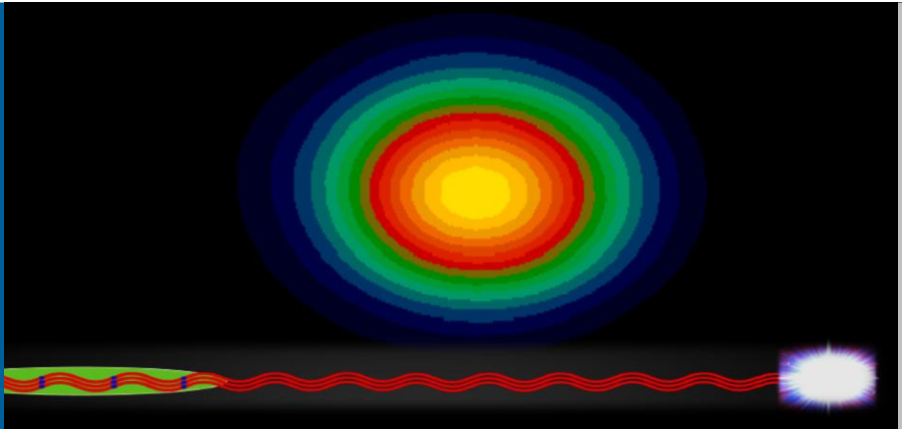
2022 UST GLOBAL MENTORING CONFERENCE

COHERENCE 의 추구



김광제

Univ. of Chicago
Argonne National Lab

U.S. DEPARTMENT OF ENERGY
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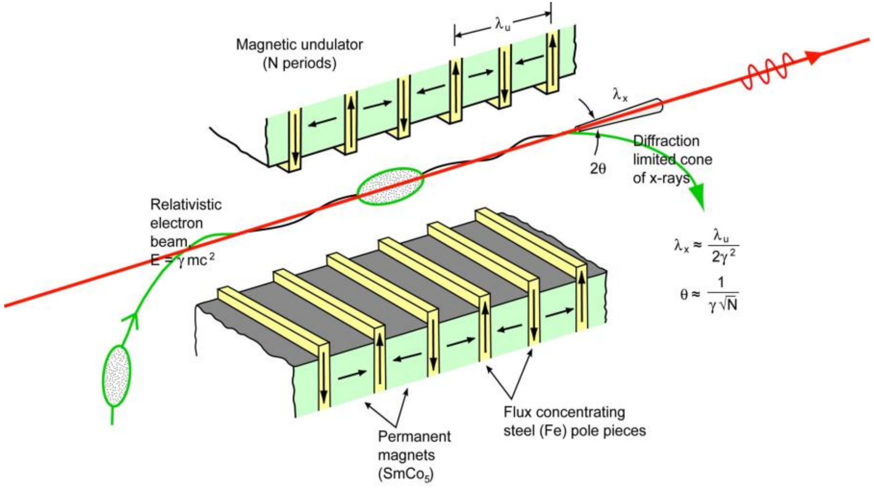
July 14, 2022

THE THEME: ELECTRON BEAM GOING THROUGH A LONG UNDULATOR



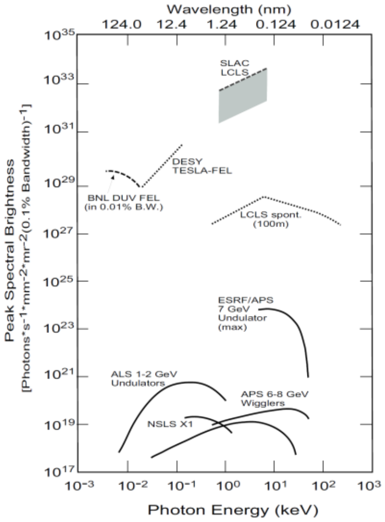
With Klaus Halbach, the inventor of PM-based undulator

- Undulator radiation is bright and partially coherent
- It can become a free-electron-laser, which can be fully coherent

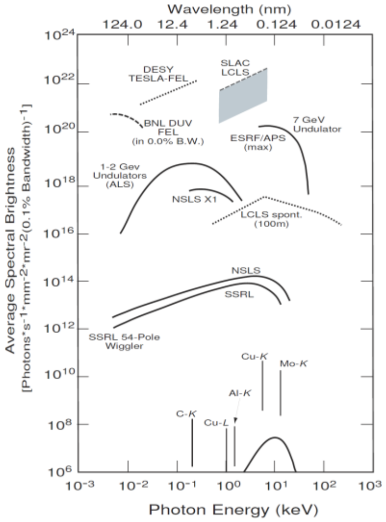
CONTENTS

- **Undulator radiation**
  - Brightness as quality factor and phase space density
  - Transverse (spatial) coherence
  - Polarization coherence
- **Temporal coherence and phase coherence**
- **Free electron laser**
  - Theory development
  - SASE (High-gain): quasi coherence
  - XFELO: full coherence
  - Ultimate: X-ray comb

BRIGHTNESS CURVE IN UNITS OF PHOTONS /(MM^2 MRAD^2 S 0.1% BW)] BECAME THE STANDARD IN COMPARING MERITS OF SR FACILITIES



(a) peak brightness



(b) average brightness

- During1980s, I served as the brightness $\mathcal{B}$ curve factory for many proposed synchrotron radiation facilities
- A war on nomenclature; **Brightness** or **Brilliance?**
- However, what is brightness $\mathcal{B}$ really?

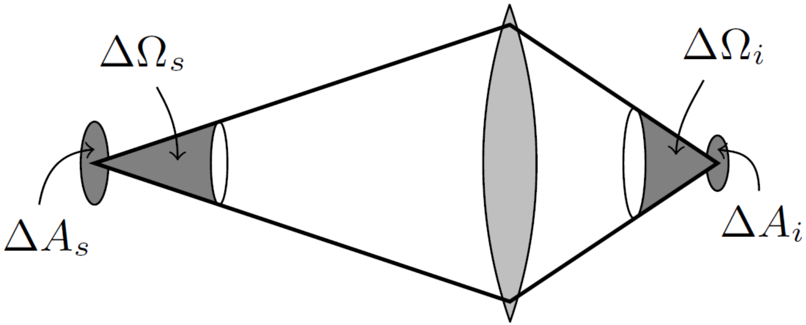
BRIGHTNESS IS A MEASURE OF SOURCE STRENGTH INDEPENDENT FROM OPTICAL MANIPULATION

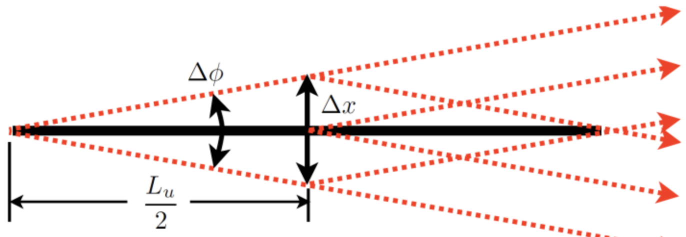
- $\Delta A_i = M \Delta A_s \, , \Delta\Omega_i = \frac{\Delta\Omega_s}{M}$

  $\therefore \Delta A_s \times \Delta\Omega_s = \Delta A_i \times \Delta\Omega_i$

→ Define $\mathcal{B} = \frac{d\mathcal{F}}{\Delta A \Delta\Omega \, d\omega} = \frac{d\mathcal{N}_{ph}}{\Delta A \Delta\Omega d\omega dt}$

- $\frac{d\mathcal{F}}{\Delta\Omega d\omega} = |\mathcal{E}(\boldsymbol{\phi})|^2$ from textbooks
- How do we compute ΔA ?
  - ΔA =electron beam size (Ken Green)?,
  - Add the depth of field effect for an extended source (undulator) ?





INTRODUCE $\mathcal{B}(x, \phi; z)$ AS PHASE SPACE DISTRIBUTION OF PHOTONS

- **Is there an expression $\mathcal{B}(x, \phi; z)$ such that**

A. $\int dx\, \mathcal{B}(x, \phi; z) = |\mathcal{E}(\phi)|^2$ (angular distribution)

B. $\int d\phi\, \mathcal{B}(x, \phi; z) = |E(x)|^2$ (spatial distribution) ; $E(x) = \frac{1}{\lambda}\int d\phi\; \mathcal{E}(\phi) e^{ik\phi x}$

- **After struggling for for several days, find the following expression will do**

$$\mathcal{B}(\boldsymbol{x}, \boldsymbol{\phi}; z) = \text{const} \int d\boldsymbol{\xi} \left\langle \mathcal{E}^*\left(\boldsymbol{\phi} + \frac{1}{2}\boldsymbol{\xi}; z\right) \mathcal{E}\left(\boldsymbol{\phi} - \frac{1}{2}\boldsymbol{\xi}; z\right)\right\rangle e^{-ik\boldsymbol{\xi}\cdot\boldsymbol{x}}$$

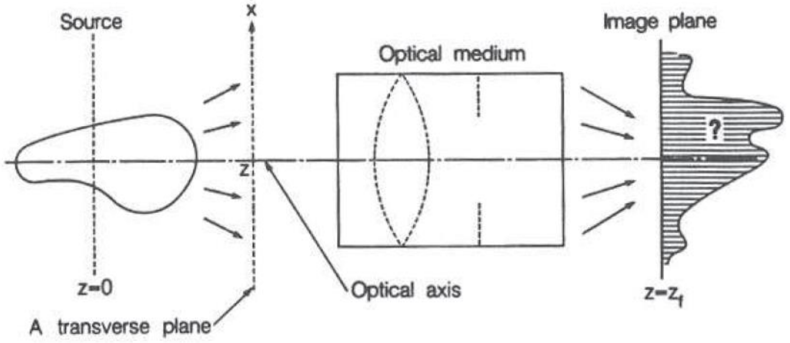
- **$\mathcal{B}(x, \phi; z)$ has further nice properties**
  - Through optical elements it behaves exactly as in geometrical optics of photons!
  - For an electron beam it is the convolution of the single electron $\mathcal{B}_e(x, \phi; z)$ with the electron distribution function $f(x, x'; z)$

- **Although $\mathcal{B}(x, \phi; z)$ can be negative (thus not a true photon distribution), it is positive if integrated over an area $\Omega \geq \lambda/2$ (minimum area)**

Excited with the discovery in 1984

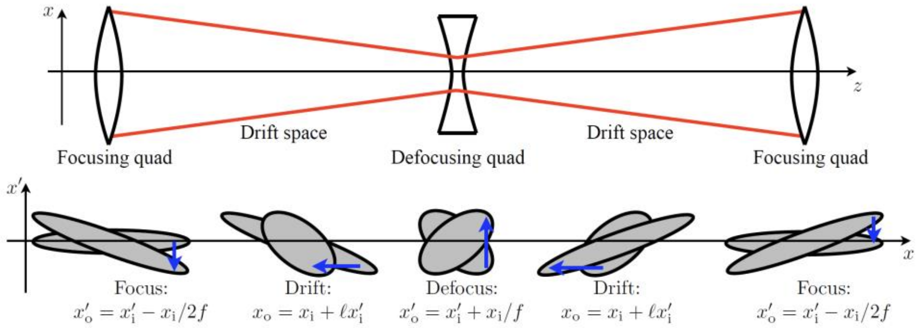
But Wigner discovered it in 1932 for quantum optics (M. Blume)!

$\mathcal{B}(x, \phi; z)$ PROVIDED THE LOGICAL BASIS FOR SR OPTICS



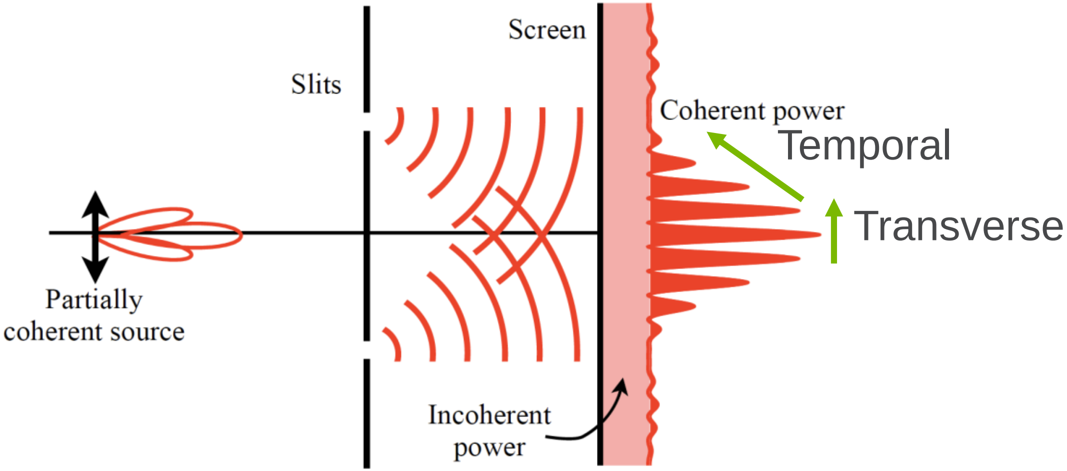
General problem of optics

Transformation of $\mathcal{B}(x, \phi; z)$



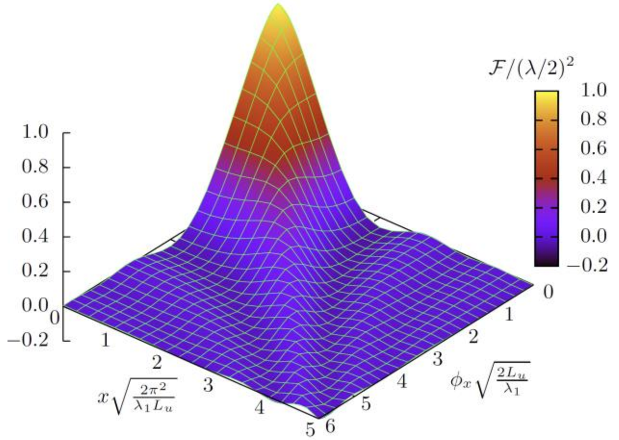
YOUNG'S INTERFERENCE FORMS WHEN THE SOURCE PHASE SPACE AREA IS NEAR MINIMUM

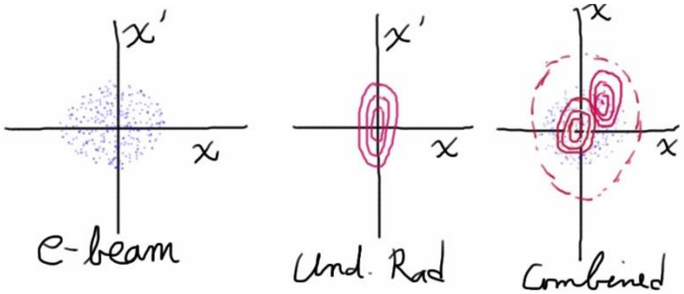
- Transverse: $\Omega_{trans} = \Delta x \Delta \phi = 2\pi \Sigma_x \Sigma_{x'} \approx \frac{\lambda}{2}$
- Temporal : $\Omega_{trans} = c\Delta t \Delta\lambda/\lambda = 2\pi c \Sigma_t \Sigma_{\Delta\lambda/\lambda} \approx \frac{\lambda}{2}$



UNDULATOR RADIATION

- Radiation from a undulator can be approximated by coherent Gaussian field with $Z_R = \frac{L_u}{2\pi}$
  - $\sigma_r = \frac{\sqrt{2\lambda L_u}}{4\pi}$, $\sigma_{r'} = \sqrt{\frac{\lambda}{2L_u}}$
- Those from electron beam are obtained by convolution
  - $\Sigma_x = \sqrt{{\sigma_x}^2 + {\sigma_r}^2} = \sqrt{\varepsilon_x \beta^* + \varepsilon_r Z_R}$,
  - $\Sigma_{x'} = \sqrt{{\sigma_{x'}}^2 + {\sigma_{r'}}^2} = \sqrt{\varepsilon_x / \beta^* + \varepsilon_r / Z_R}$
  - Matched if $\beta^* = Z_R \rightarrow \Sigma_x \Sigma_{x'} = \sqrt{{\varepsilon_x}^2 + {\varepsilon_r}^2}$
  - **Coherent (transversely) if $\boldsymbol{\varepsilon_x \leq \varepsilon_r = \frac{\lambda}{4\pi}}$**





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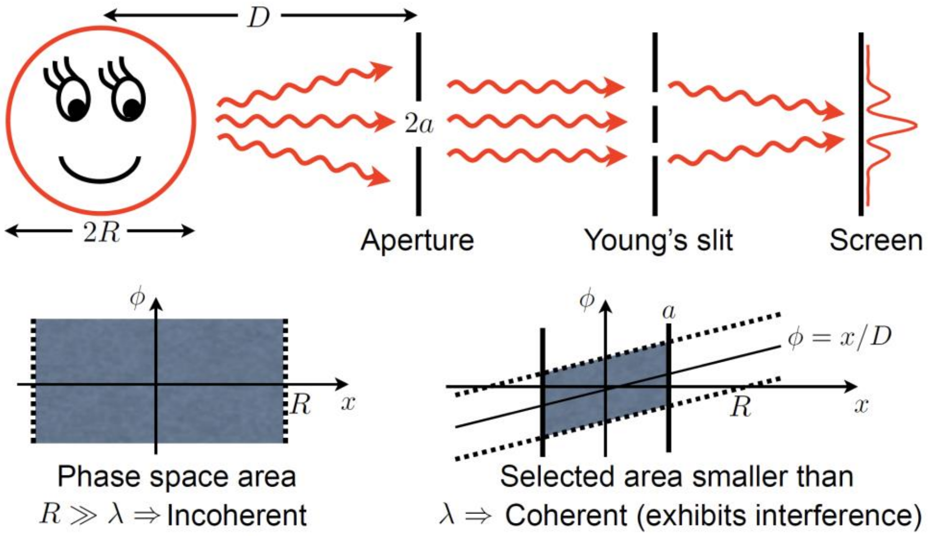
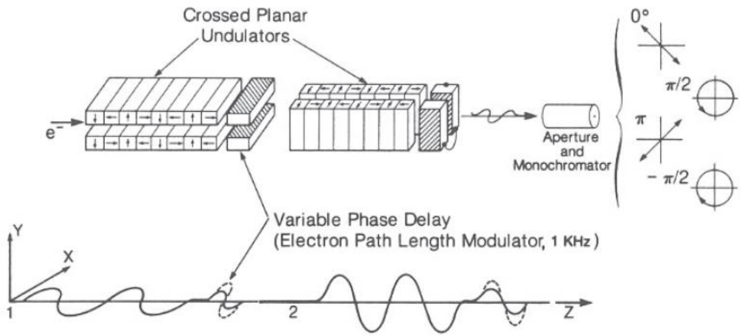
INCOHERENT SOURCE CAN BE MADE COHERENT BY PROPAGATION AND WITH AN APERTURE



- The same criteria can be obtained by requiring that that the path length from any points in the sun to any point in the aperture is less than $\frac{\lambda}{4}$

POLARIZATION COHERENCE



Kwang-Je Kim of Berkeley, after listening to the first days' discussions, has proposed an alternate scheme which, from the viewpoint of the insertion device and machine physics working group at least, is the major break through of the Workshop. His proposal is illustrated in Figure 3, and consists of a pair of

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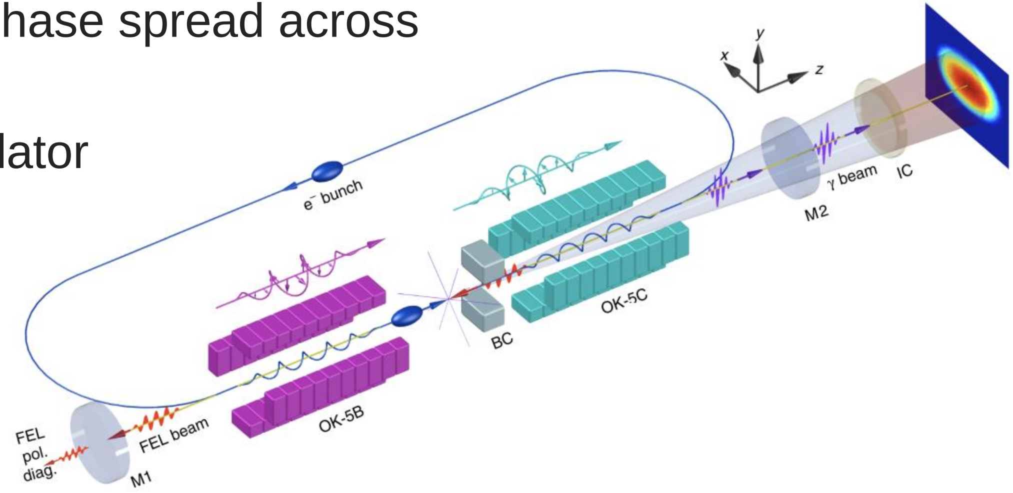
- During the1983 New Ring's Workshop at SLAC, I. Tinoco (RNA structure) gave a keynote talk on the usefulness of circularly polarized undulator radiation whose helicity can be switched.
- While listening, I recalled the Transverse Optical Klystron in which two planar undulators placed in tandem exploits the interference of two U-radiation for higher FEL gain
- **If the second undulator is rotated by 90 degree, the interference will occur in polarization →switchable circular polarization→ crossed undulator!**
  - Impromptu poster session by H. Winick for new ideas for a box of wine (one bottle finally)
  - Discovered earlier in Russia (N. M. Nikitin, Sov. Phys. J. 21, 332 (1978))!
  - Inspired other proposals for switchable undulator magnets (APPLE by Sasaki)

CROSSED UNDULATOR

- For short wavelengths, the polarization from a crossed undulator washes out due to the phase spread across electron distribution
- It should work well in an FEL oscillator



  - Linear degree of polarization of 99.7 % was observed at Duke FEL/ Gamma ray facility—Y. Wu, et. al., *Nature Photonics* **volume 13**, 629–635 (2019)
  - Should also work for the future X-ray FEL oscillator (XFELO)

TEMPORAL COHERENCE AND PHASE COHERENCE

- *A single sinusoidal wave train is **temporally coherent**:* $\Omega_t = cT\frac{\Delta\omega}{\omega} = \lambda$

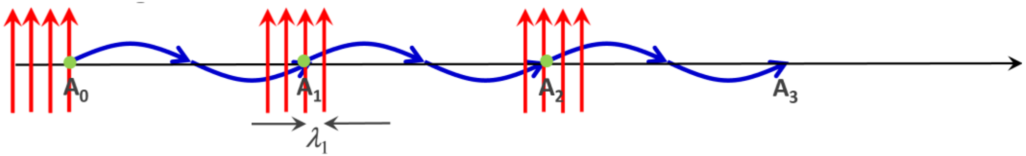
- *A collection of N_e wavetrains is **temporally coherent*** $\Omega_t = cT\frac{\Delta\omega}{\omega} \approx \lambda \; if \; T \gg \Delta T$.
  *However, it is **not phase coherent**→ intensity* $\propto N_e$

- *If $c\Delta T \ll \lambda$, then it is **phase coherent**→ intensity* $\propto N_e{}^2$

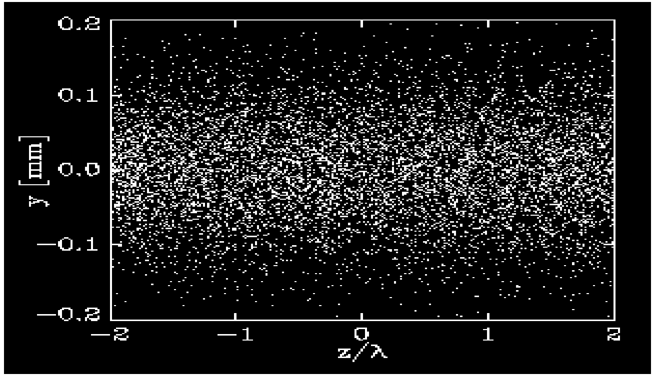
- ***This is what happens in a free-electron lasers (FELs)***

GAIN MECHANISM WITH BRIGHT E-BEAM IN AN UNDULATOR

- When the input EM wavelength = λ_1(undulator rad wavelength), an electron sees the same EM field in the successive period→ sustained energy exchange



- Energy modulation at period length λ_1 → density modulation→ enhanced radiation→ FEL gain



John Madey
1943-2016

1D SELF-AMPLIFIED SPONTANEOUS EMISSION (SASE)

- Pendulum equations for electron motion and Maxwell equation (W. Colson, thesis at Stanford U)
- Bonifacio, Narducci, and Pellegrini simplified the FEL system as an interaction between three collective variables and showed that the EM field can grow exponentially in a long undulator--SASE
  - Feasibility of X-ray FEL without mirror
- Struggled to develop a theory smoothly joining initial undulator radiation and SASE (competition with BNL)
- The FEL interaction can be studied by borrowing the Vlasov-Maxwell equation from plasma physics (learned from auditing Berkely class and reading Ichimaru)
  - $f(\theta,\eta;z) = V(\eta)+ \tilde{f}(\theta,\eta;z)$
  - $V(\eta)$=coarse-grained background distribution, $\tilde{f}(\theta,\eta;z)$ contains discreteness and E are regarded perturbation
  - Solve the initial value problem by the Laplace transform
- **Key insight on one lonely day in 1986: Klimontovich distribution for point particles (1985)**
  - $\boldsymbol{f(\theta,\eta;z) = \sum_j \delta\left(\theta - \theta_j(z)\right) \delta\left(\eta - \eta_j(z)\right)}$
  - Still regarding $\tilde{f}(\theta,\eta;z)$ and E as perturbation
- The spectral power evolution could be written as

$$\frac{dP}{d\omega} = g_A e^{z/L_G} \exp\left[-\frac{(\omega-\omega_m)^2}{2(\omega_m\sigma_\nu)^2}\right]\left(\left.\frac{dP}{d\omega}\right|_0 + g_S\frac{\rho\gamma mc^2}{2\pi}\right)\ ;\ \sigma_\nu \approx \sqrt{\frac{0.8\,\rho}{z/\lambda_u}}$$

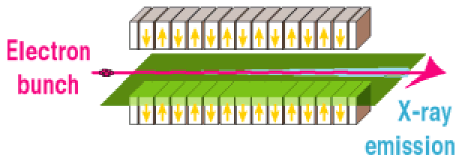
3D FEL THEORY

- **Pendulum equations include electrons' betatron motion and Maxwell equation include diffraction (1986)**
  - In electron phase equation $\frac{d\theta}{dz} = 2k_u\eta - \frac{k}{2}\left(\boldsymbol{p}^2 + k_\beta{}^2\boldsymbol{x}^2\right)$, the missing red term was later corrected by Yu and Krinsky
- **High-gain solutions**
  - Variational solution (L. H. Yu and S. Krinsky)
  - Generalize Van-Kempen's non-Hermitian eigenvalue problem
    - Optical guiding due to the dominant eigen mode
  - **Agreement of the "analytic" solution with simulation enhanced the theory and the simulation, became an important factor for laboratories to consider seriously the construction of SASE FEL**
  - Ming Xie's fitting formulae greatly expedited the parameter design of an SASE FEL
- **Low-gain solution (1992)**
  - An iterative solution gave, for the first time, a precise formula for gain, incorporating diffraction and betatron motion (handled previously with "filling factors")
  - Madey's theorem generalized to 3D involving a convolution of brightness functions $\mathcal{B}_E$ and $\mathcal{B}_U$, and $\bar{F}$:

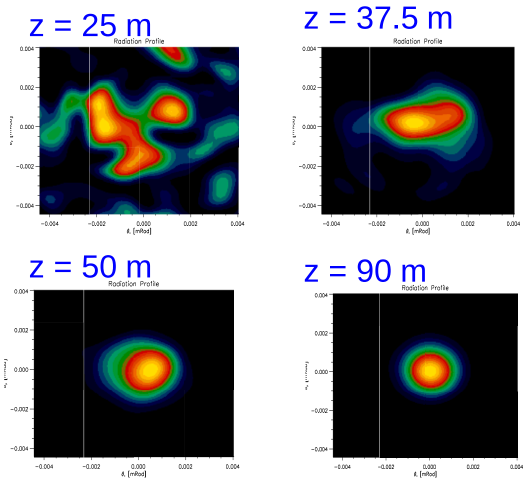
$$G = \frac{n_e \kappa_h \chi_h}{\lambda^2} \frac{\int d\eta d\boldsymbol{p} d\boldsymbol{\phi} d\boldsymbol{x} d\boldsymbol{y}\ \mathcal{B}_E(\boldsymbol{y},\boldsymbol{\phi})\mathcal{B}_U(\eta, \boldsymbol{x}-\boldsymbol{y}, \boldsymbol{\phi}-\boldsymbol{p})\frac{\partial}{\partial\eta}\bar{F}(\eta,\boldsymbol{x},\boldsymbol{p})}{\int d\boldsymbol{\phi} d\boldsymbol{y}\ \mathcal{B}_E(\boldsymbol{y},\boldsymbol{\phi})}$$

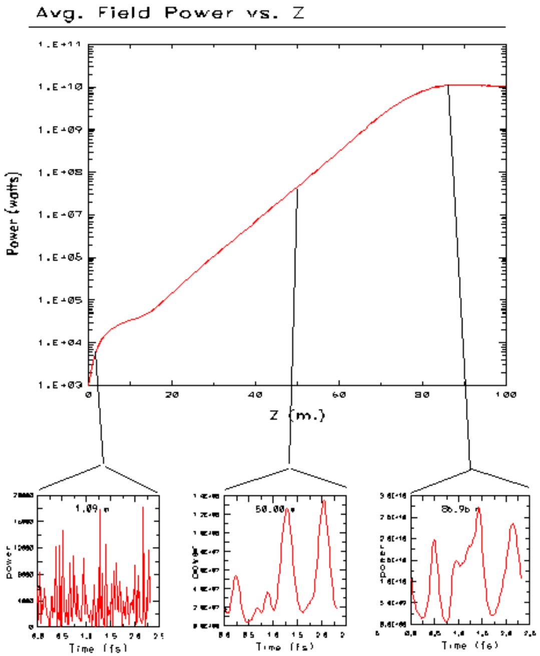
  - **Became the basis of an X-ray FEL oscillator**

A HIGH-GAIN FEL (G~10^6) PRODUCES SELF-AMPLIFIED-SPONTANEOUS EMISSION (SASE)→ ROUTE TO FEL FOR X-RAYS FOR WHICH MIRRORS ARE NOT AVAILABLE



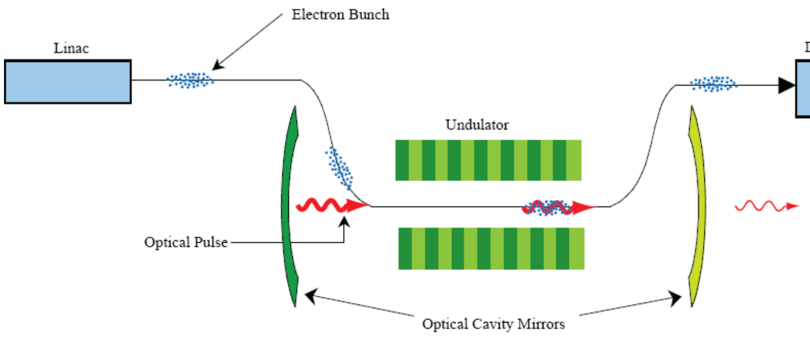
Transverse mode

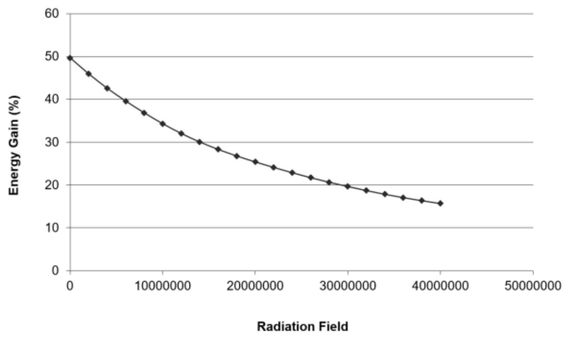




Transversely coherent. Temporally, random collection of short phase-coherent regions

LOW-GAIN FEL OSCILLATOR PRINCIPLES





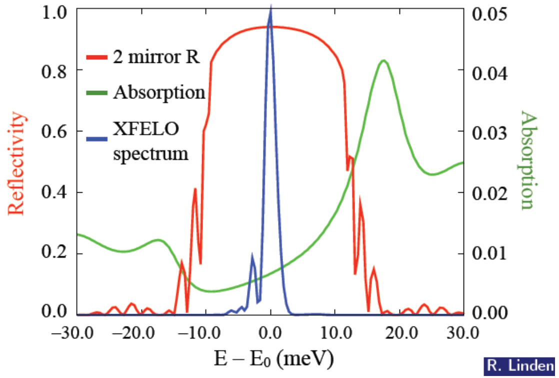
- Synchronism: The spacing between e bunches =cavity round trip time/n
- Lasing starts if: $(1+G_0)\ R_1\ R_2 > 1$ ($R_{1,2}$: mirror reflectivity), G_0= initial gain

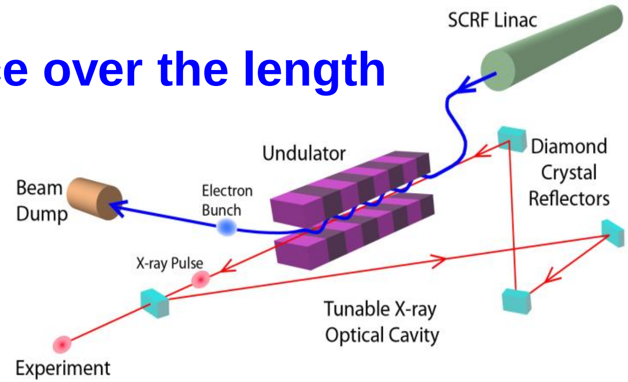
→Need high-reflectivity, normal-incidence mirrors

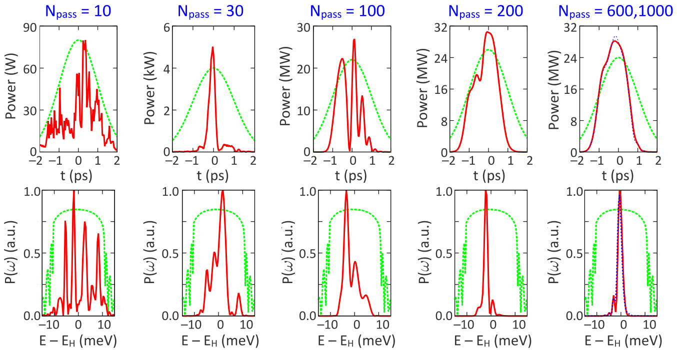
- The gain G decreases as the intra-cavity power increases. A steady state ("saturation") is reached when $(1+G_{sat})\ R_1\ R_2 = 1$
- FELOs have been built for IR, visible, and UV wavelengths

X-RAY FREE-ELECTRON LASER OSCILLATOR (XFELO) CAN PROVIDE FULL COHERENCE AND STABILITY

- Bragg reflectivity of crystals (synthetic diamond) high is high over narrow bandwidth
- Full coherence (transverse, phase-coherence over the length of the pulse)

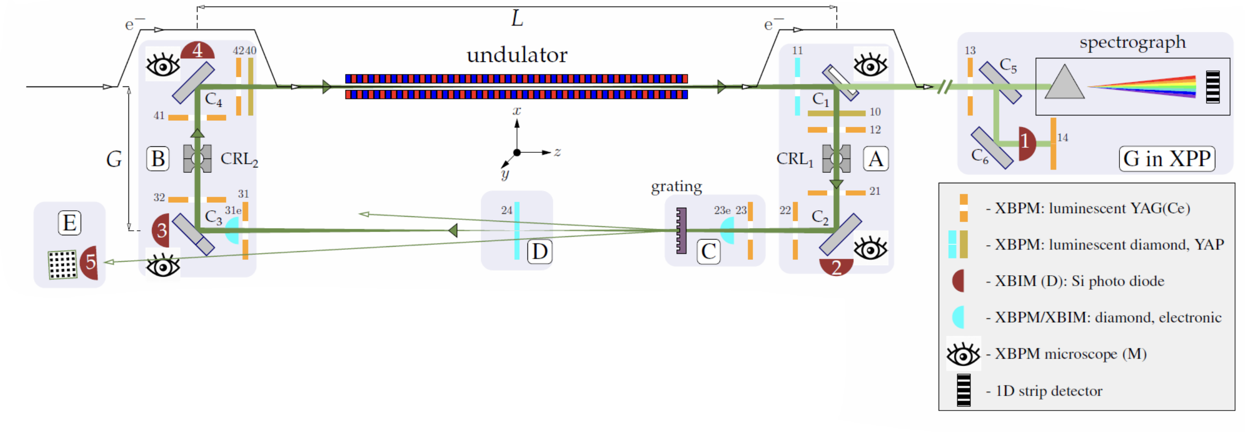


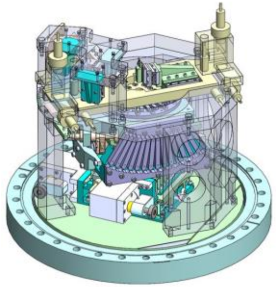


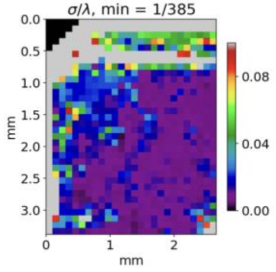


- R. Collela and A. Luccio (1983)
- Revived by KJK, Y. Shvyd’ko, S. Reiche (2008)
- Bragg reflectors for hard x-rays (5 –25 keV)

CAVITY-BASED XFEL PROJECT TO TEST THE XFELO PRINCIPLES





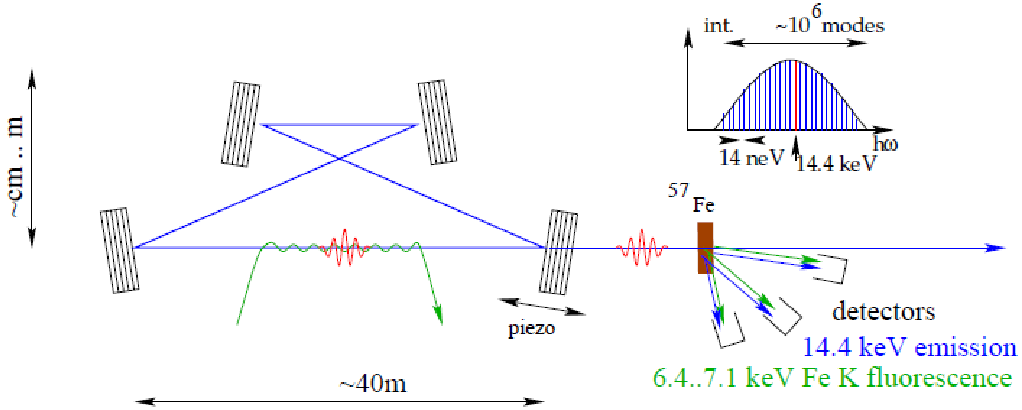


- Construct a rectangular cavity enclosing 7 LCLS II undulator sections
- Produce double bunches separated by cavity roundtrip time from the copper linac
- Measure single pass gain and ring down
- To be completed in FY 2024

ULTIMATE COHERENCE: X-RAY COMB

(B.W. ADAMS AND KJK, 2015)

- **The XFEL-O output pulses are copies of the same circulating intra-cavity pulse → By stabilizing cavity RT time to less than 0.01λ/c, the spectrum of XFELO output becomes a comb**
- **The extreme-stabilized XFELO will**
  - **establish an x-ray-based length standard**
  - **Revolutionize the quantum optics of nuclear states**
  - **have applications in fundamental physics such as x-ray Ramsey interferometer to probe quantum gravity, etc.**



I LEARNED FROM YUAN T. LEE THAT

- Kwang (광, 光)--light
- Je (제， 齊)– ordered
- **Kwang-Je → *Coherent Radiation***