2022 가속기 및 빔라인 미래인재 양성교육단 여름학교

# X선 회절과 결맞음성

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# Why X-ray?



#### X-rays



#### (1) Classical Description

transverse Electro-Magnetic wave : Light!



 $\lambda$ (wavelength) : 0.01 nm ~ 10 nm c(speed of light) :  $3 \times 10^8$  m/sec v(frequency) :  $v = \frac{c}{\lambda}$  $\omega$ (angular frequency) :  $\omega = 2\pi v$ 

Scatter, Interfere, Diffract 🔶 Superpose

(2) Quantum Mechanical Description (particle nature)

$$E = hv = \hbar \omega = h \frac{c}{\lambda}$$
  
h (Planck constant) =  $6.63 \times 10^{-34}$  Joule · sec  
 $E \cdot \lambda = 12400 \text{ eV} \cdot \overset{\circ}{\text{A}}$   
(Hard) x-ray Energy : 124 keV ~ 1.24 keV

#### (3) Maxwell's Equations (wave nature)

 $\nabla \cdot \vec{E} = 4\pi\rho$  Coulomb's law  $\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$  Faraday's law  $\nabla \cdot \vec{B} = 0$  Absence of free magnetic poles  $\nabla \times \vec{B} = \frac{4\pi}{c} J + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$  Ampere's law

Solution : 
$$\vec{E}(\vec{r},t) = \vec{E}_o \exp(i(\vec{k}\cdot\vec{r}-\omega t))$$
  
 $\left|\vec{k}\right| = \frac{2\pi}{\lambda}$  : wave vector

#### Wave equation :

$$\nabla^{2}\vec{E} = \frac{1}{c^{2}}\frac{\partial^{2}\vec{E}}{\partial t^{2}}$$
  
Spherical coordinate:  
$$\left[\frac{1}{r^{2}}\frac{\partial}{\partial r}\left(r^{2}\frac{\partial}{\partial r}\right) + \frac{1}{r^{2}\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^{2}\sin^{2}\theta}\frac{\partial^{2}}{\partial\varphi^{2}}\right]\vec{E} = \frac{1}{c^{2}}\frac{\partial^{2}\vec{E}}{\partial t^{2}}$$

Cartesian coordinate:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

Plane wave solution :

$$\vec{E}(\vec{r},t) = \vec{E}_o \exp(i(\vec{k}\cdot\vec{r}-\omega t))$$

$$|\vec{k}| = \frac{2\pi}{\lambda} : \text{wave vector}$$

Spherical wave solution:

$$\sim \frac{1}{r} \exp\left[i\left(kr - \omega t\right)\right]$$



#### (4) Polarization



$$\mathbf{E}(\mathbf{r},t) = \hat{x}E_0 \exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t)) + \hat{y}E_0 \exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t + \frac{\pi}{2}))$$
$$= E_o(\hat{x} + i\hat{y})\exp(i(\mathbf{k}\cdot\mathbf{r} - \omega t))$$

• Polarization : the direction of electric field always perpendicular to the direction of the propagation. parallel to the acceleration of the electric charge

• Phase : the argument in the exponential

#### Coherence length : the length scale within which the phase of the electromagnetic field is correlated

А

 $2L_{\tau}$ 



If two waves are in-phase at a point, how far do we have to go away before **the waves are out of phase?**  $\rightarrow$  coherence length (L<sub>L</sub>, L<sub>T</sub>)

# Use of X-ray

 Imaging: Real Space Measurement: I(r) (Medical application)









Differences in Absorption

Computed Tomography scan

✓ X-ray Scattering or Diffraction: Reciprocal Space Measurement :  $I(q), I(\theta)$ 



# Nobel Prizes for Research with X-Rays

1901 W. C. Röntgen in Physics for the discovery of x-rays. 1914 M. von Laue in Physics for x-ray diffraction from crystals. 1915 W. H. Bragg and W. L. Bragg in Physics for crystal structure determination. 1917 C. G. Barkla in Physics for characteristic radiation of elements. 1924 K. M. G. Siegbahn in Physics for x-ray spectroscopy. 1927 A. H. Compton in Physics for scattering of x-rays by electrons. 1936 P. Debye in Chemistry for diffraction of x-rays and electrons in gases. 1962 M. Perutz and J. Kendrew in Chemistry for the structure of hemoglobin. 1962 J. Watson, M. Wilkins, and F. Crick in Medicine for the structure of DNA. 1979 A. McLeod Cormack and G. Newbold Hounsfield in Medicine for computed axial tomography. 1981 K. M. Siegbahn in Physics for high resolution electron spectroscopy. 1985 H. Hauptman and J. Karle in Chemistry for direct methods to determine x-ray structures. 1988 J. Deisenhofer, R. Huber, and H. Michel in Chemistry for the structures of proteins that are crucial to photosynthesis.

**2009** Venkatraman Ramakrishnan, Thomas A. Steitz and Ada E. Yonath in Chemistry for "studies of the structure and function of the ribosome" X-ray crystallography

## **Structure-Function Relationships**



→ Materials Control and Engineering

#### Structure Determination





Hard X-rays:  $\lambda \le 1 \text{ nm}$ ,  $E \ge 1 \text{keV}$ Soft X-rays:  $\lambda \ge 1 \text{ nm}$ ,  $E \le 1 \text{keV}$ 

William & Lawrence Bragg Novel Prize for Physics 1915

- X-ray interaction with matter
- Scattering cross-section
- Diffraction
- Why Synchrotron?
- Coherent X-rays

# X-ray interaction with matter

### **Interaction Mechanisms**



X-rays interact with electrons via an electromagnetic interaction.
 (cf. Neutrons interact with atomic nuclei via very short range (~ fm) forces.
 Neutrons interact with unpaired electrons via magnetic dipole interaction.)

# Penetration in Matter

8 keV X-rays, Thermal Neutrons, & Low Energy Electrons



For X-rays : Decreasing penetration as Z increases

(cf. For Neutrons

: H/D difference

: Cd, B, Sm

: No systematic Z dependence)

# X-rays interacting with electrons

- (1) Photoelectric absorption X-rays kicks electron from shell to continuum
  - Leads to fluorescent X-ray emission when hole in shell is filled from outer shell
  - Goes as 1/E<sup>3</sup> but with sharp steps at shell energies when new channel opens



# X-rays interacting with electrons

(2) **Thomson scattering** : elastic and coherent

$$\underset{E}{\overset{}} \bullet \overset{} \overset{} \overset{} \overset{} E' = E$$

$$\left|\vec{k}\right| = \left|\vec{k'}\right|, \ E = E'$$



(3) **Compton scattering** : inelastic and incoherent

$$|\vec{k}| \neq |\vec{k}'|, E \neq E'$$

## **Refractive Index**



$$n = 1 - \delta + i\beta$$

## Visible vs. X-ray



Monochromator: grating Mirror: Metal coated glass

l(E), l(ω), l(λ)

Monochromator: Si single crystal Mirror: Si single crystal

 $I(\theta), I(q)$ 

# Scattering cross-section

#### Intrinsic cross-section



#### **Reciprocal Space Measurement**

Schematic layout of a scattering experiment to determine the differential cross-section



#### Intrinsic cross-section



#### For bound electrons



#### One atom



The resultant phase difference:  $\Delta \phi(\vec{r}) = (\vec{k} - \vec{k'}) \cdot \vec{r} = \vec{Q} \cdot \vec{r}$ 

$$\left|\vec{k}\right| = \left|\vec{k}'\right|$$
 : elastic  $\left|\vec{Q}\right| = 2\left|\vec{k}\right|\sin\theta = \frac{4\pi}{\lambda}\sin\theta$ 

Volume element  $d\vec{r}$  at  $\vec{r}$  :  $r_0 \rho(\vec{r}) d\vec{r}$  with phase factor  $e^{i\vec{Q}\cdot\vec{r}}$ 

$$r_0 f(\vec{Q}) = r_0 \int \rho(\vec{r}) e^{i\vec{Q}\cdot\vec{r}} d\vec{r}$$

Atomic Form Factor : Fourier transform of charge density (distribution of electrons)

$$f(\vec{Q}) = \int \rho(\vec{r}) e^{i\vec{Q}\cdot\vec{r}} d\vec{r}$$
  

$$Q \to 0: \text{ all of the different volume elements scatter in phase}$$
  

$$f^o(\vec{Q} = 0) = Z$$
  

$$f^o(\vec{Q} \to \infty) = 0$$

$$f(Q, \hbar\omega) = f^{0}(Q) + f'(\hbar\omega) + if''(\hbar\omega)$$

**Anomalous Dispersion Corrections** 

f' : Real part of scattering length Element Specific f'' : Dissipation

### One molecule



### **Reciprocal Lattice**



#### **Reciprocal Space**

- An array of points (hkl) that is precisely related to the crystal lattice



 $a^* = 2\pi (b \ x \ c) / V_0$ , etc.

A single crystal has to be aligned precisely to record Bragg scattering

A crystal

$$F_{\text{crystal}}(\vec{q}) = \left(\sum_{j=1}^{N} f_j(\vec{q}) e^{-i \vec{q} \cdot \vec{r}_j}\right) \cdot \left(\sum_{n=1}^{M} e^{-i \vec{q} \cdot \vec{R}_n}\right)$$
Unit Cell Structure Factor Lattice Sum
$$\sum_{n=1}^{M} e^{-i \vec{q} \cdot \vec{R}_n} \approx \begin{cases} M >> 1 \quad \text{for} \quad \vec{q} \cdot \vec{R}_n = 2\pi \times \text{integer} \\ 0 \quad \text{otherwise} \end{cases}$$

$$\frac{\text{Reciprocal Lattice:}}{\vec{R}_n = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3}$$

$$\vec{a}_i \cdot \vec{a}_j^* = 2\pi \delta_{ij} \quad i, j = 1, 2, 3$$

$$\vec{G}_{hkl} = h \vec{a}_1^* + k \vec{a}_2^* + l \vec{a}_3^*$$

$$\vec{G}_{hkl} \cdot \vec{R}_n = 2\pi (hn_1 + kn_2 + ln_3)$$



$$S(\vec{q}) = \left| F_{\text{crystal}}(\vec{q}) \right|^2$$

### The measured intensity from a crystallite





#### Compton Scattering by a free electron (inelastic scattering)

• Assumption: view the incident x-ray as a beam of photons.

Electron is at rest and is free.

In a collision, energy will be transferred from the photon to the electron, with the result that the scattered photon has a lower energy than that of the incident one.

: Compton Effect





Momentum conservation

$$\vec{k}' - \vec{k} = \vec{q}'$$
$$\frac{k}{k'} = 1 + \lambda_c k (1 - \cos \psi) = \frac{E}{E'} = \frac{\lambda'}{\lambda}$$



- Vary slowly with scattering angle (more important at higher scattering angle).
- Effective when the x-ray energy approaches the rest mass of electron.
- In a diffraction experiment, it gives rise to a smoothly varying background.

mc<sup>2</sup>=511 keV
# Diffraction

# X-Ray Reflectivity vs. Diffraction



 $2d\sin\theta = m\lambda$ 

Bragg's Law



## **Scattering Geometry**



#### X-ray Diffractometer (4-circle)



Some aspects of real crystals that go beyond these assumptions:

- thermal motion of atoms
- lattice strain
- finite size of crystal
- defects
- disorder

In addition, we had assumed that the photons only scatter once before leaving the sample. In the case of reflectivity, we already know that multiple reflection have to be taken into account when the reflectivity is close to 1.

We have to consider such multiple reflection in Bragg condition.

# X-ray Crystallography



- X-ray crystallography
  - : the size of the atoms, the length and types of chemical bonds, the nature and differences of various materials on the atomic scale.
  - : knowledge of crystals (the appearance and the symmetry of the crystals)
  - : X-rays are just electromagnetic radiation as visible light but of short wavelength







- $\theta$  tilt of helix (angle from perpendicular to long axis)
- h = 3.4 Å (Distance between bases)
- p = 34 Å (Distance for one complete turn of helix; Repeat unit of the helix)





ded at LCLS (left, seen from the

# **Powder Diffraction**





Cubic	$\frac{1}{d^2} = \frac{h^2 + k^2 + l^2}{a^2}$
Tetragonal	$\frac{1}{d^2} = \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}$
Hexagonal	$\frac{1}{d^2} = \frac{4}{3} \left( \frac{h^2 + hk + k^2}{a^2} \right) + \frac{l^2}{c^2}$

# Why synchrotron radiation?

# Synchrotron sources offer

- ✓ Brightnesss (small source, collimated)
- ✓ Tunability (IR to hard x-rays)
- ✓ Polarization (linear, circular)
- ✓ Time structure (short pulses)

### Intensity !!!



### B = photons/source area, divergence, bandwidth F<sub>c</sub> ~ $\lambda^2$ B





# 방사광가속기의 진보





1세대: Parasitic SR mode from existing high-energy machines 2세대: Dedicated SR sources (with IDs later) 3세대: Fully optimized to adopt insertion devices 4세대: Multi-Bend Achromat (MBA)

#### **Intensity and Spectrum**



### **Brilliance & Emittance**



# Synchrotron vs. XFEL

	Synchrotron	XFEL
Time Resolution	~ 80 picosec	10–100 femtosec
Rep. Rate	500 MHz	0.1–10 KHz
# Photons/pulse	10 <sup>9</sup>	1012
Coherent Flux	10 <sup>8</sup> –10 <sup>9</sup> photons/sec	10 <sup>14</sup> –10 <sup>16</sup> photons/sec
Spatial Coherency	0.1%	~100%
Time Coherency	No	0.5%
Bandwidth		0.3 x10 <sup>-3</sup>
Photon degeneracy at 9keV	2.2×10-4	3×10 <sup>8</sup>
Stability	Stable	Not stable

	3세대 방사광	차세대 (4GSR)
		•
<b>휘도(</b> Brilliance]	1020	>1022
빔크기 (Source size)	280μm×11μm (H×V)	8μm×10μm (H×V)
결맞는 X-선 (Transverse coherence lengths (9keV))	12μm×340μm (H×V)	420μm×340μm (H×V)
펄스폭 (Pulse length)	80ps	35–70ps
반복율 (Repetition rate)	~650MHz	10-100 MHz

미국 APS와 APS-U의 사양 비교

- ✓ 높은 에너지의 X-선 빔이 가능 : 원자 수준의 구조 분석
  - : 실제 작동 상황에서 측정
- ✓ 작고, 센 X-선 빔 가능 : 공간 분해능: 수 나노 미터 : 실 상황에서 측정 용이
- ✓ 결맞음성이 좋은 X-선 빔 획득 : 나노미터 크기의 프로브 : 이미징

  - : 동역학 연구



# 국내 현황

3세대 방사광가속기



- 포항 PLS-II
- 3 GeV (둘레 280m)
- ~20 keV
- 35 Beamlines
- Emittance: 5800 pm.rad (Hor.)
- 500 MHz
- Pulse duration ~100 ps



4세대 XFEL

#### 4세대 다목적방사광가속기



#### • 오창 (2028년 운영 목표)

- 4 GeV (둘레 800 m)
- ~30 keV (100 keV 까지도 가능)
- ~40 Beamlines
- Emittance: 58 pm.rad (Hor.)
- 500 MHz
- Pulse duration ~100 ps
- 3세대 대비 수 백배 이상의 휘도







■ 포항 PAL-XFEL

3 Stations

Pulse 120Hz

• 10 GeV (길이 1.1 km)

~100% coherence

2-15 keV. 0.25-1.25 keV

Pulse duration 10-40 fs





# Coherent X-rays

### Coherence



Visibility of fringes is a direct measure of the coherence of a beam. If beam is coherent across the spacing of the slits a Fourier Transform of the slit structure is observed downstream.

### Coherence



# Coherence: Laser Speckle



A. L. Schawlow "Laser Light" Scientific American, 219 (3), p. 120, (1968)



Laser Speckle: Interference pattern arising from randomly distributed scatterers

# First Speckle: Exner,

### 1877 (using candle light)



K. Exner: Sitzungsber. Kaiserl. Akad. Wiss. (Wien) 76, 522 (1877)

# First Speckle Photo:

von Laue, 1914 (using arc discharge lamp)



M. von Laue: Sitzungsber. Akad. Wiss. (Berlin) 44, 1144 (1914)

### First X-ray Speckle



#### Speckle of (001) Cu<sub>3</sub>Au M. Sutton, et.al., Nature **352**, 608 (1991).

 $I \mid |F(q)|^2$ 



FIG. 4 Speckle patterns measured using a 2.5- $\mu$ m, b, 5  $\mu$ m and c, 50- $\mu$ m collimating pinholes. The analysing pinholes used were 50, 25 and 100  $\mu$ m, respectively. Representative error bars are indicated, and the solid lines simply connect the data points. The (001) Bragg angle,  $2\theta_{\mu}$ , is ~23.9°.

# Coherent vs. Incoherent

Small-angle X-ray scattering pattern of silica nanospheres



#### **Coherent scattering**

- Speckle pattern directly related to the electron density
- Speckle arising from the path length difference of X-rays in the sample
- Local information

#### **Incoherent scattering**

- Statistical average of incoherently scattering regions
- No speckle
- Local information lost

# "Speckle" ⇒ Coherence based Techniques



Speckle of (001) Cu<sub>3</sub>Au M. Sutton, et.al., Nature **352**, 608 (1991).

# X-ray Photon Correlation Spectroscopy



# Application: XPCS

# Bulk

# Hard

Equilibrium critical fluctuations

 Decomposition kinetics
 Antiphase-domain dynamics

 Ordering Kinetics
 Domain Fluctuations

• Coarsening

# Soft

Diffusion
Phase Transition
Hydrodynamic Fluctuations
rdering & relaxation dynamics

- Ordering & relaxation dynamics
  - Atomic Diffusion

# Surface

- Liquid crystal (Smectic Membranes)
- Liquid (Water, Glycerol, etc)
- Polymer Films: Confinement? Selective detection?
- Organic Semiconductors, etc.

## Nano dynamics: Soft materials & Nano system



# Surface Dynamics



Organic Semiconductors, (OLED, OTFT, OPV)

GISAXS, GID

Surface Dynamics of Polymer Films, PRL 90, 068302 (2003). Viscoelastic Effects in Ultrathin Films, PRL 98, 227801 (2007). Surface Dynamics near Tg in High q, PRL 101, 246104 (2008). Surface Dynamics of "Dry" Homopolymer Brushes, Macromolecules 42, 737 (2009).





### In-situ growth

High Energy E=25.75 keV MOCVD, GaN growth



Coherent X-ray spectroscopy reveals the persistence of island arrangements during layer-by-layer growth, **Nature Physics (2019)** 

# Imaging Regime of Coherent X-rays



### **Bragg Coherent Diffraction Imaging**



Sciences and Chemical Engineering 2022

https://doi.org/10.1016/B978-0-12-823144-9.00082-0

# **Diffraction vs. Interference**

**Double Slit** 



# Phase Retrieval Algorithm



3D displacement field distribution



3D scanned diffraction pattern at Bragg (1-11) coherent diffraction peak



#### 34IDC, APS

## Structural Sensitivity & In situ Measurement

- Achievable at the nanoscale with sub-angstrom sensitivity to crystalline lattice distortion.
- ✓ In situ measurements are easily available.
- Gold nanoparticle by adsorption of propane thiols
   : ~ pico meter level distortion



- Single zeolite crystal of organic residue effect with "in situ" temperature dependence : ~ 0.1nm scale lattice distortion
  - Spatial derivative of the displacement along the entire size of the crystal:10<sup>-4</sup>



Nat. Mater. 12, 729 (2013).
#### Operando techniques



#### **Operando Study**



Science 348, 1344 (2015).

Nano Lett. 14, 5123-5127 (2014)

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연료전지, Battery, 에너지 소자
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#### In situ Catalysis



#### **Imaging Lattice Dynamics**



Ultrafast three dimensional imaging of lattice dynamics in individual gold nanocrystals J. N. Clark, et.al. (2013). *Science*, *341*(6141), 56–59

#### **Imaging Lattice Dynamics**



### Ptychography



PNAS 108, 13393 (2011)

Advanced coherent/polarized X-ray scattering beamline (ACPXS)



## 차세대 방사광가속기의 새로운 과학기술

✓ Local quantity measurements (nano probes & coherence)
⇒ Discovery of New Materials?
Understanding & Control Defects
✓ Flux-hungry techniques available! (Inelastic scattering, Resonant magnetic Hard x-ray scattering…)

✓ Operando, in situ study

 $\cdots$  need more

New Idea New Instrumentation New People!



# 감사합니다.