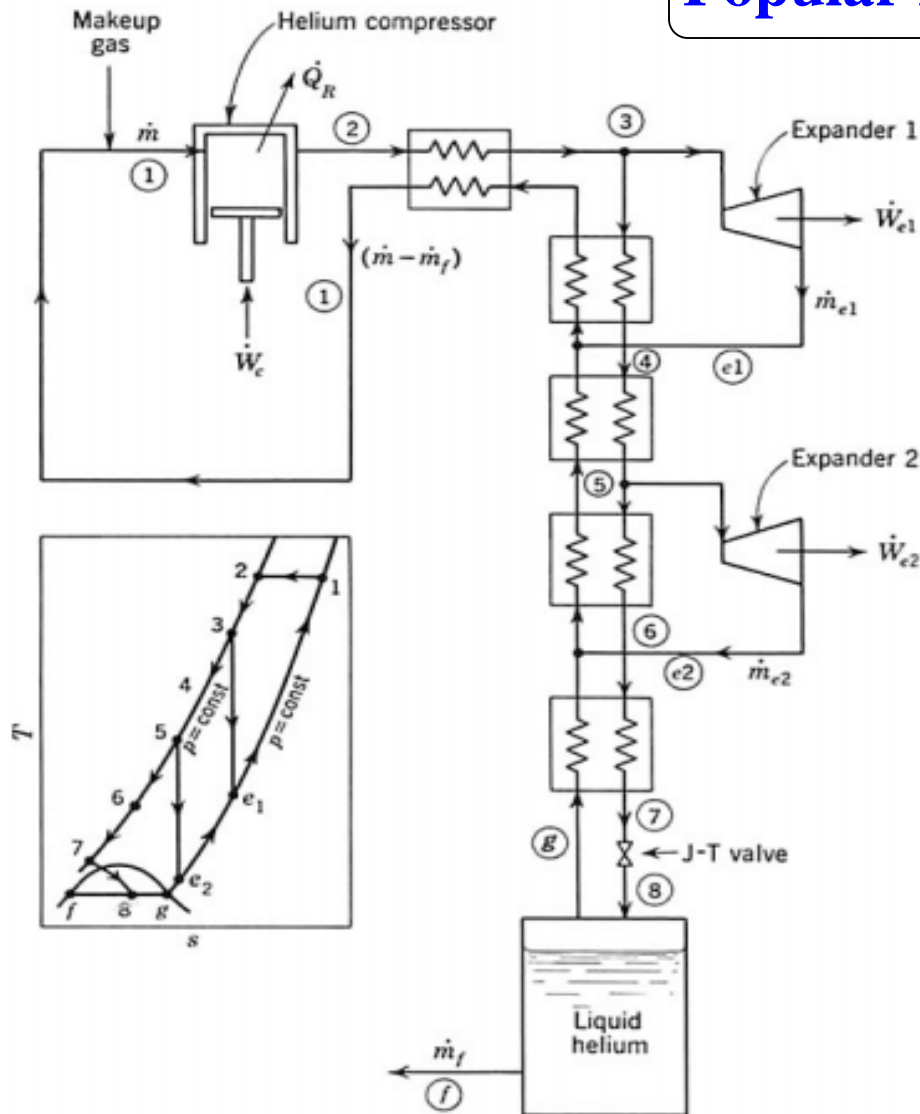
1. In 1902, Claude, Frenchmen, first proposed the liquefaction cycle with expander, so the liquefaction cycle with expander was called the Claude cycle
2. The Claude cycle is generally used in small and medium sized air separation units. Claude cycle with precooling has sufficient cooling capacity and is used in large and medium scale LO_2 and LN_2 equipment.



Claude cycle

Collins Liquefier

Popular During 1950-80



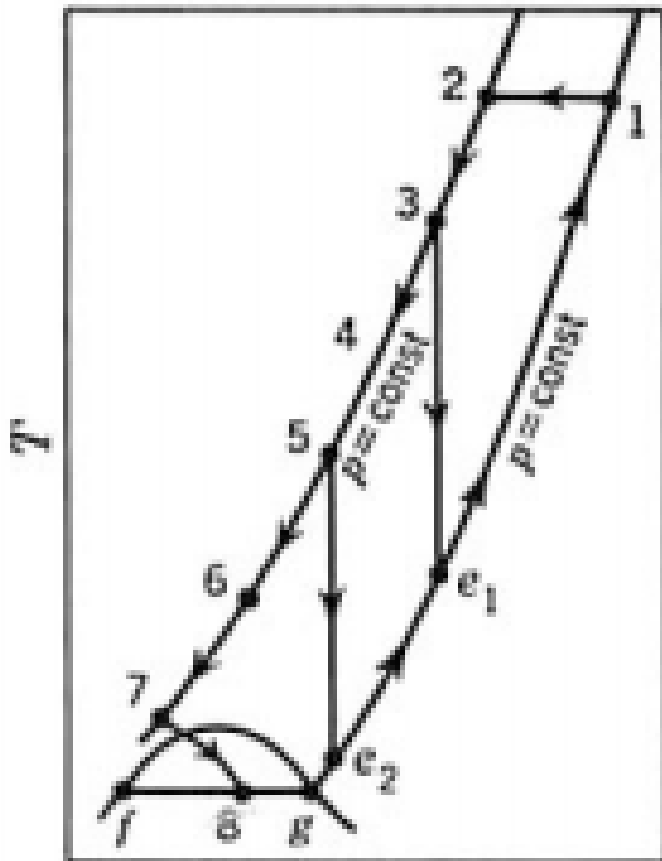
- Introduced by Sam Collins (MIT) in 1952
- Optimized performance via expander flow rates and temperatures
- LN_2 pre-cooling increases yield by factor of 3.

The yield and work requirement are given by



$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) + x_1 \left(\frac{h_3 - h_{e1}}{h_1 - h_f} \right) + x_2 \left(\frac{h_5 - h_{e2}}{h_1 - h_f} \right)$$

$$\frac{-W_{net}}{\dot{m}} = \left(T_1 (s_1 - s_2) - (h_1 - h_2) \right) - x_1 (h_3 - h_{e1}) - x_2 (h_5 - h_{e2})$$



Here Yield can be optimized with T_3 , T_5 , x_1 and x_2 at different pressure (p_2)

For standard Helium Liquefier
 $T_3 = 60 \text{ K}$, $T_5 = 20 \text{ K}$
 $x_1 = 15 \%$, $x_2 = 40 \%$



HELIUM LIQUEFIER WITH TURBINE

Capacity can be further enhanced by using two Expander and precooled with liquid Nitrogen. This is called Modified Claude Cycle.

Initial year, isentropic expansion was achieved by Reciprocating Piston type and later Turbine

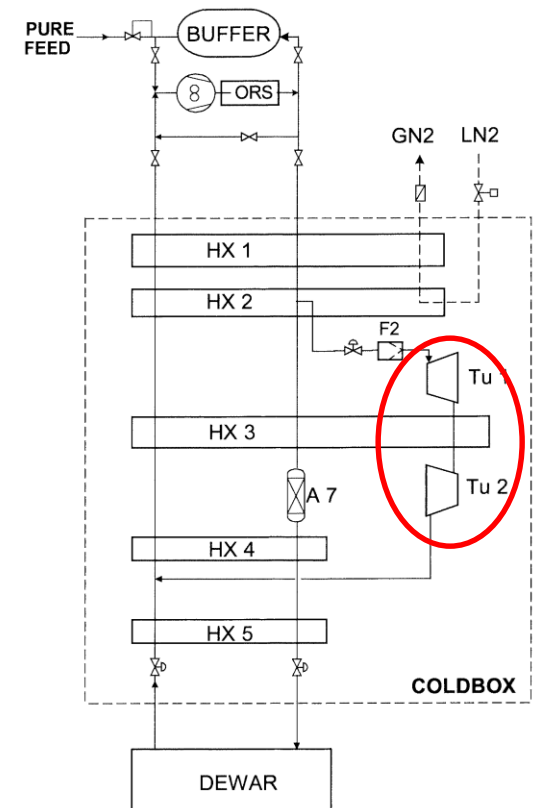
Advantage : Turbines are more reliable than the expansion engines because the latter are susceptible to performance deterioration due to contamination.

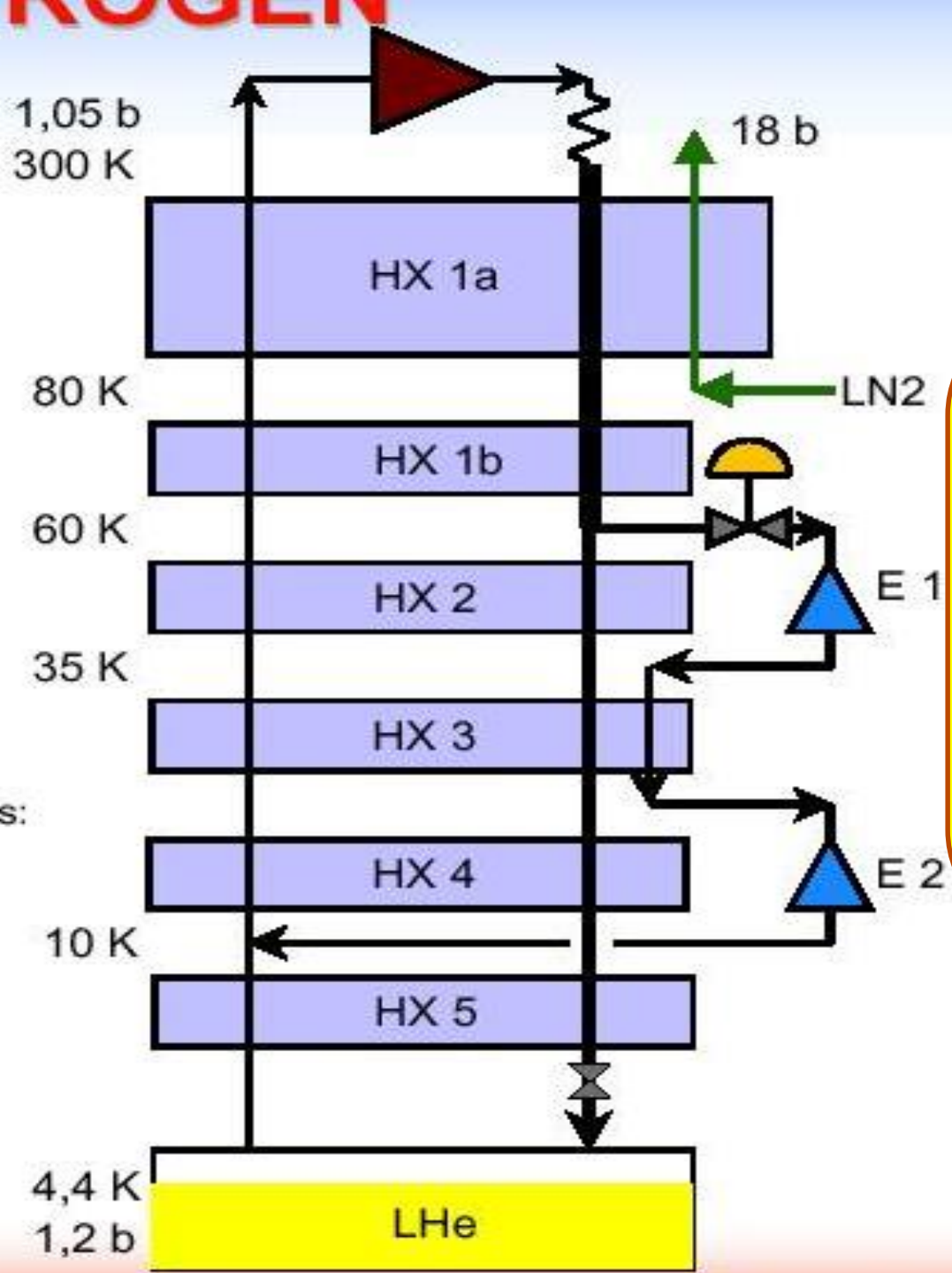
Disadvantage : Turbines do not have a limited expansion ratio, that is a **limited ratio of inlet to outlet pressures**.

That was the reason Turbine are used in series rather than parallel for Reciprocating Expander

Standard 1 kW class Refrigerator

300 litres/hr





Major Components :

1. Compressor
2. Heat Exchanger
3. Expander
4. JT Valve

Most Popular Helium Liquefier in Low Temperature Laboratories (1965 – 1980)

ADL to CTI 1400



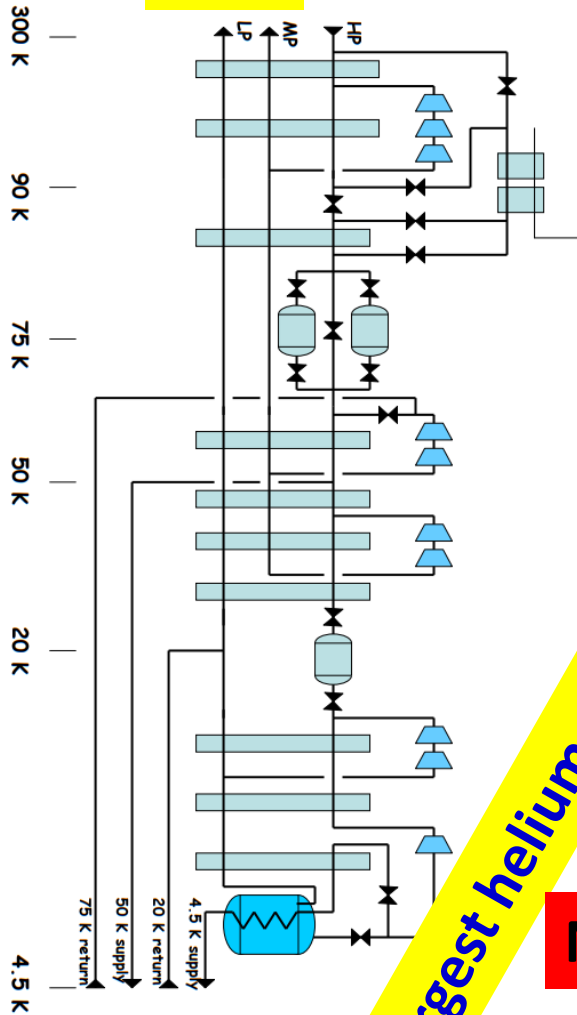
Capacity : 10 litre/hr (single Compressor) : Reciprocating 2 stage; 100 Nm³/hr , Reciprocating Expansion Yield = .08

CTI 1410 → KOCH → PSI → LINDE (L 1410)

**Automatic Engine speed, Screw Compressor, Modular
Production rate : 10 litres/ hr to 50 litres/ hr**

18 kW at 4.5 K Helium Refrigerator for CERN LHC Project

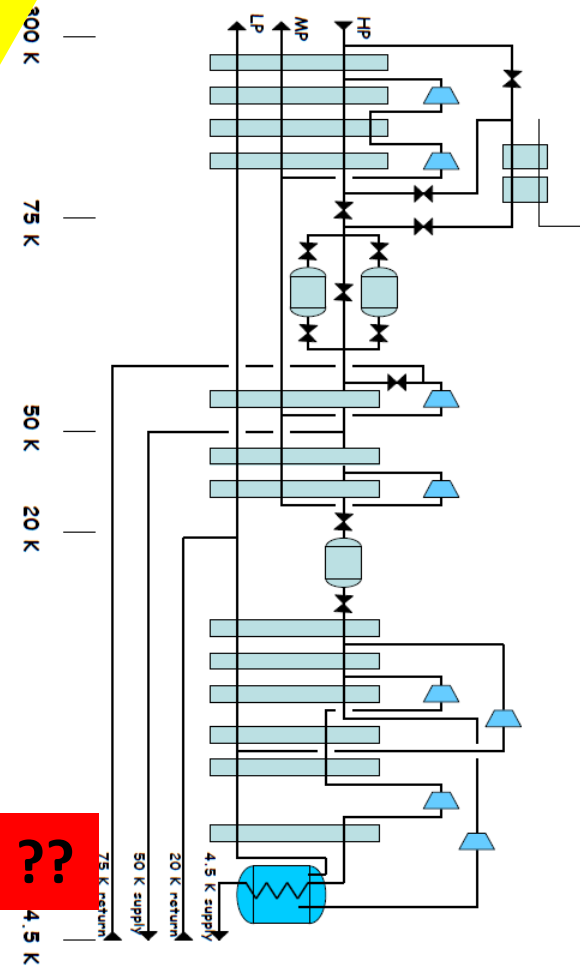
LINDE



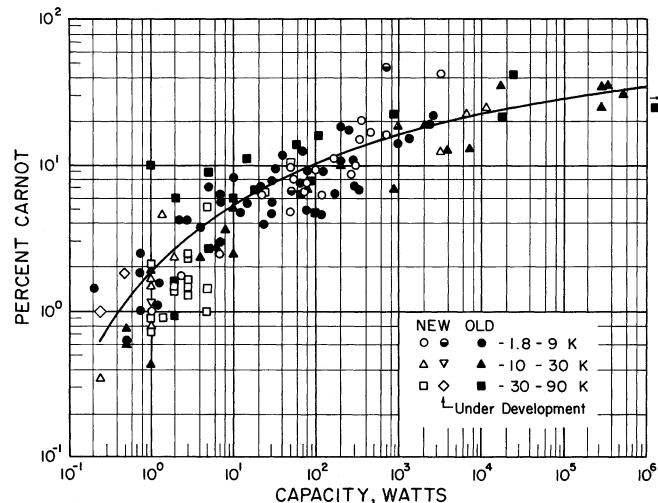
Largest helium refrigerator with Single Cold Box

Next 20 / 25kW ??

AIR Liquide



Strobridge Survey (Efficiency) 1974



GM Cryocooler ; 4 kW for 1W refrigeration at 4.2 K

**Laboratory Scale Helium Refrigerator (~ 100 W) :
700 W for 1 W**

Medium Range 1kW Class (300 W for 1W Range)

**Large Helium Refrigerator (18 kW for CERN :
225 W for 1 W**

Whether we have reached peak value of 225 W ??

Efforts are on to replace the screw compressor by dry centrifugal compressors: deletion of the Oil Removal System and reduction of the electrical consumption by 12% to 20% depending on the number of compressor stages.

Isothermal efficiency of centrifugal compressors is higher than screw compressors, and this efficiency increases with the number of compressor stages.



Standard 1 kW at 4.3 K Helium Refrigerator needs a Compressor with capacity 100g/s and discharge pressure at 13 bar (g)

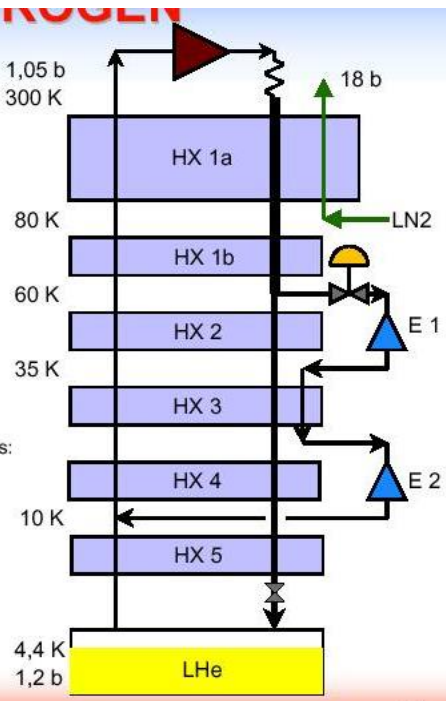
Isothermal Operation : Work required

$$W = \frac{m}{M} RT \ln(p_2/p_1) = 150kW$$

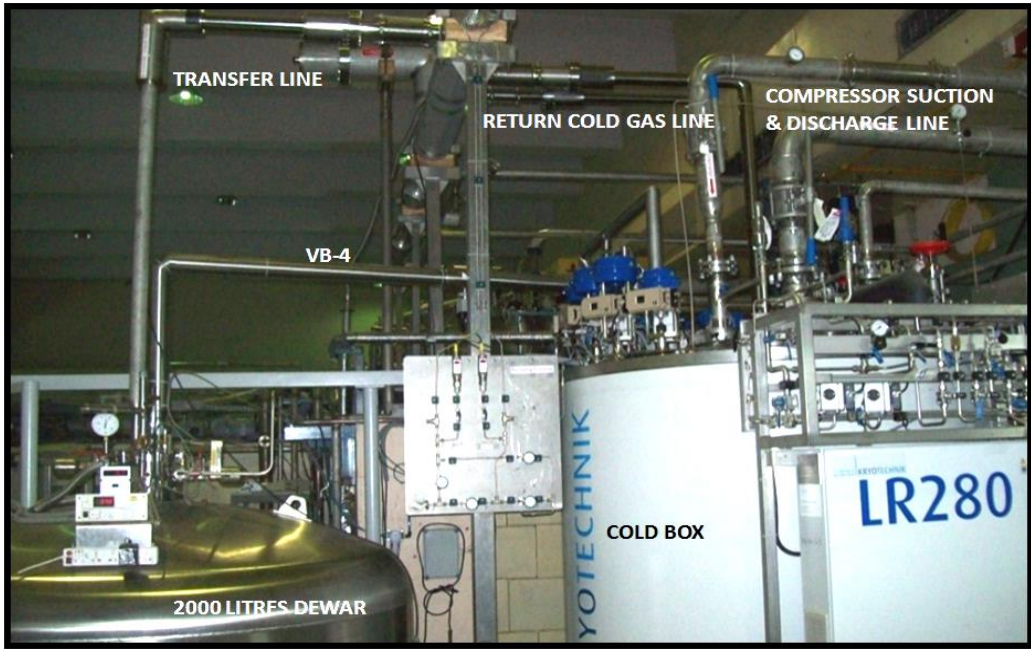
Actual Plug Power ; 305 kW

Compressor Efficiency : 49%

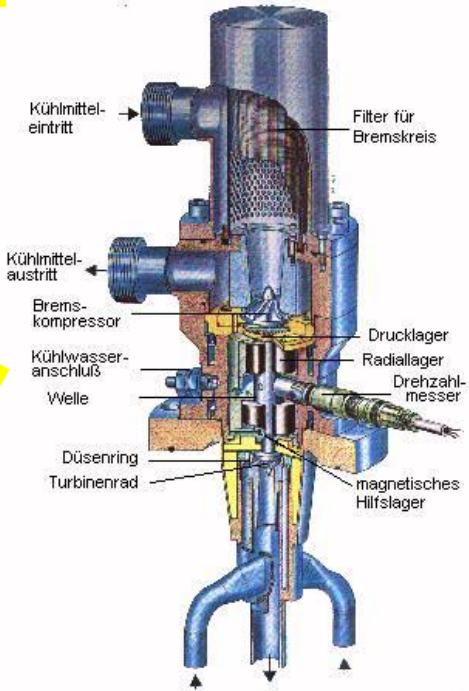
Inverse COP = 300 W/ W



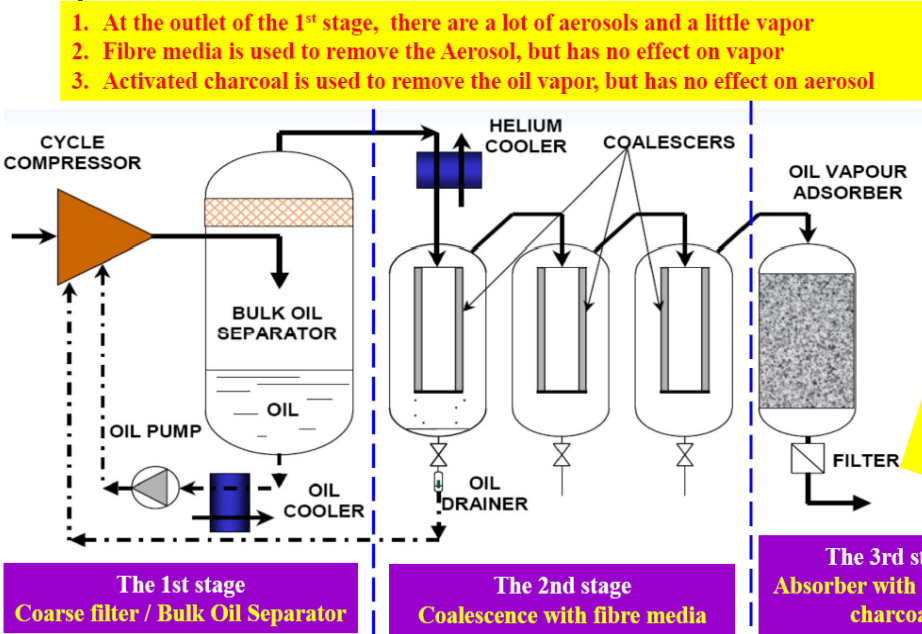
Major Components of Helium Refrigerator



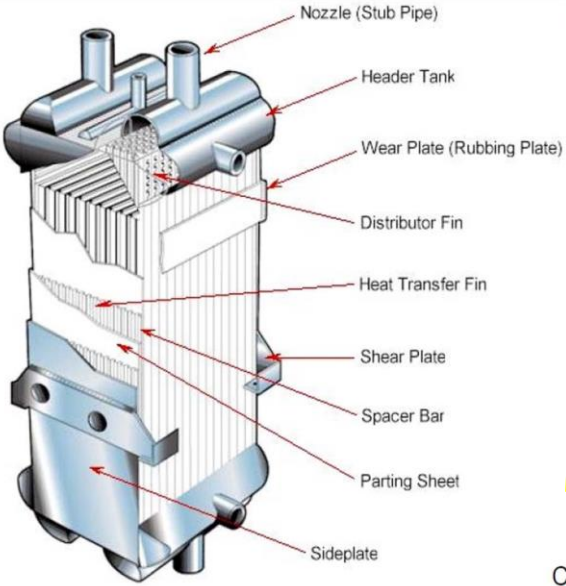
Turbine

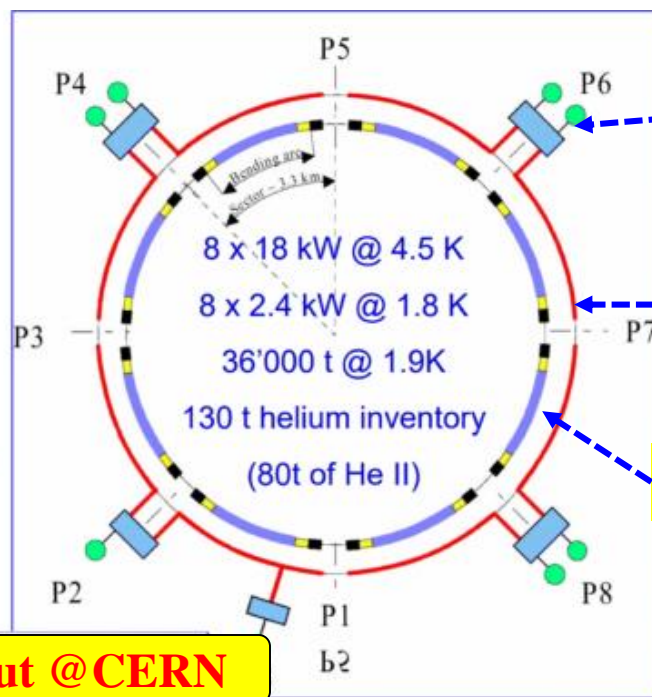
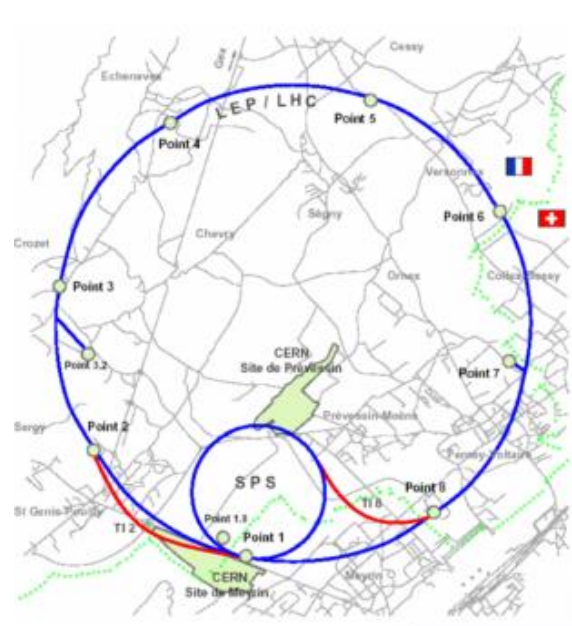


Oil Removal System



Heat Exchanger



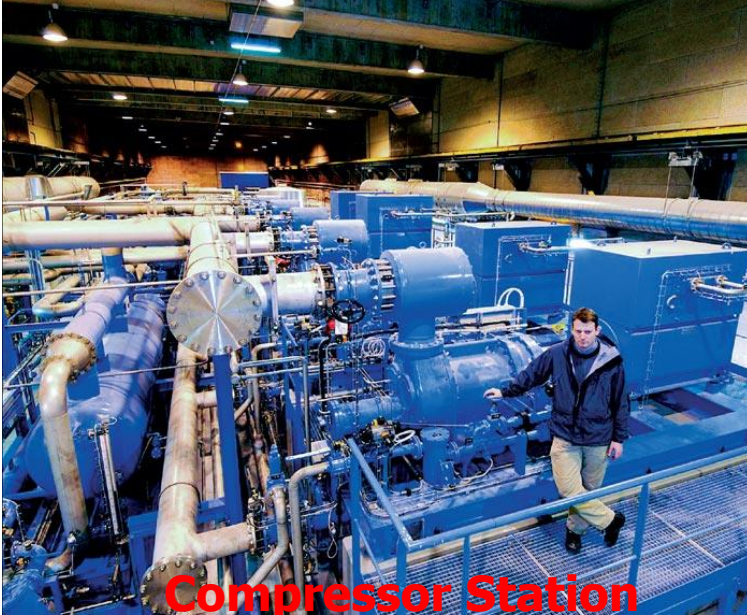


18 kW Refrigerator

LHe Distribution Line

Beam Line

Helium Refrigerator Layout @CERN



Compressor Station



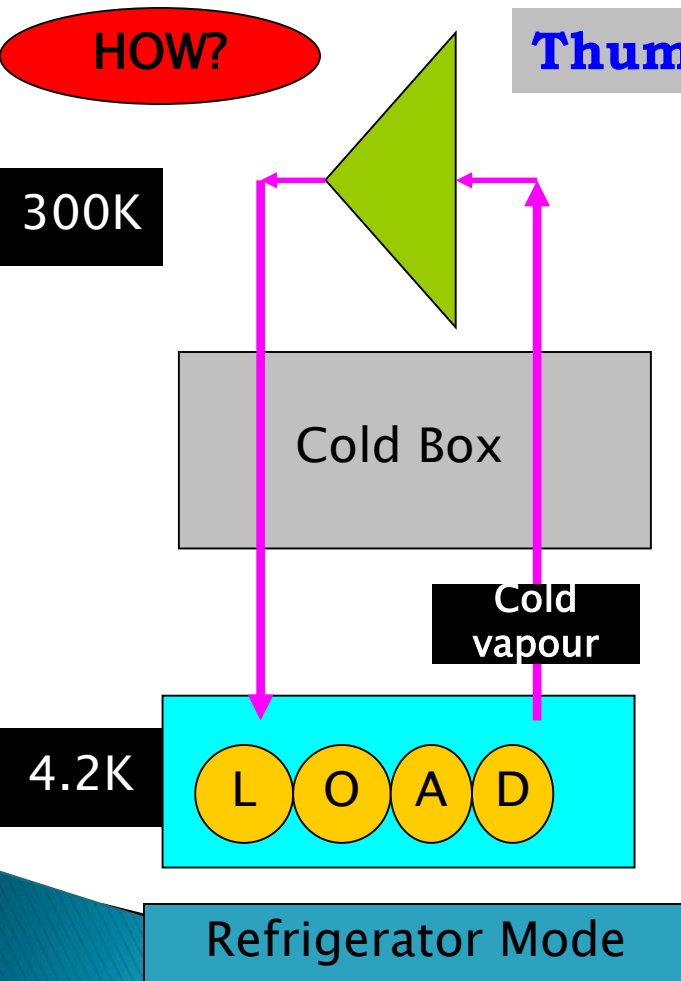
Cold Box

Liquefaction Vs. Refrigeration

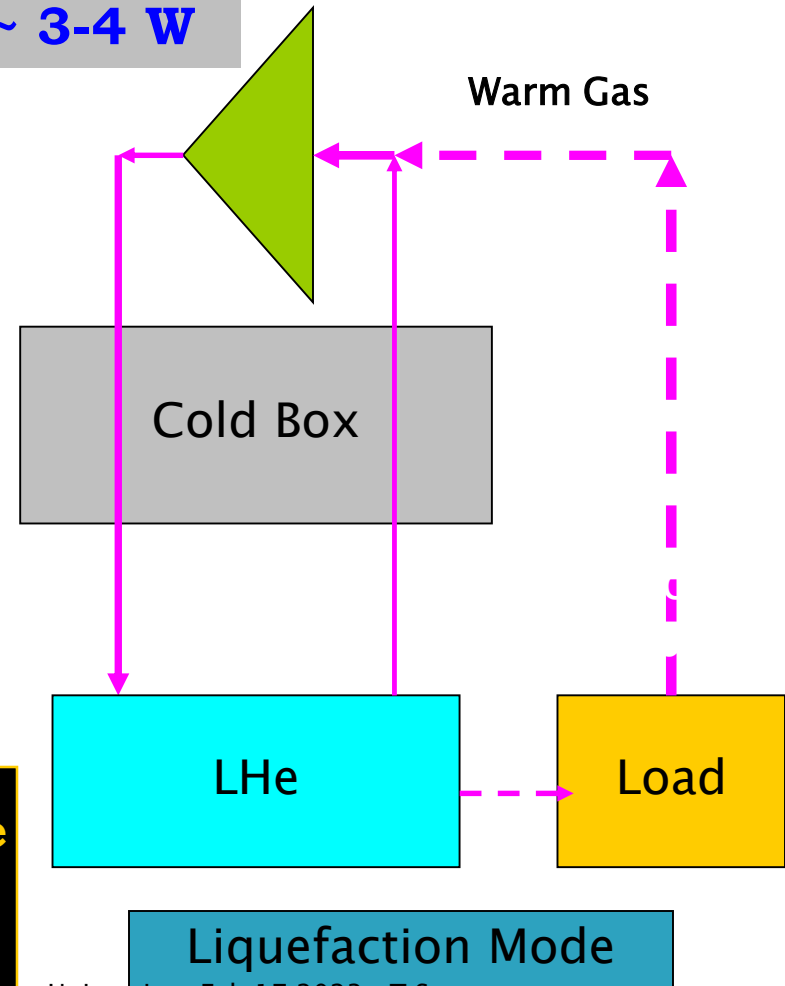
Same Thermo dynamical Cycle as Liquefaction Cycle. Difference in way of operation

HOW?

Thumb Rule: 1L/hr ~ 3-4 W



**Load may be the
S.C. magnets
& Cavities**



Liquefaction Vs. Refrigeration



$$100 \text{ L/hr} = 3.33 \text{ g/sec} = 3.33 \times 20 \text{ j/g} = 66.6 \text{ W} \ll 300 \text{ W}$$

Thumb Rule: 1L/hr ~ 3-4 W

Why ???

1. Cold Enthalpy 4.2- 300 K is used

2. JT Temp will be lower and hence yield

Transition from He I to He II (Super fluid)

78 K

4.2 K



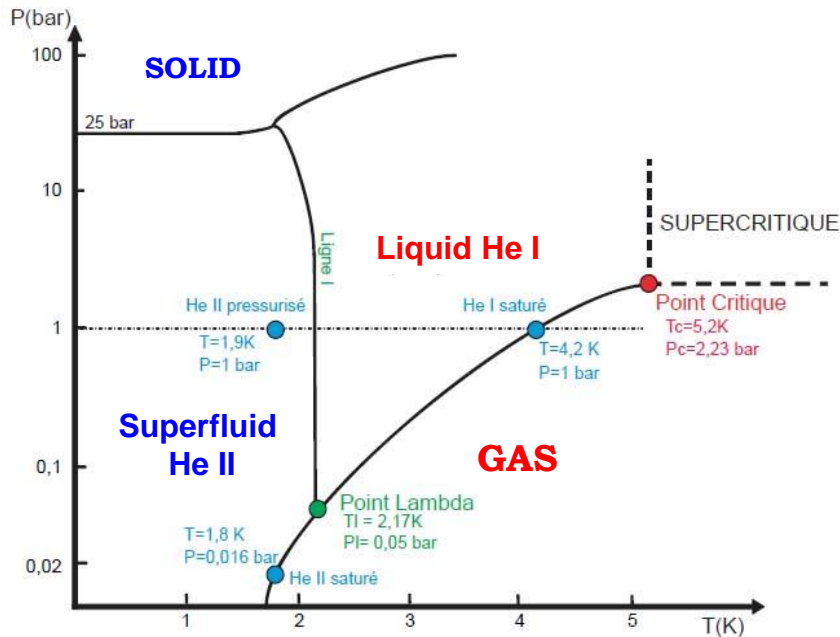
2 K



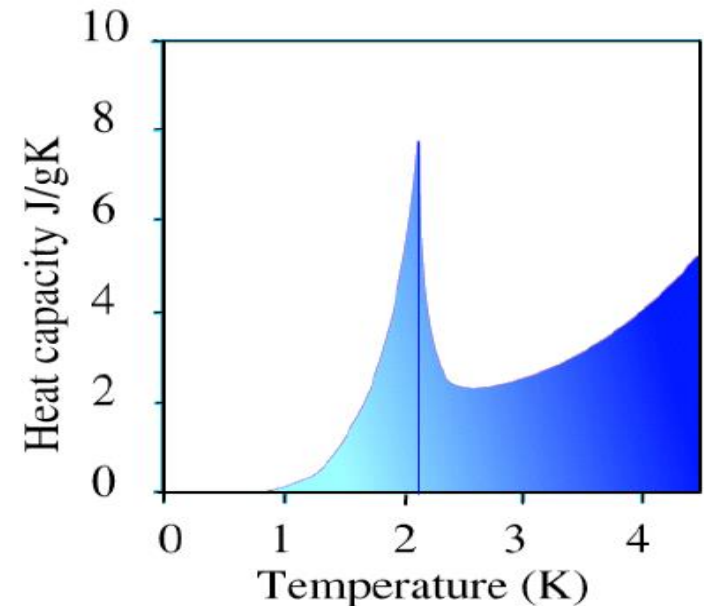
**Must for Present
& Future High
Power
Accelerator**

Super-fluidity is the characteristic property of a fluid with zero viscosity which therefore flows without loss of kinetic energy (no Pressure drop)

TRANSITION TO A SUPER-FLUID PHASE BELOW THE λ -point (2.17K)



1. Low Viscosity
2. High Conductivity
3. High Specific Heat



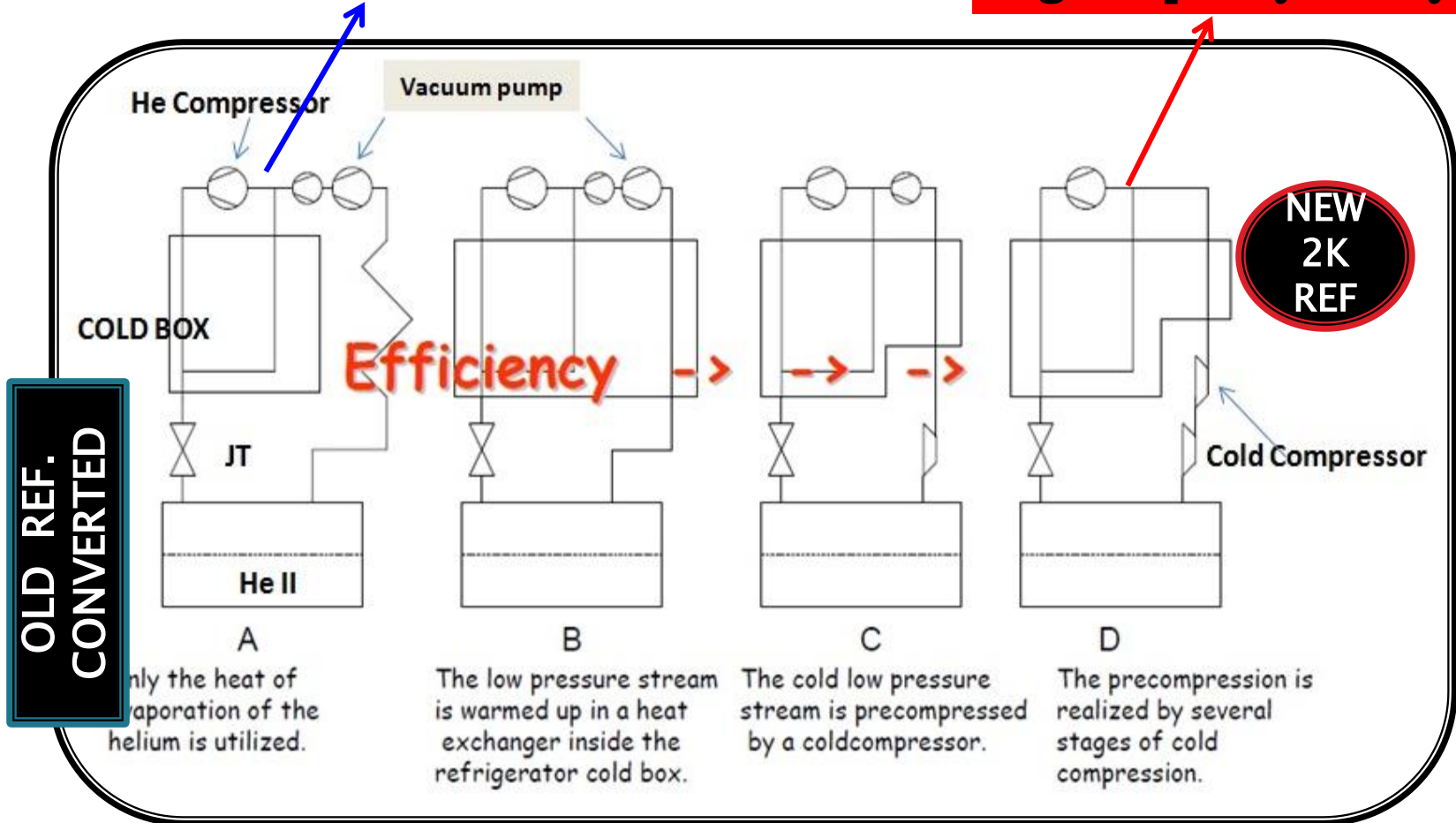
Advantages

1. Super-fluid Helium can easily flow through SC strand /Cable
2. Small temperature rise with a heat input (specific heat)
3. Large Conductivity maintain equal temperature. SC Magnet is stable

2 K Helium Refrigerator

Small capacity 2 K system

High Capacity 2 K system

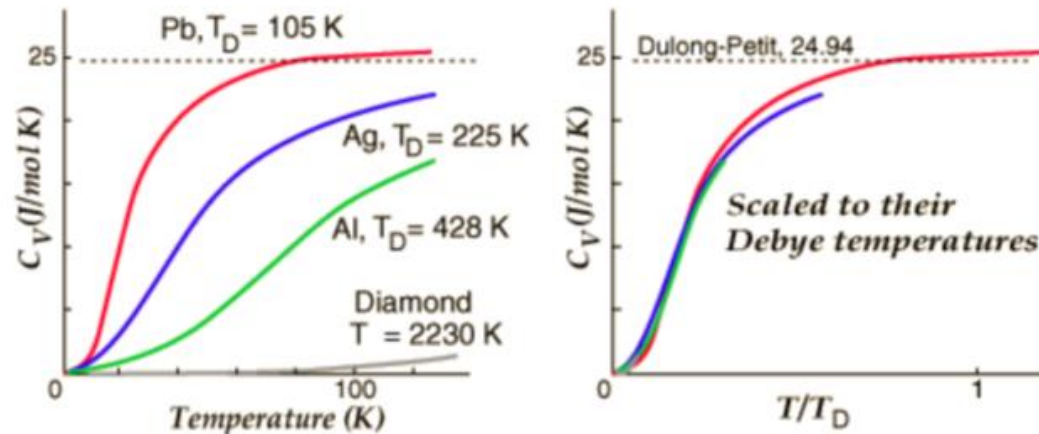


$$Q_{4.5} = 3Q_{2K+} + Q_{4.5K} + 0.1Q_{60K}$$

There are some time left

Material Properties at Cryogenic Temperature

At the left of Figure below, the experimental results of specific heats of four substances are plotted as a function of temperature and they look very different. But if they are scaled to T/T_D , they look very similar and are very close to the Debye theory.



The Specific heat of copper is 0.386 J/gm K and that of lead is only 0.128 J/gm K). Why are they so different? The difference is mainly because it is expressed as energy per unit mass; if you express it as energy per mole, they are very similar. It is in fact that similarity of the molar specific heats of metals which is the subject of the Law of Dulong and Petit.

$$\text{Copper } 0.386 \text{ J/gm K} \times 63.6 \text{ gm/mole} = 24.6 \text{ J/mol K}$$

$$\text{Lead } 0.128 \text{ J/gm K} \times 207 \text{ gm/mole} = 26.5 \text{ J/mol K}$$

Specific Heat of Materials at Low Temperature

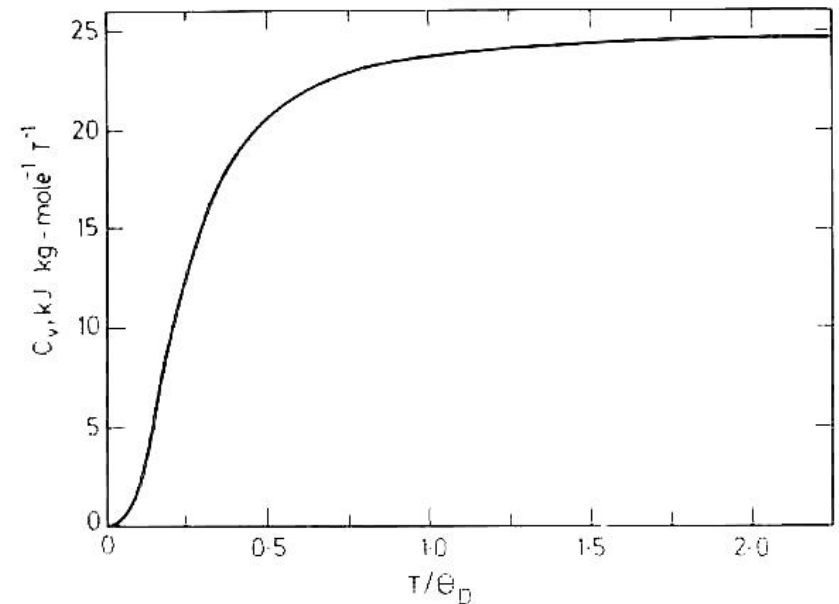
Energy (Joule) Required to change the temperature of 1g / 1 g.Mole substance by one degree (J/ g.K) or / J/ Mole.K

$$C_v = 9R \left(\frac{T}{\theta_D} \right)^3 \int_0^{\theta/T} \frac{x^4 e^x}{(e^x - 1)^2} dx$$

At Room Temperature and Above
 C_v (kg/ mole.K) = 3 R constant

At low Temperature

$$C_v = \frac{12\pi^2 R}{5} \left(\frac{T}{\theta_D} \right)^3$$



While cooled down from Room Temperature to Liquid Nitrogen, Specific heat reduced drastically

Electronic Specific Heat

The temperature dependence of Einstein model is just T . It becomes significant at low temperatures and is combined with the above lattice specific heat in the Einstein-Debye specific heat.

$$C_{metal} = C_{electron} + C_{phonon} = \frac{\pi^2 N k^2}{2 E_f} T + \frac{12 \pi^4 N k_B}{5 T_D^3} T^3$$

$$C_{metal} = C_{electron} + C_{phonon} = \gamma T + \alpha T^3$$

At normal temperature , the electronic contribution to the total specific heat is quite small because of small value of γ .

At very low temperature ($T < 1K$) does becomes important because it varies as T while the Phonon contribution varies as T^3

Role of Specific Heat on Cool Down

To cool 1 gm mass from Room temperature to 4.2 K

$$Q = m C_p (300 - 4.2) \text{ J}, C_p = f(T)$$

At low temperature, C_p is less, less heat to be extracted at low Temperature. Cryogen (M_c) is required to cool 1 Kg mass

$$M_c \cdot L = m \cdot C_p (300 - 4.2)$$

Requirement of Cryogen To cool Down

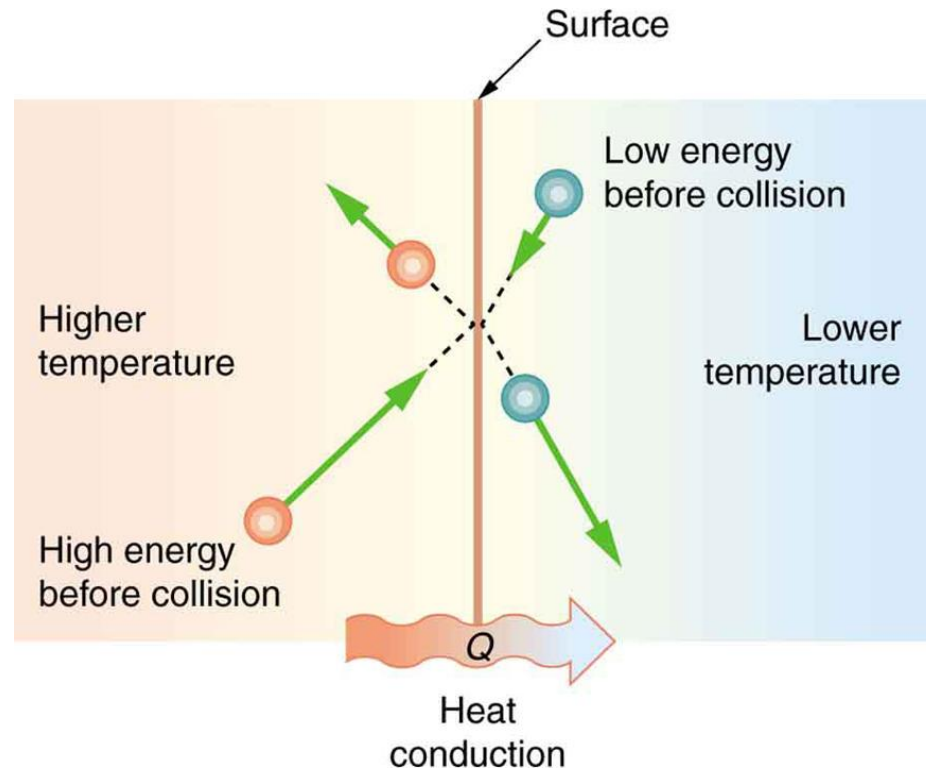
	300- 4.2K (Lhe) L/kg	78- 4.2 K (LHe) L/kg	(300- 4.2) With Cold Gas	300-78 K (LN2)
SS	33	1.44	0.80	0.53
Al	66	3.2	1.60	1.0

1. C_p of Aluminium is double of SS
2. Precooling with LN2 saves Lhe
3. Slow Cooling uses cold enthalpy of gases

Remember Latent Heat of LHe is only 20 against 200 J/gm for LN2

Thermal Conduction Mechanism

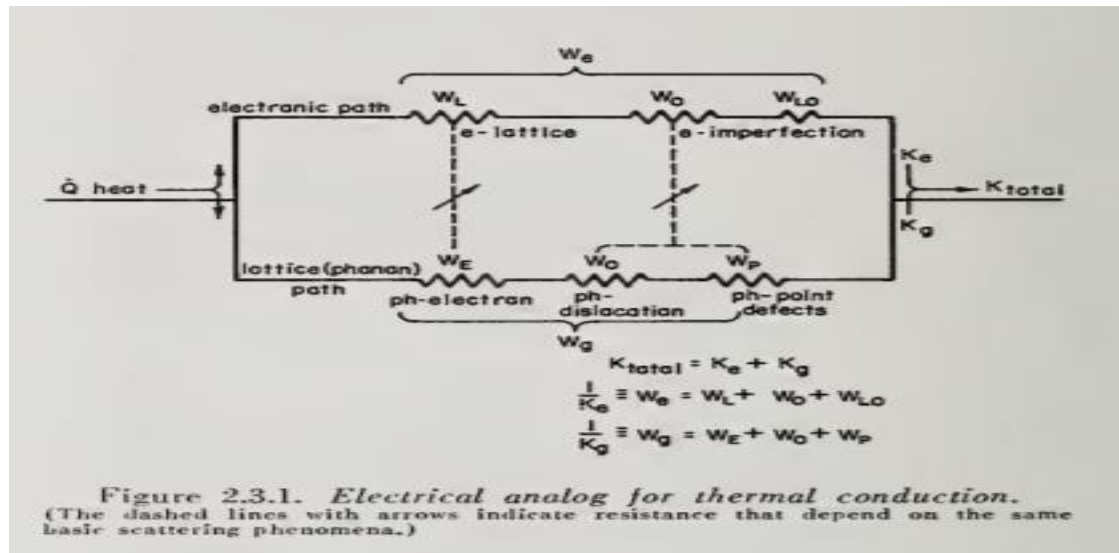
The (average) kinetic energy of a molecule/ Free electron in the hot body is higher than in the colder body. If two molecules/ electron collide, an energy transfer from the hot to the cold molecule occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. We call this transfer of heat between two objects in contact thermal conduction.



Two parallel mechanisms are primarily responsible for the transport of heat in a metal at low temperatures.

- 1. The most important is the electronic thermal conduction, the transport of thermal energy by the motion of conduction electrons.**
- 2. The second is the lattice thermal conduction, heat transport by vibrations (called phonons) of the thermally excited interacting lattice ions.**
- 3. For pure metals the lattice thermal conductivity is very much smaller than the electronic thermal conductivity. For Alloys / Non Metal , Lattice Contribution is comparable with Electronic Contribution**
- 4. Impurities and lattice defects scatter electrons and decrease thermal conductivity.**

K, is the sum of two terms, the electronic conductivity K_e and the lattice conductivity, K_g ; that is, $K = K_e + K_g$.



THERMAL PROPERTIES

➤ Thermal conductivity

- **For pure metals:**

- k_{ph} is negligible
- k has a maximum at low temperature
- At low T° , k is affected by impurities
- The more is the purity of the material,
 - the higher is this maximum
 - the lower is the T° of this maximum
- $k \propto T$ at low temperature

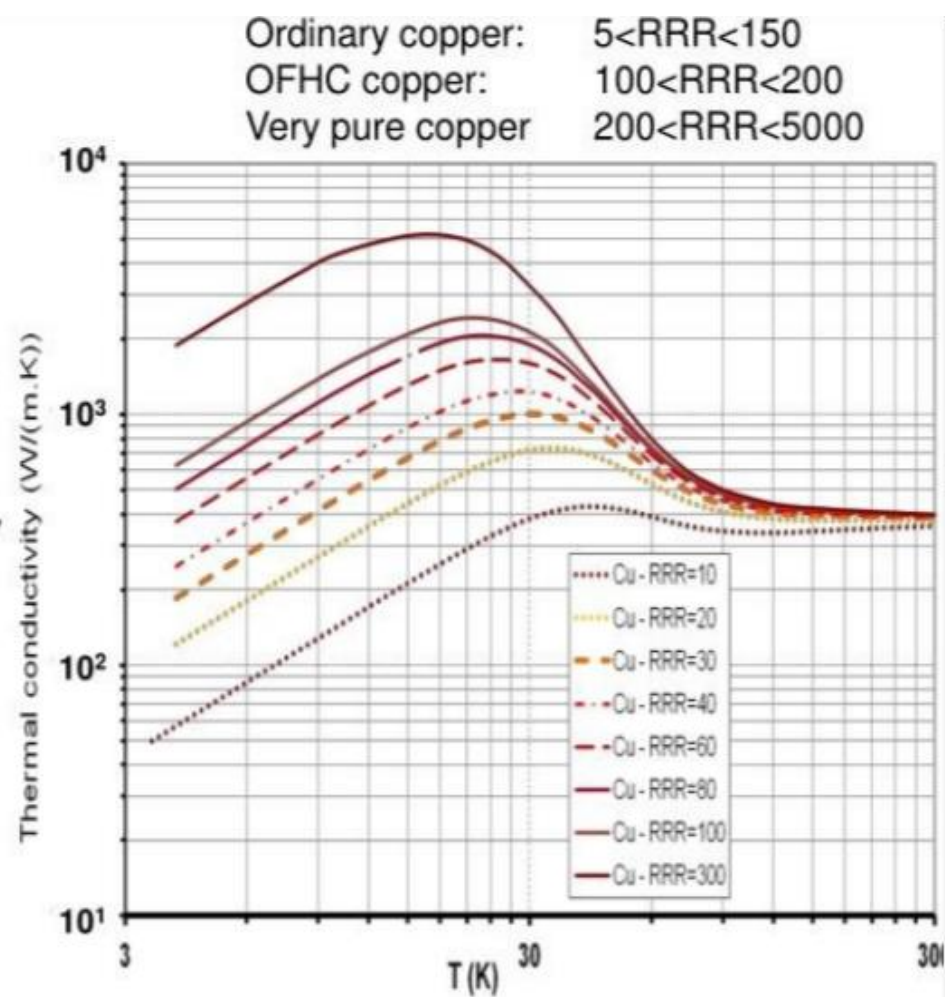
- **For metallic alloys:**

- k decreases as T decreases
- $k \propto T$ at low temperature
- Wiedemann-Franz law:

relates k_e and the electric resistivity ρ : $\rho \cdot k_e / T = 2.445 \cdot 10^{-8} \text{ (W} \cdot \Omega / \text{K}^2 \text{)}$

- **For superconductors:**

- $T > T_c$ (normal state) \Rightarrow cf. behaviour of metals
- $T < T_c$ (Meissner state): $k_s \propto T^3$ and $k_s(T) \ll k_n(T) \Rightarrow$ thermal interrupter

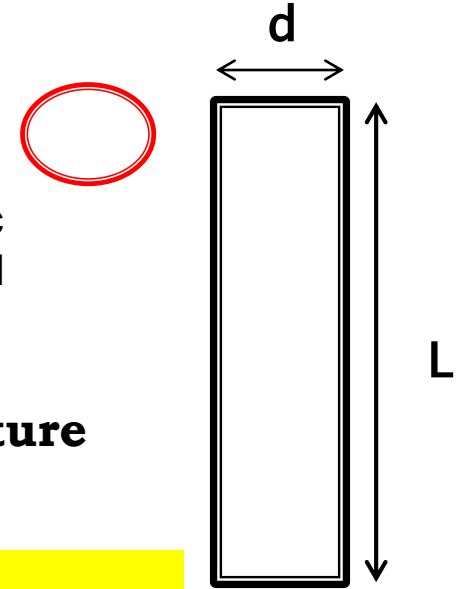


HEAT TRANSFER MECHANISM

A. Solid conduction heat transfer : Necktube, Support structure .

$$Q_c = K_m \frac{A_c}{L} (T_h - T_c)$$

A_c = Cross sectional area (example for a rod of dia (d), $A_c = \pi d^2/4$, Similarly for a pipe of outer diameter d and thickness 't', $A_c = \pi dt$, L = Length of rod/ pipe,



Conductivity varies a lot between Room Temperature and Low Temperature (78 K or 4.2 K), $K = f(T)$

$$Q_c = \frac{A_c}{L} \int_{T_c}^{T_h} k dT$$

Table for Integral Value of Different materials are available with reference to $T_c = 4.2$ K

$$\int_{78}^{300} k dT = \int_{4.2}^{300} k dT - \int_{4.2}^{78} k dT$$

Conduction is reduced

- **Thin Pipe**
- **Long Length**
- **Insulating Material**

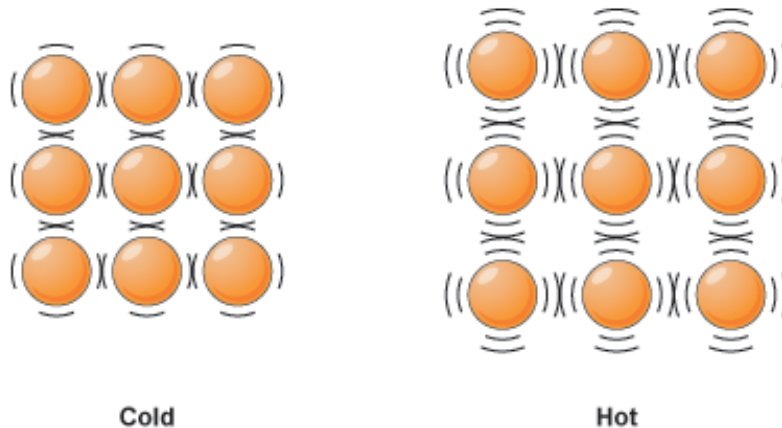
Thermal conductivity integrals

$T_c = 4 \text{ K}$

T_2	ETP copper	Aluminium 1100	Austenitic stainless steel	Glass	PTFE
[K]	[W cm ⁻¹]	[W cm ⁻¹]	[W cm ⁻¹]	[mW cm ⁻¹]	[mW cm ⁻¹]
10	33.2	6.07	0.0293	6.81	4.4
20	140	27.6	0.163	20.0	16.4
30	278	59.2	0.424	36.8	32.3
40	406	96.2	0.824	58.6	50.8
50	508	134	1.35	84.6	71.6
60	587	170	1.98	115	93.6
70	651	202	2.70	151	116
80	707	232	3.49	194	139
90	756	258	4.36	240	163
100	802	284	5.28	292	187
120	891	330	7.26	408	237
140	976	376	9.39	542	287
160	1060	420	11.7	694	338
180	1140	464	14.1	858	390
200	1220	508	16.6	1030	442
250	1420	618	23.4	1500	572
300	1620	728	30.6	1990	702

- ▶ **Reduction of heat flow to the cold boundary temperature by thermal interception at intermediate temperature**

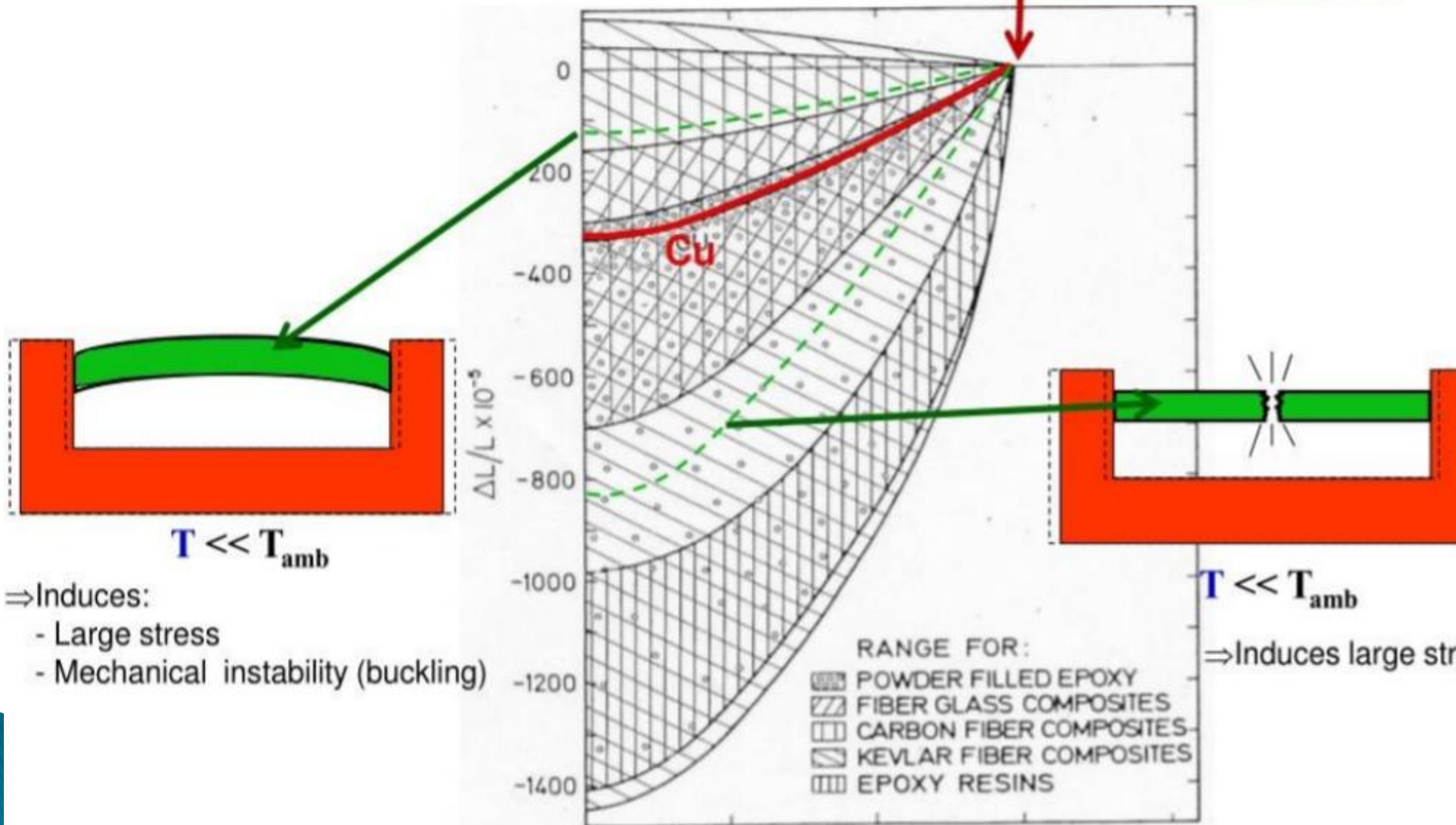
Thermal Contraction/ Expansion



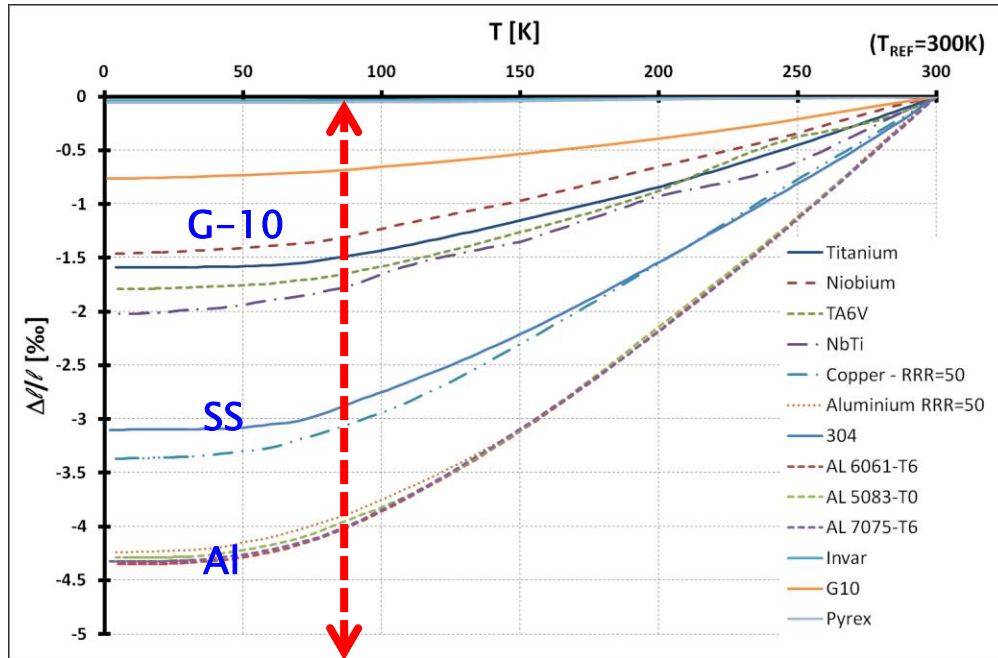
When heated, solids (and liquids and gases) gain thermal energy. The particles start to move about more – their vibrations take up more space, so there is expansion in all directions. The opposite is true when the temperature falls – the material will get smaller (contract).

➤ Thermal expansion/contraction of solids

• Example:



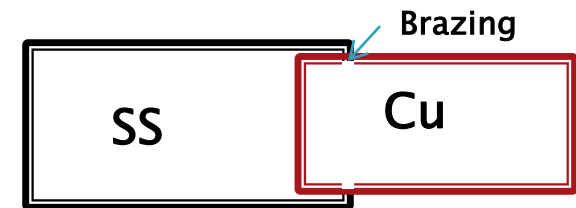
Thermal Contraction at Low Temperature



$$\Delta L = \alpha L (T_2 - T_1)$$

$$\text{Stress} = E \cdot \alpha \cdot (T_2 - T_1)$$

E = Young Modulus
 α = Thermal expansion Coefficient



- **90 % of Contraction happens between 300 K to 78 K. Contraction Between LN2 Temperature and LHe Temperature is bare minimum**
- **Non Metal (G- 10) have less thermal Expansion Co- efficient than Metal (CU, SS)**
- **Approx 3- 4 mm contraction on cooling a rod of SS, Cu, Al of Length 1 Meter**
- **Different Thermal Expansion co-efficient between Material can be used constructively or there can be design failure**

Mechanical Properties

Sr. No.	Property
1	Yield and Ultimate Strengths
2	Fatigue Strength
3	Impact Strength
4	Hardness and Ductility
5	Elastic Moduli

MECHANICAL PROPERTIES

➤ Introduction

- Tensile test:

Ultimate tensile strength
UTS

Yield tensile strength
YS

Slop:
Young modulus
 $E = R_e \times L/\Delta L$

NB: stiffness $k=EA/L$

Stress
 $\sigma = F/s_0$ ($\text{N/m}^2 \equiv \text{Pa}$)

cross section s_0

Fracture

0.2%
offset line

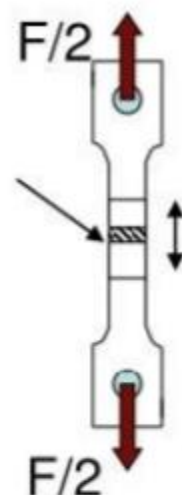
$YS_{0.2}$

Plastic deformation
(irreversible)

Necking

Elastic deformation
(reversible)

Strain
 $\Delta L/L$ (%)



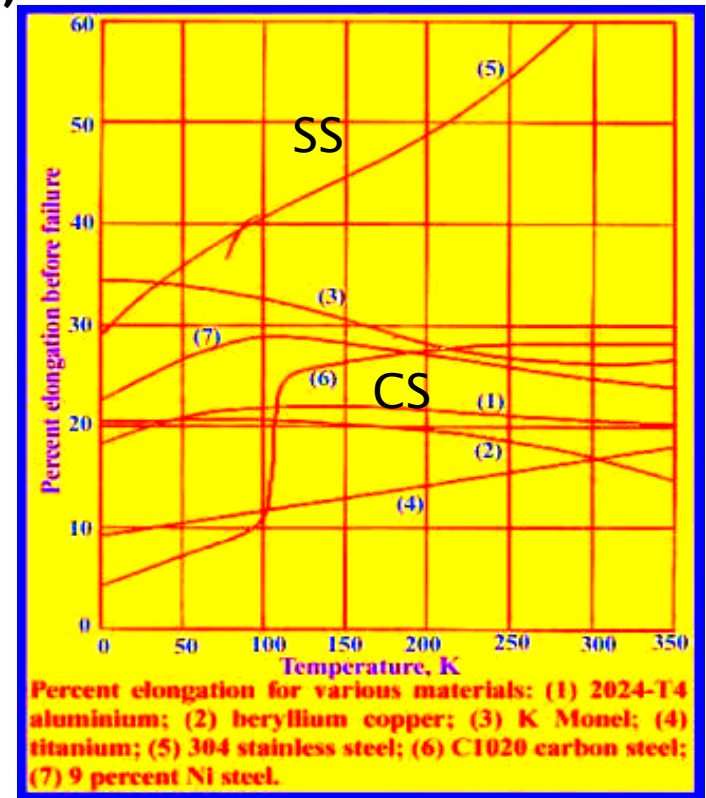
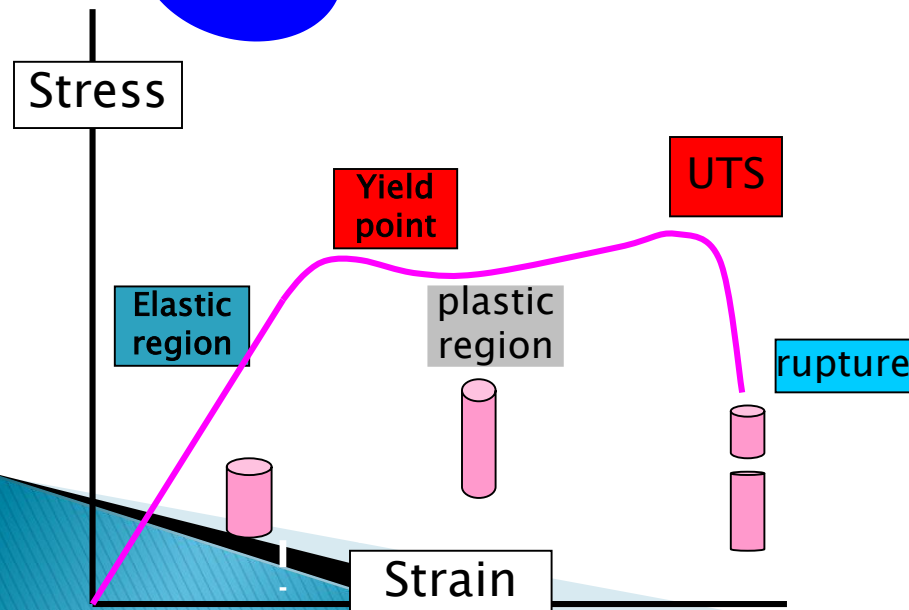
Materials Used at Cryogenic Temperature

Significant Change on Property (Mechanical, Thermal) on cooling it down from RT to Cryogenic Temperature

Ductile to Brittle : Percentage of Elongation Before failure reduced significantly (Yield strength and UTS almost same)

NO Carbon Steel, Nylon, Plastic Material

Yes SS, Cu, Al, Teflon



Contact mail : tsdatta59@gmail.com

Thanks



Namaste

