

Beam Position Monitors

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Why do we need the **B**eam **P**osition **M**onitors?

- Not knowing the position of the beam is like driving blindfolded
- Beam position is essential information for beam quality assurance

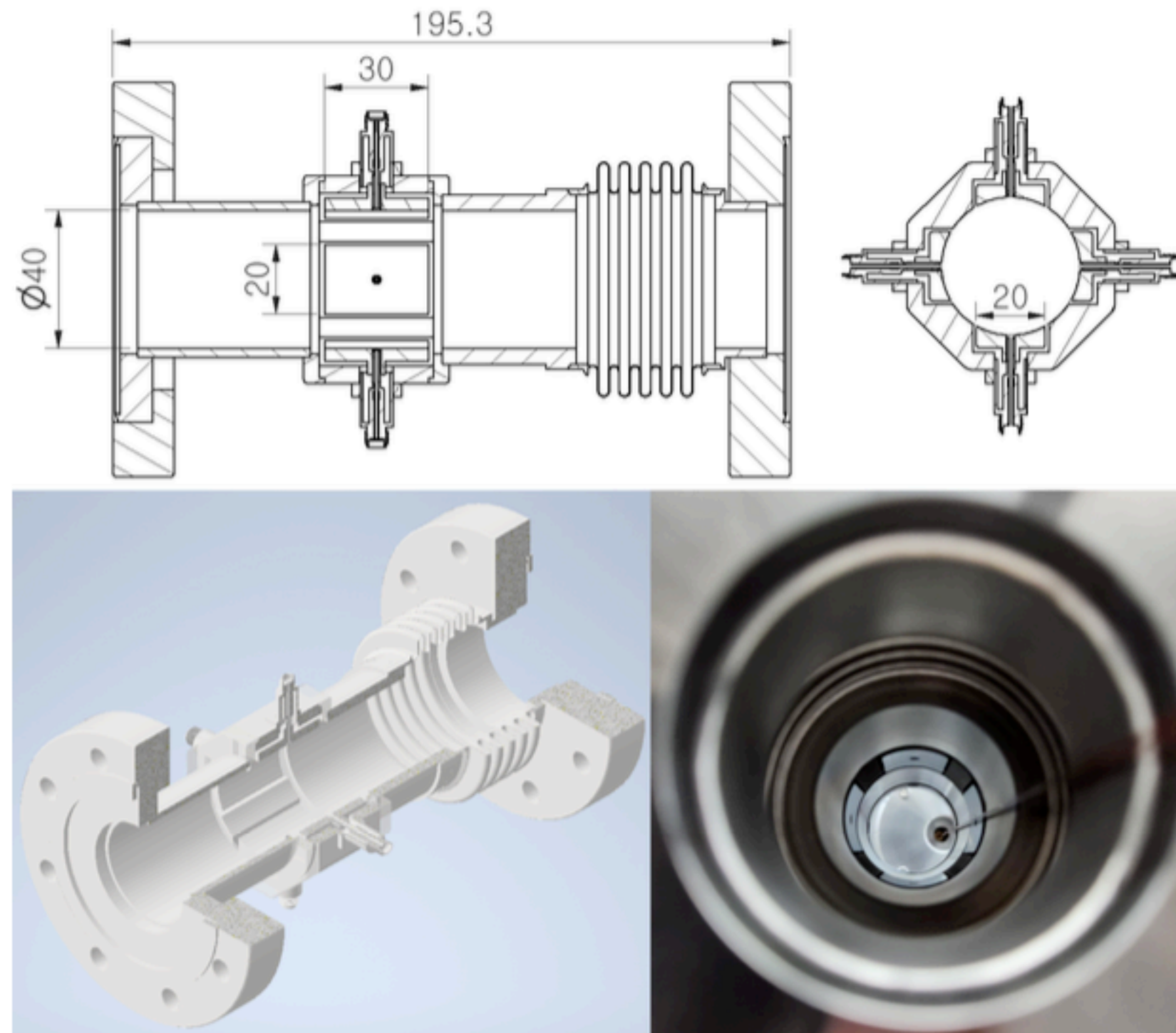


ChatGPT

F1

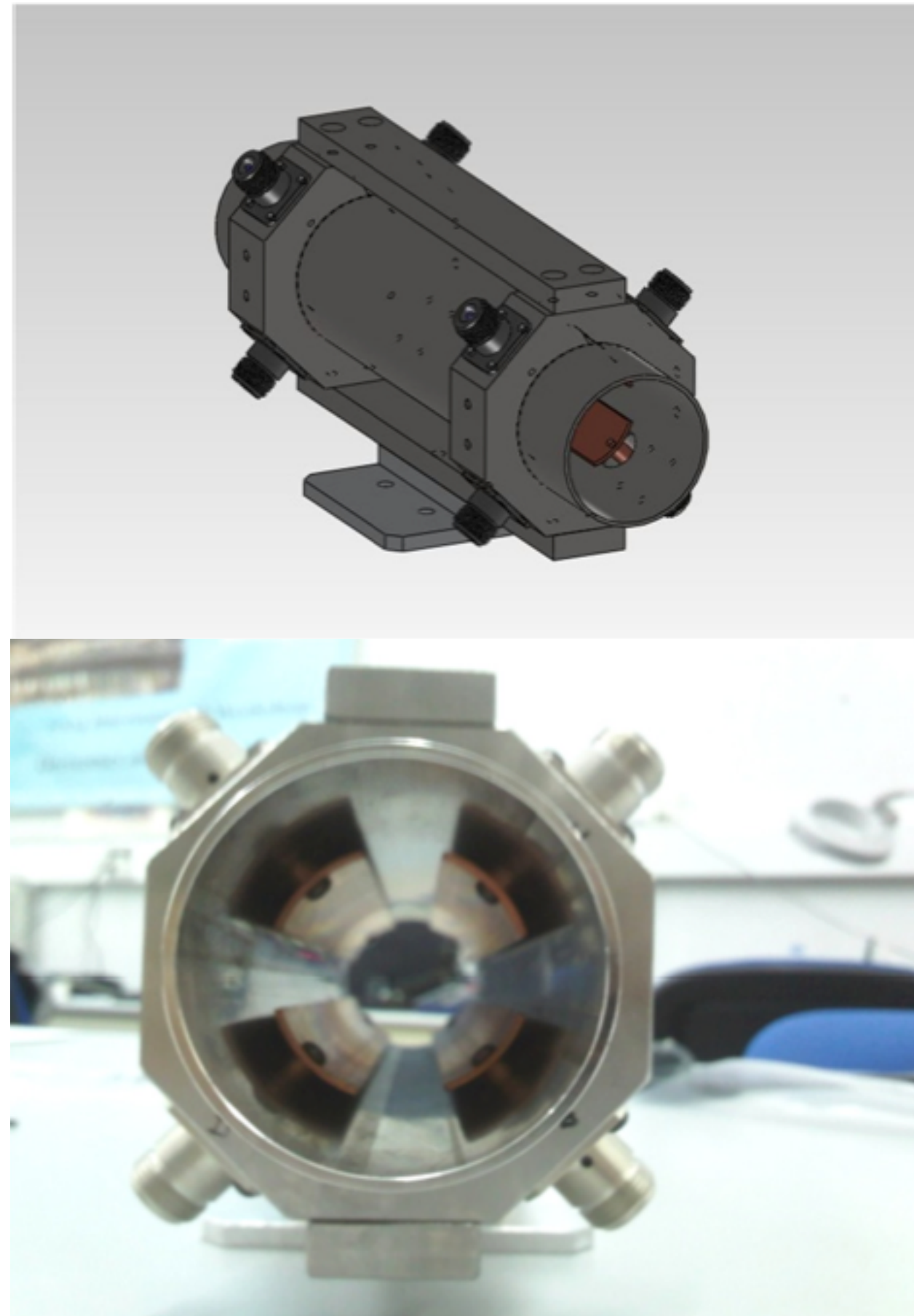
Types of BPMs

- Most popular types: Button, Stripline, and Cavity BPMs



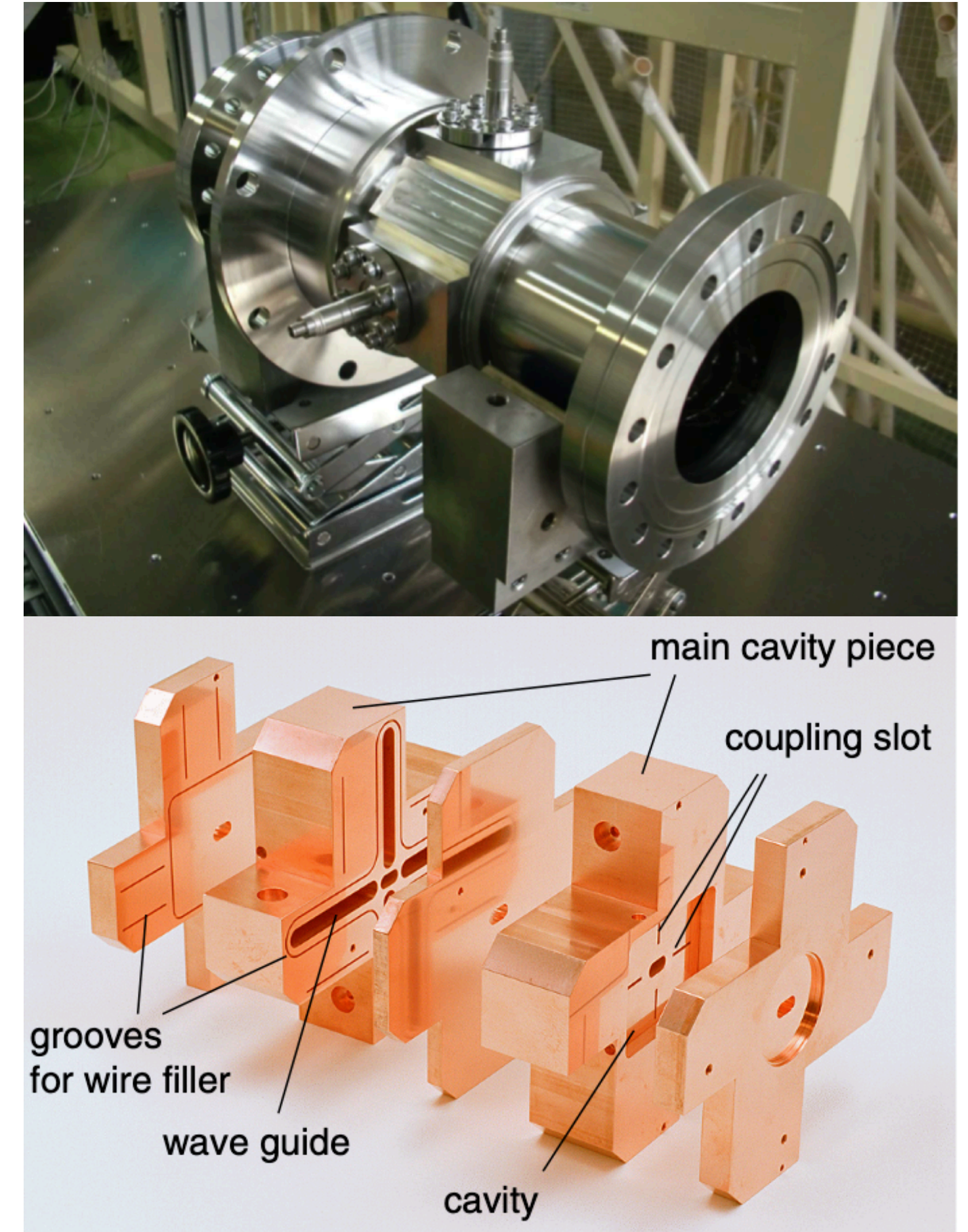
Button BPM

JINST 19, T06006 (2024)



Stripline BPM

IBIC2012, MOPA31 (2012)



Cavity BPM

ILC TDR and PhysRevSTAB 11, 062801 (2008)

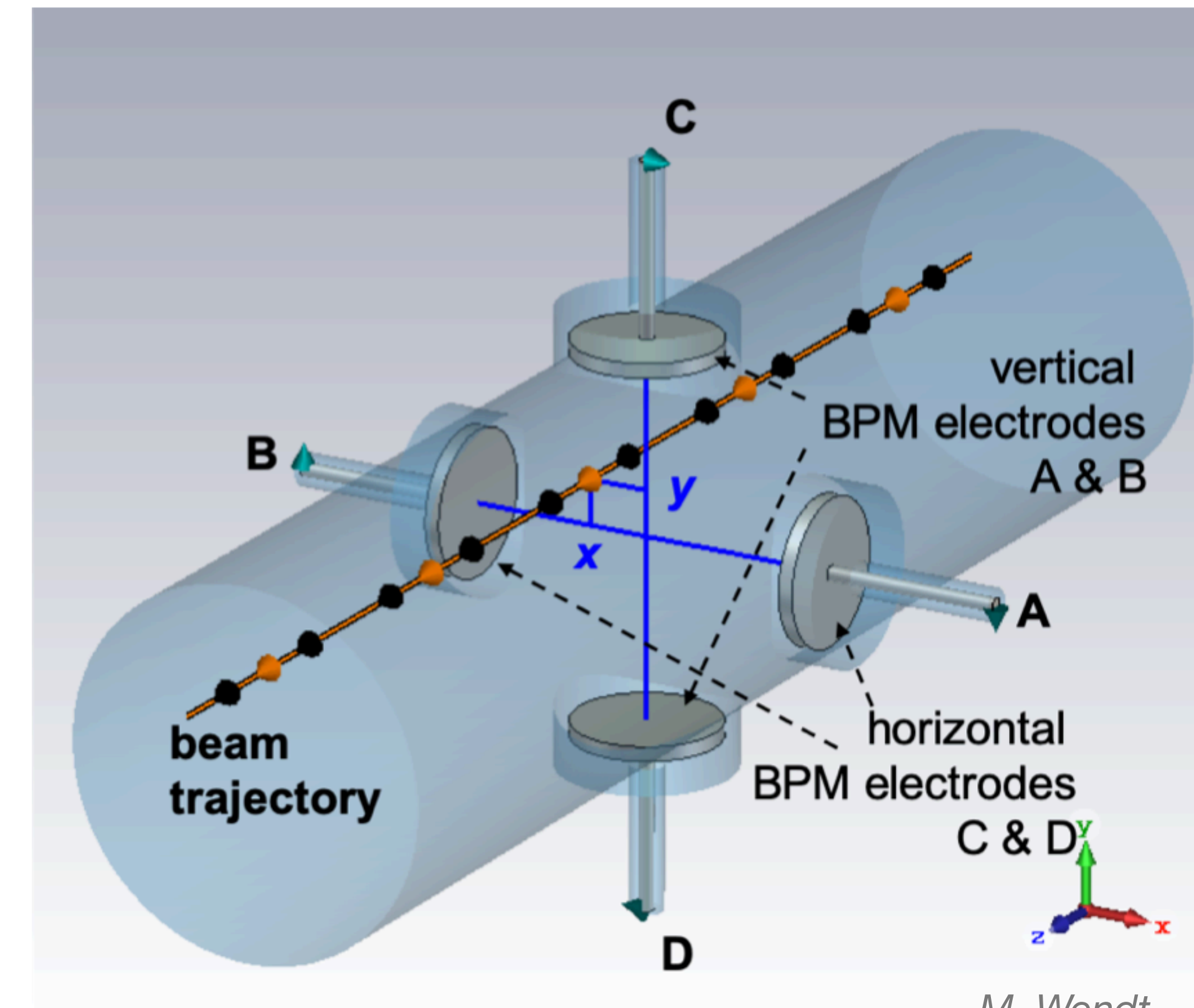
Types of BPMs

- Most popular types: Button, Stripline, and Cavity BPMs

	Button BPM	Stripline BPM	Cavity BPM
Coupling to the beam	Capacitive	Electromagnetic	Resonant
Signal shape	Bipolar differentiated pulse	Doublet pulse pair	Decaying sinusoid (ring-down)
Transfer impedance	$\sim 0.5 - 2 \Omega$	$\sim 1 - 5 \Omega$	$R_s \sim \text{k}\Omega/\text{mm}$
Frequency response	Broadband	Broadband	Narrowband
Typical resolution	10 - 50 μm (single bunch) $\sim 1 \mu\text{m}$ (multi-turn average)	1 - 10 μm ($< 1 \mu\text{m}$ with optimization)	100 nm - 1 μm (typical) Best: 8.7 nm (KEK ATF2)
Beam coupling impedance	Low	Medium	High
Electrode size	Small	Medium	Large
Optimal bunch length	Short bunches	Long bunches	Short bunches
Beam type	Electron beam	Proton and Ion beams	Electron beam
DC/debunched beam	No	No	No
Primary use cases	Storage rings, damping rings, collider rings	Proton linacs, Ion accelerators, hadron synchrotrons	FEL undulator sections, high-precision linac sections, IP

Principle of Operation

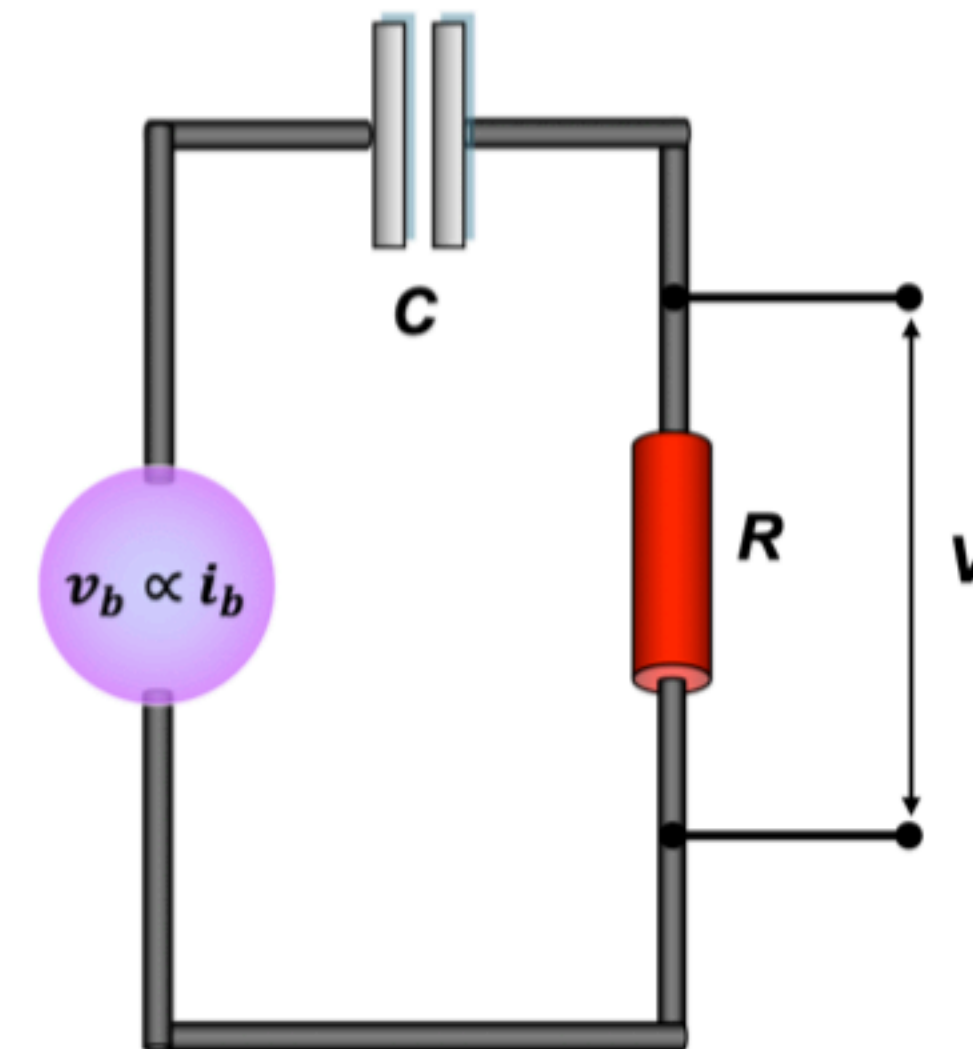
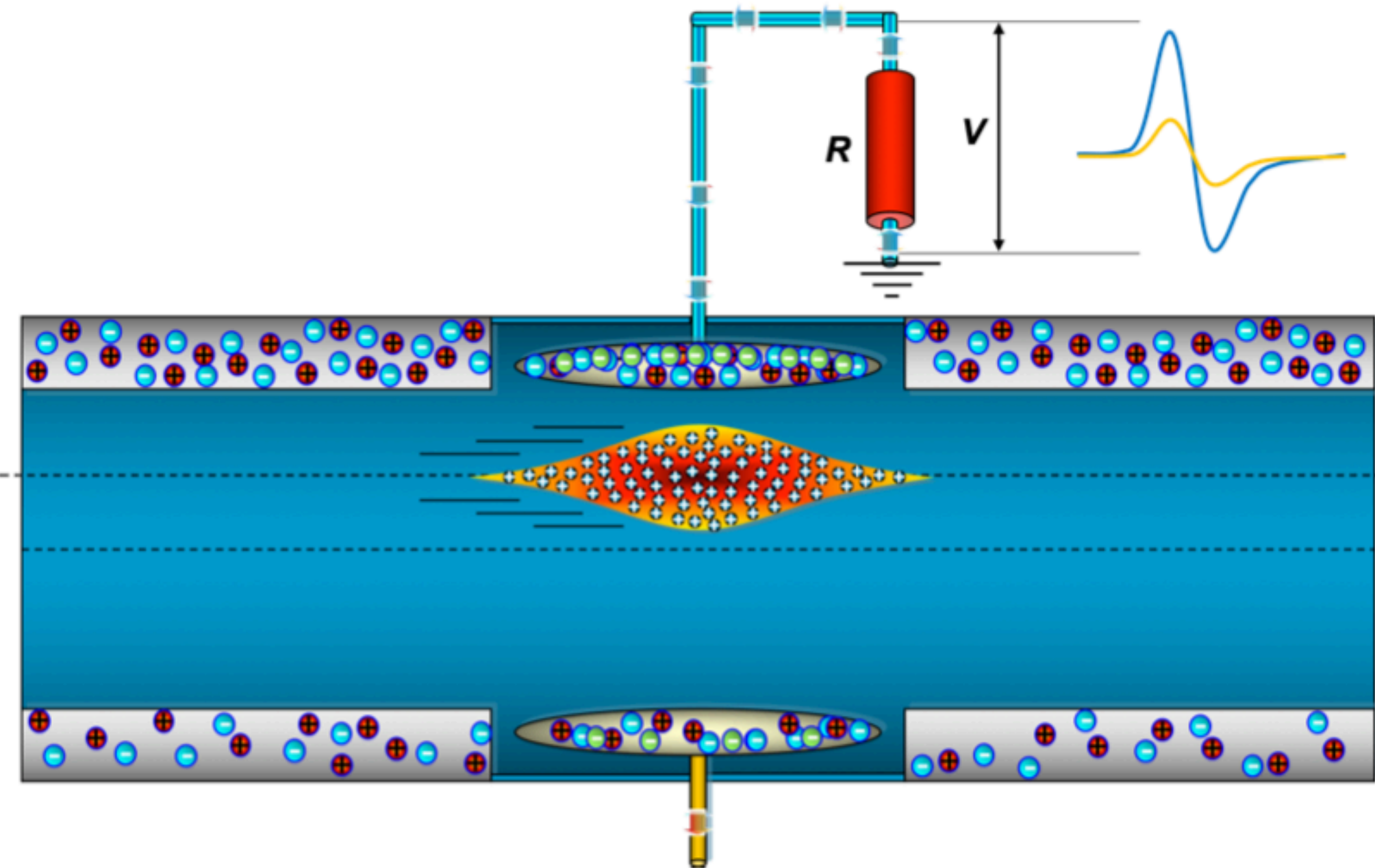
- Coupling to the electromagnetic field of the beam
 - Beam is a current → accompanied by both a magnetic and an electric field
- Symmetrically arranged a pair of electrode
 - e.g. “button” BPM: round, metallic, coin-like electrodes
 - Each pair measures the beam position along each axis
 - Horizontal and vertical offset of the beam trajectory with respect to the center of the vacuum chamber
- Combining with a readout electronics, it should monitor the beam with high resolution, accuracy, and repeatability



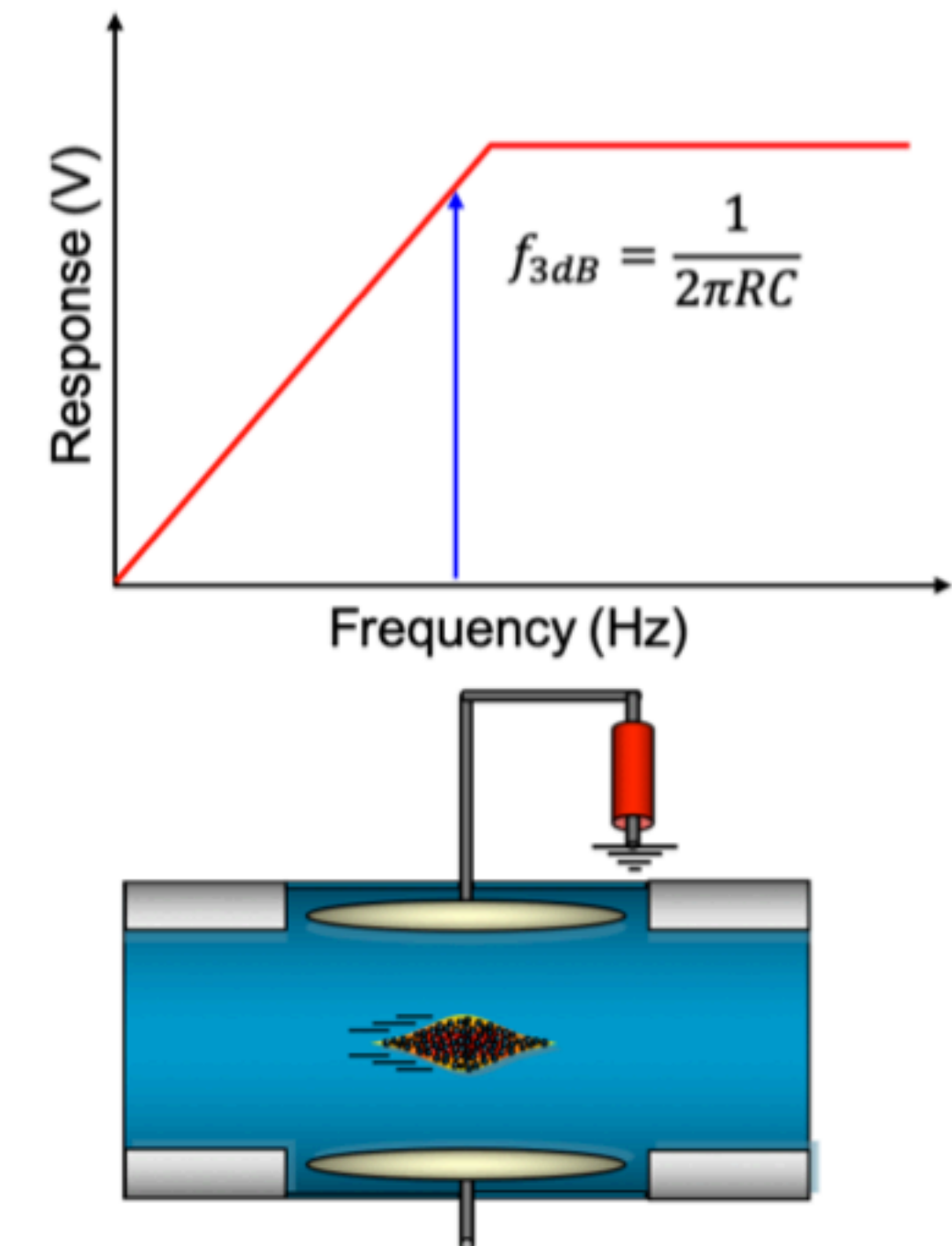
M. Wendt

Principle of Operation

- Broadband BPM pickups work with the image current (or image charge) by the charged particle beam
 - Equivalent to a simple 1st-order high-pass filter → “Broadband”
 - No “DC-coupling” → BPM doesn’t work with debunched, DC beams



O. R. Jones



Principle of Operation

- Same waveform from the electrode pair, different signal strength

$$v_{\text{elec}} \propto (\text{position}) \times (\text{intensity})$$

- Taking the difference of signals from opposing electrodes

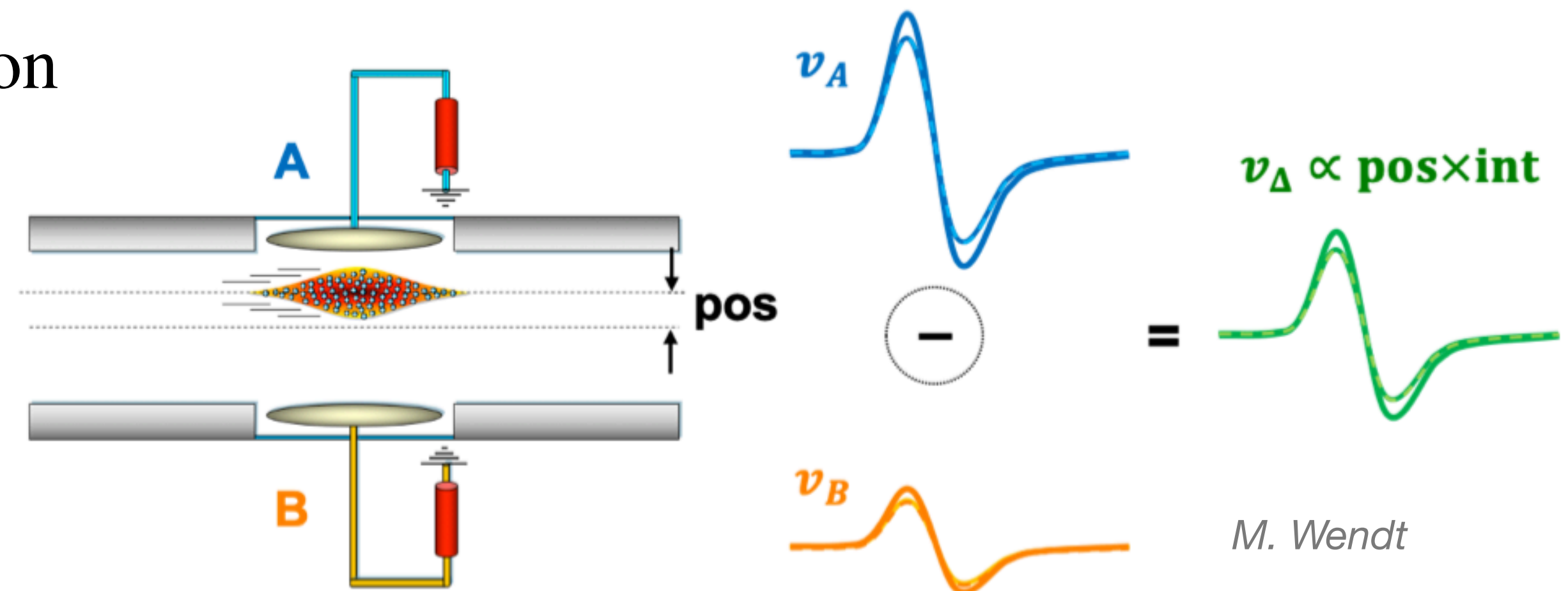
$$v_{\Delta} \equiv (v_A - v_B) \propto (\text{position}) \times (\text{intensity})$$

- Normalizing the difference by the beam intensity (v_{int})

$$\frac{v_{\Delta}}{v_{\text{int}}} \propto \frac{(\text{position}) \times (\text{intensity})}{\text{intensity}} = \text{position}$$

- Typically, $v_{\Sigma} \equiv v_{\text{int}} = v_A + v_B$, thus,

$$\frac{v_{\Delta}}{v_{\Sigma}} = \frac{v_A - v_B}{v_A + v_B} \propto \text{position}$$



BPM Pickup Signal

- Passive and linear electromagnetic coupler

Transfer impedance of a BPM pickup electrode

$$V_{\text{elec}}(x, y, \omega) = s(x, y) Z(\omega) I_b(\omega)$$

Sensitivity function (position information)

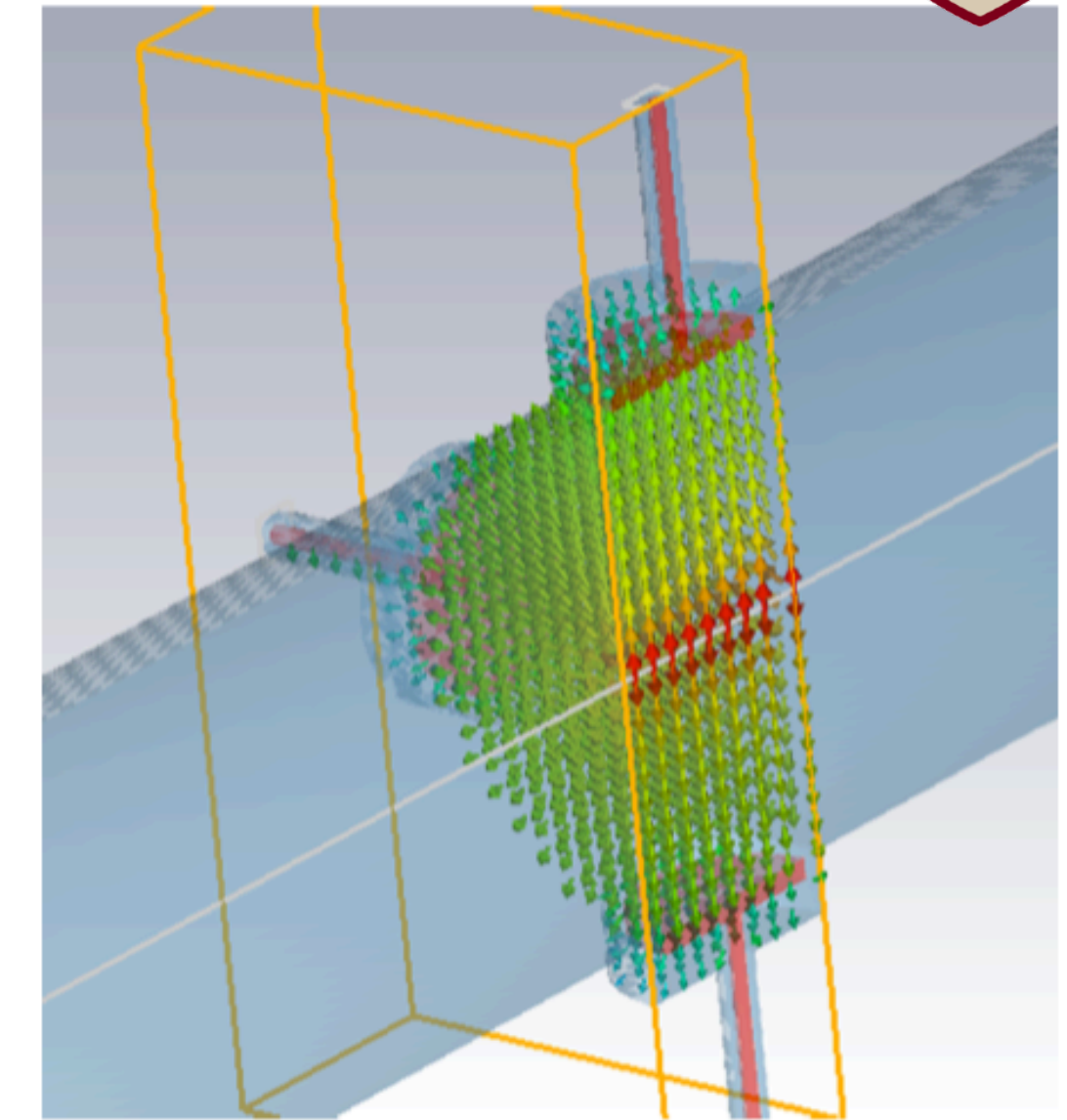
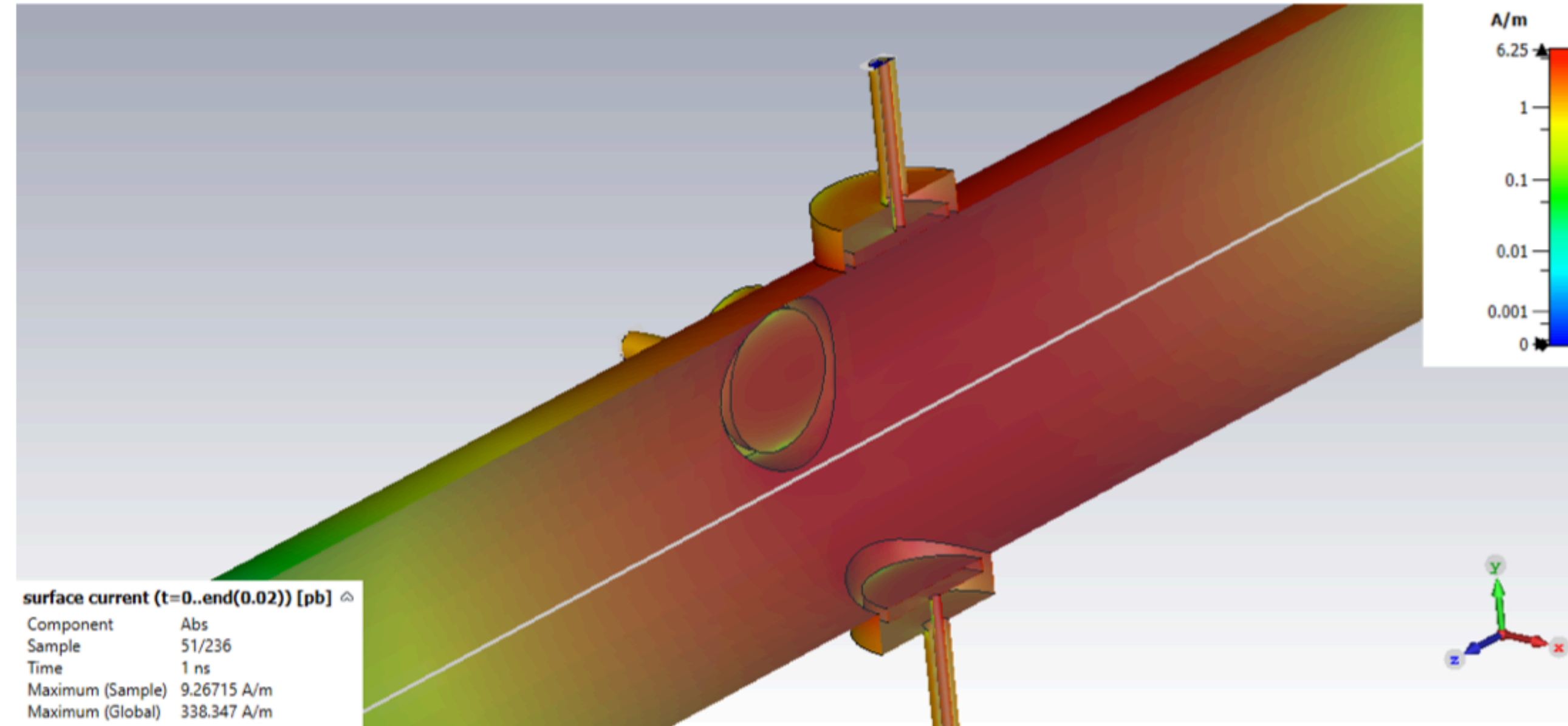
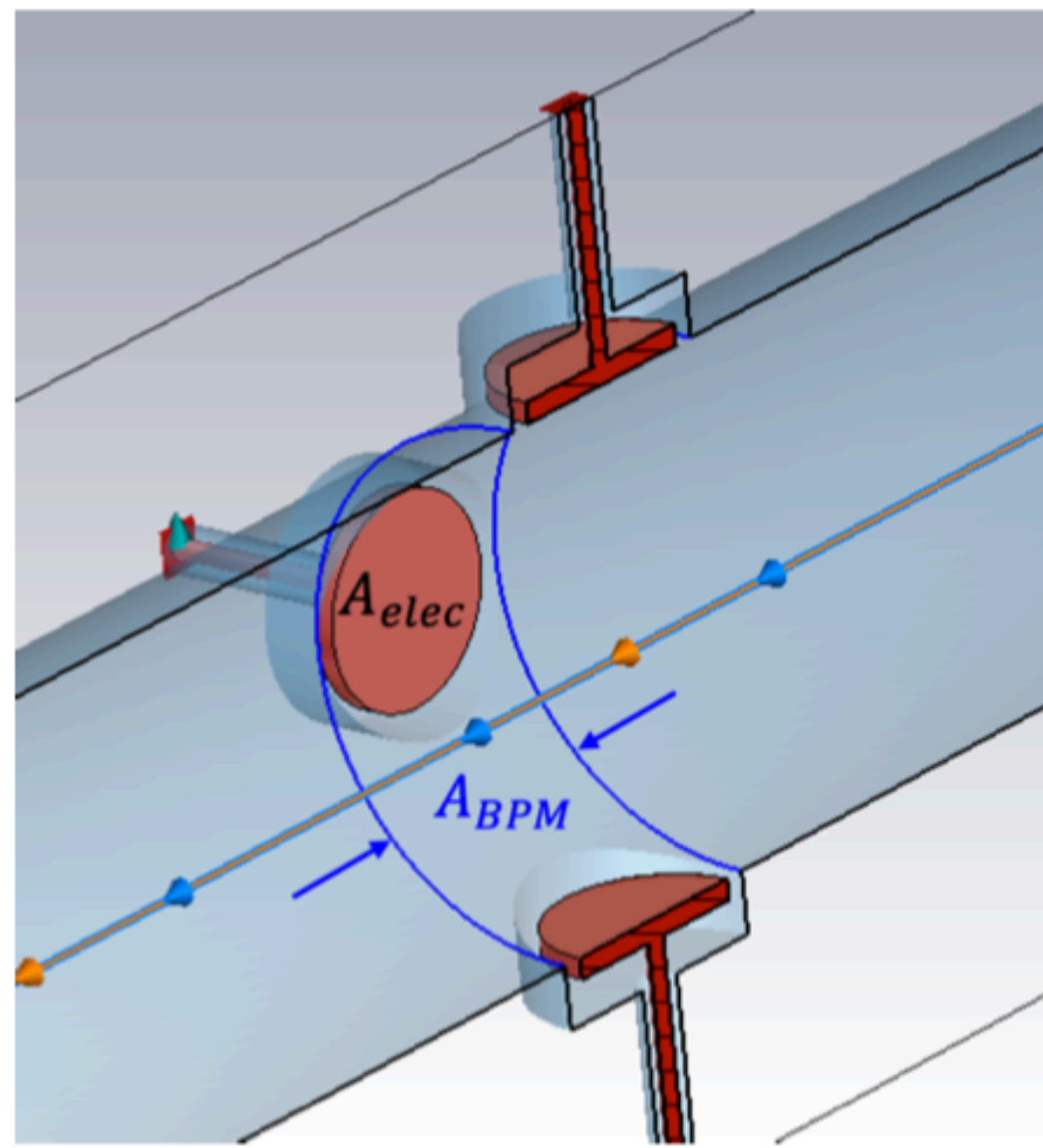
Beam (or bunch) current

- The beam position is only encapsulated in $s(x, y)$
- Transfer impedance and beam current are the response with respect to the frequency
- Transfer impedance also includes an electromagnetic coverage factor:

$$\phi = \frac{\int J_{\omega} dA_{\text{elec}}}{\int J_{\omega} dA_{\text{BPM}}}$$

- The ratio of the integrated wall current on the electrode to the whole surface of BPM region

Electromagnetic Coverage Factor



M. Wendt

- ▶ **Transfer impedance** also includes an electromagnetic coverage factor:

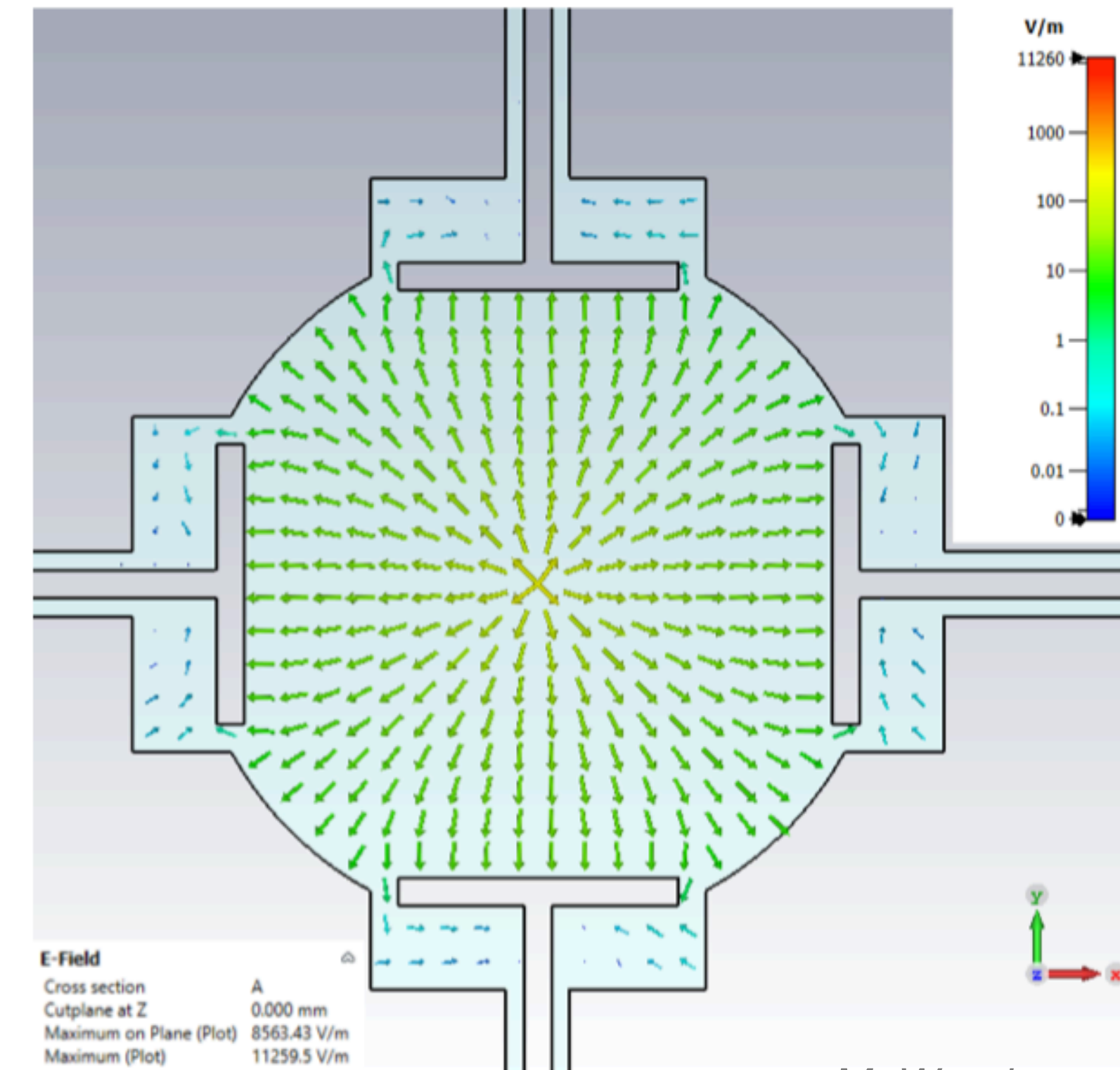
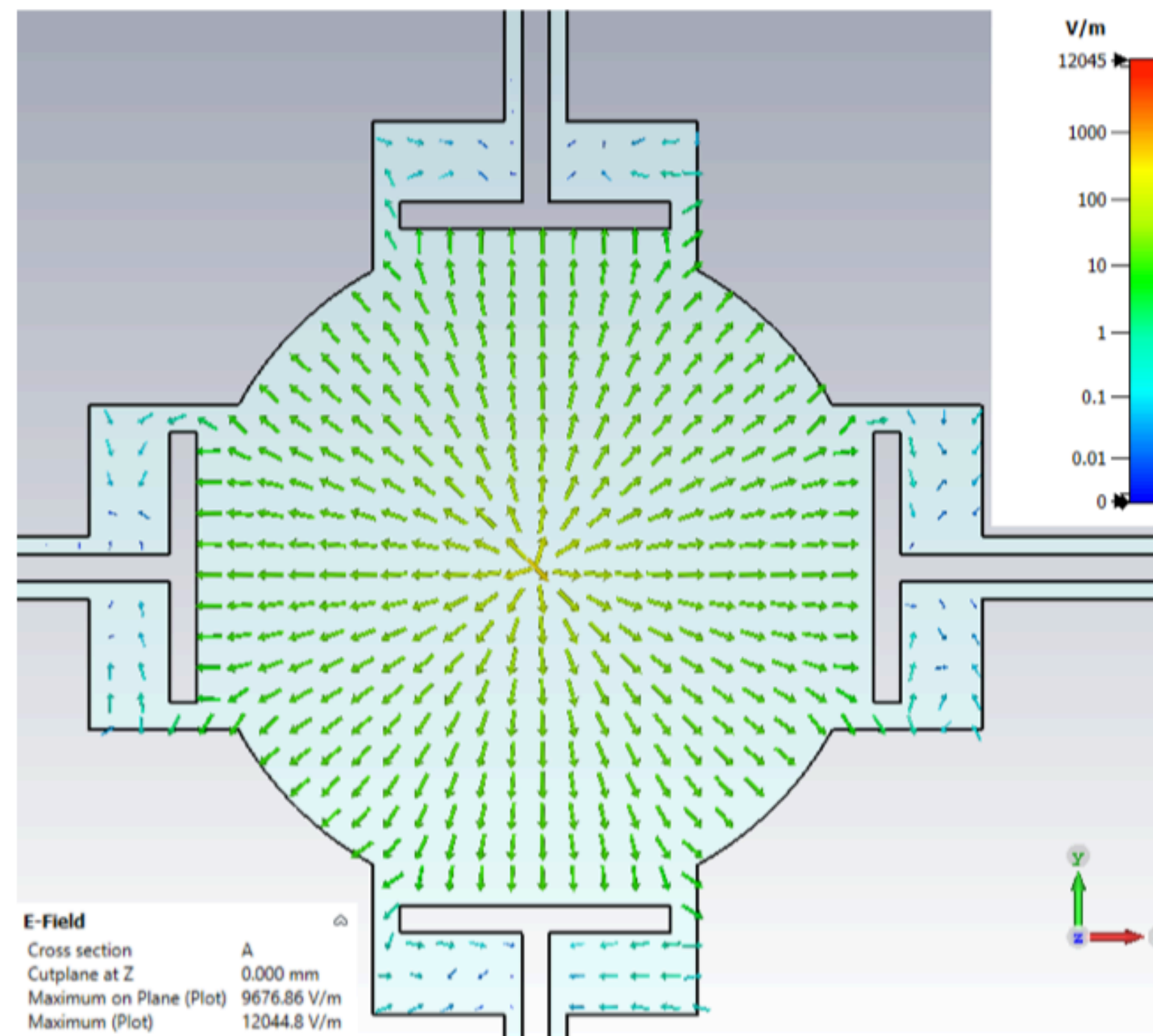
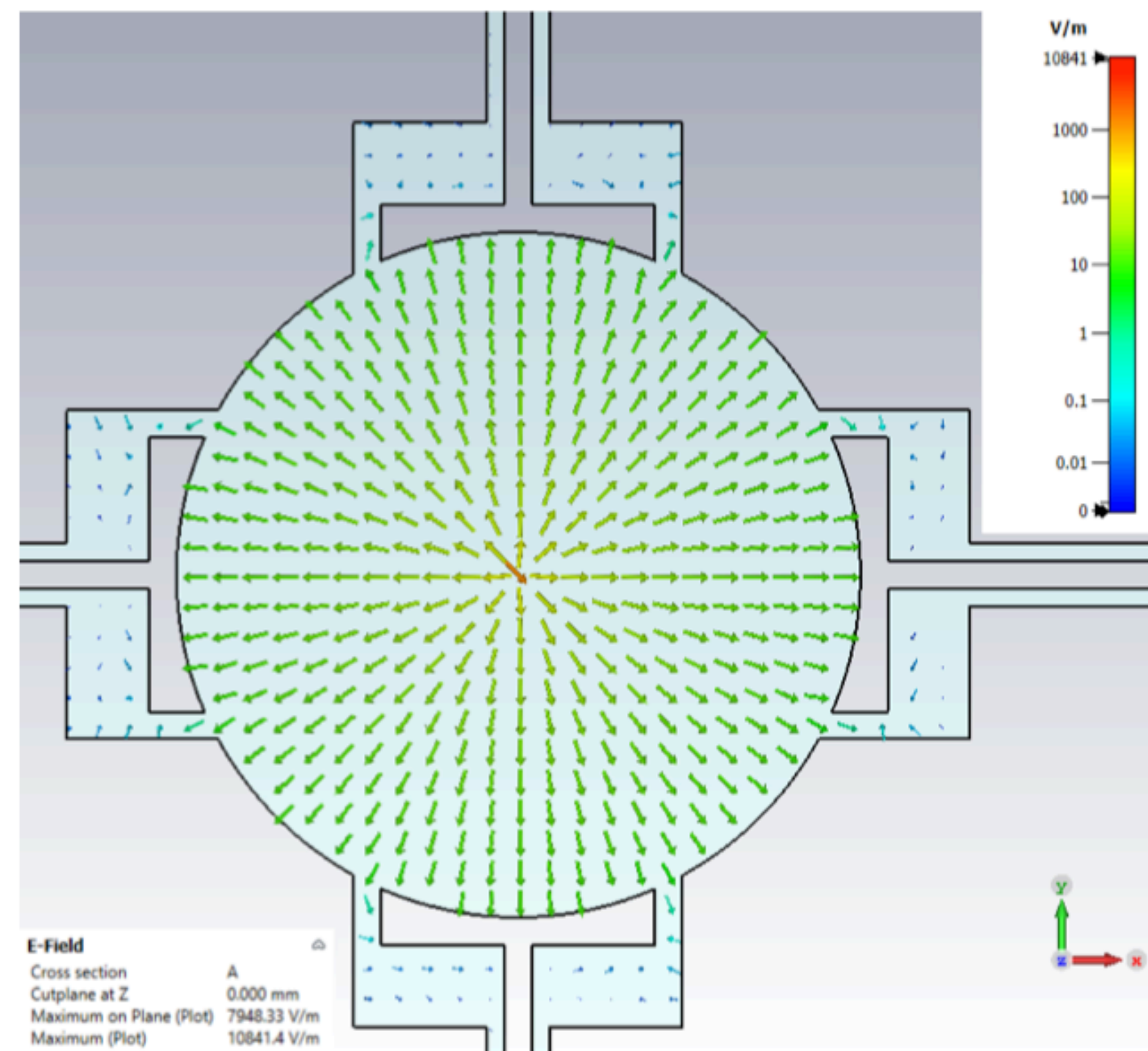
$$\phi = \frac{\int J_{\omega} dA_{\text{elec}}}{\int J_{\omega} dA_{\text{BPM}}}$$

- The ratio of the integrated wall current on the electrode to the whole surface of BPM region

Electromagnetic Coverage Factor

- Electromagnetic coverage is **NOT** identical with geometric coverage

$$\frac{\int J_{\omega} dA_{\text{elec}}}{\int J_{\omega} dA_{\text{BPM}}} \neq \frac{A_{\text{elec}}}{A_{\text{BPM}}}$$



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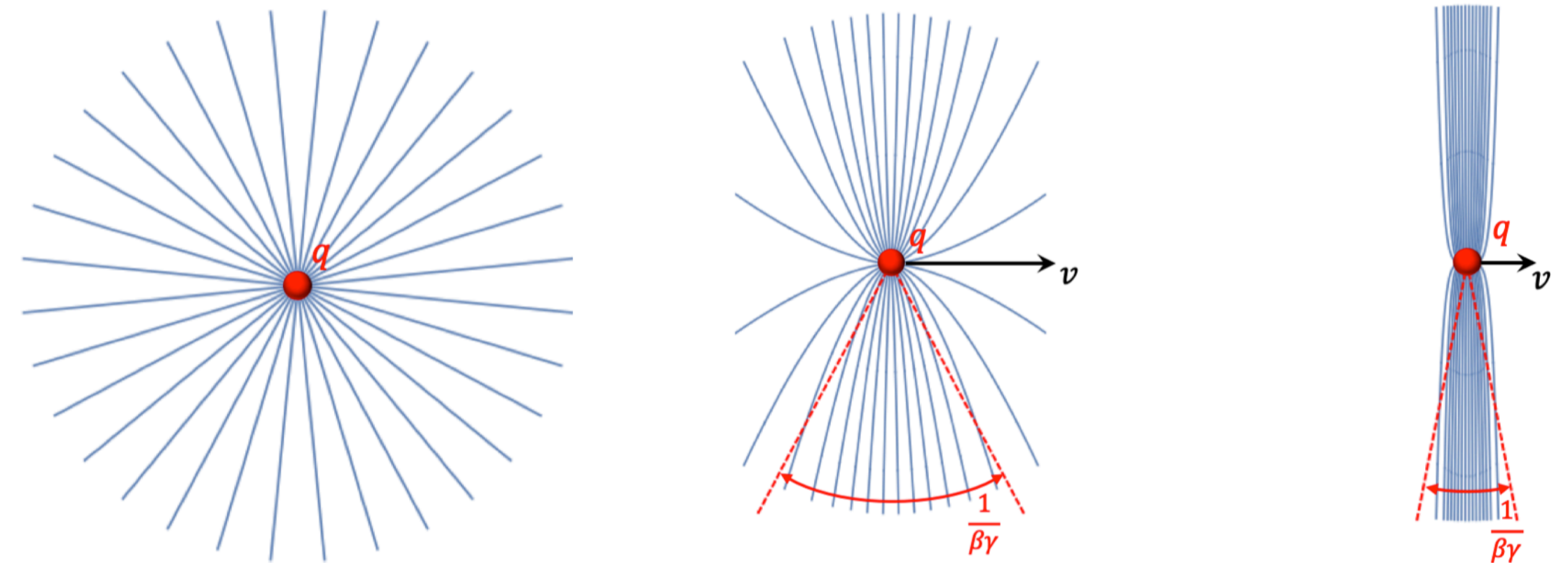
Image Charges

- If a point charge is relativistic ($v = \beta c$), from the Lorentz transformation,

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\vec{r}}{r^3} \rightarrow \frac{q}{4\pi\epsilon_0} \frac{\gamma}{(x^2 + y^2 + z^2)^{3/2}} \vec{r}$$

$$\text{with } |\vec{r}| = \sqrt{x^2 + y^2 + z^2} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

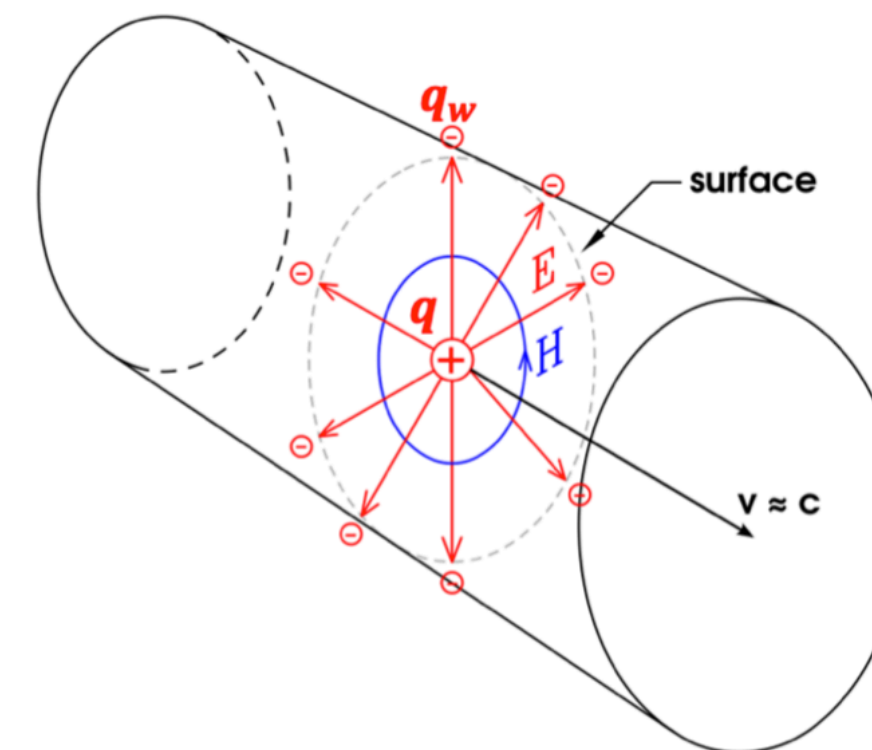
- As the point charge moves faster, the electric field gets “compressed”
 - At $\beta \rightarrow 1$, the electromagnetic field of the point charge is purely transverse
→ transverse electromagnetic (TEM) field
- This benefits to understand and to describe the BPM pickups qualitatively and quantitatively



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Opening angle $\propto \frac{1}{\beta\gamma}$

e.g. $\beta = 0.1$; $1/(\beta\gamma) = 9.95$
 $\beta = 0.6$; $1/(\beta\gamma) = 1.33$
 $\beta = 0.9$; $1/(\beta\gamma) = 0.48$



Position Characteristics

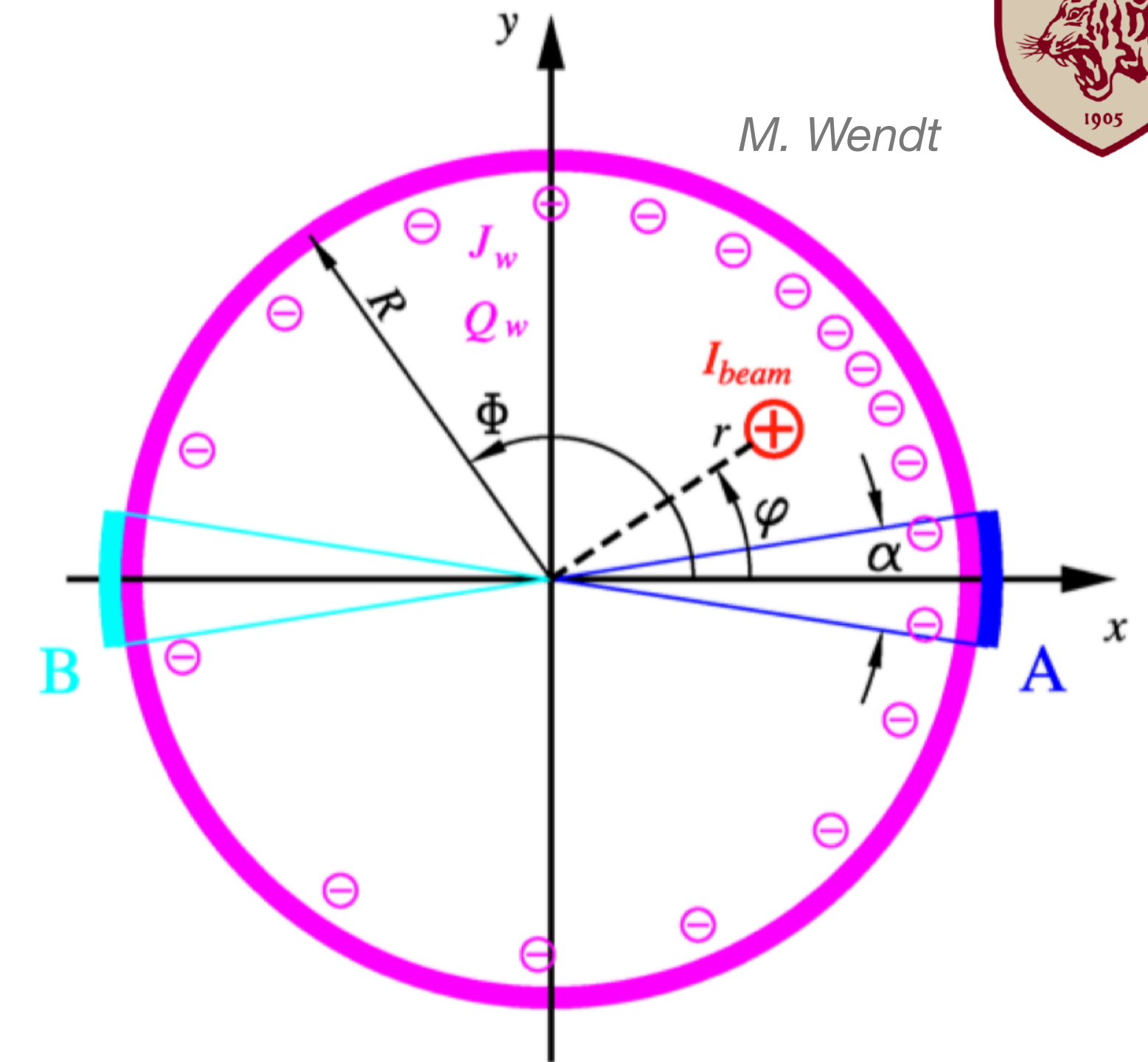
- Suppose a circular vacuum chamber of radius R and a relativistic beam ($\beta \rightarrow 1$) with a beam current I_{beam} at (r, ϕ)
- Solution for the wall current distribution:

$$J_{\omega}(R, \Phi, r, \phi) = -\frac{I_{\text{beam}}}{2\pi R} \left[\boxed{1} + \boxed{2 \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^n \cos n(\Phi - \phi)} \right]$$

Uniform distribution
(when $r = 0$), aka
“common mode”

Correction for asymmetry
(off-center beam)

or equivalently $J_{\omega}(R, \Phi, r, \phi) = -\frac{I_{\text{beam}}}{2\pi R} \frac{R^2 - r^2}{R^2 + r^2 - 2Rr \cos(\Phi - \phi)}$



Position Characteristics

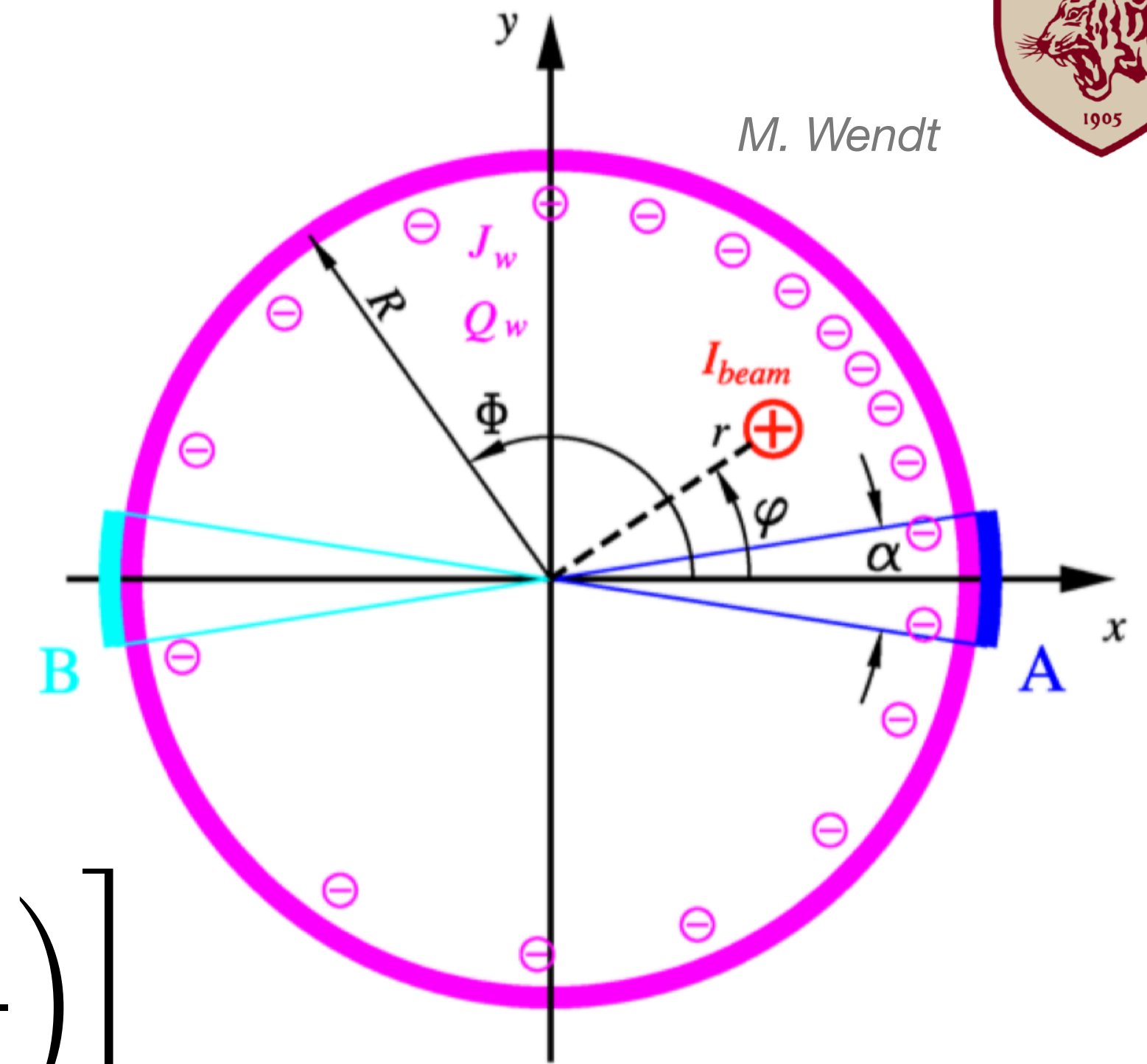
- With an BPM electrode of azimuthal angle α

$$I_{\text{elec}} = R \int_{-\alpha/2}^{+\alpha/2} J_{\omega}(R, \Phi, r, \phi) d\Phi = -\frac{I_{\text{beam}}}{2\pi} s_{\text{elec}}(r/R, \phi, \alpha)$$

where

$$s_A = \alpha + 4 \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{R} \right)^n \cos(n\phi) \sin \left(\frac{n\alpha}{2} \right)$$

$$s_B = \alpha + 4 \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r}{R} \right)^n \cos(n\phi) \sin \left[n \left(\pi + \frac{\alpha}{2} \right) \right]$$



- Remind:

$$\frac{v_{\Delta}}{v_{\Sigma}} = \frac{v_A - v_B}{v_A + v_B} \propto \text{position}$$

$$V_{\text{elec}}(x, y, \omega) = s(x, y) Z(\omega) I_b(\omega)$$

Sensitivity function (position information)

Position Characteristics

- Difference and summation:

$$\Delta \equiv v_A - v_B = \frac{8r}{R} \cos \phi \sin(\alpha/2) Z(\omega) I_b(\omega)$$

$$\Sigma \equiv v_A + v_B = 2\alpha Z(\omega) I_b(\omega)$$

- Difference-over-sum (Δ/Σ)

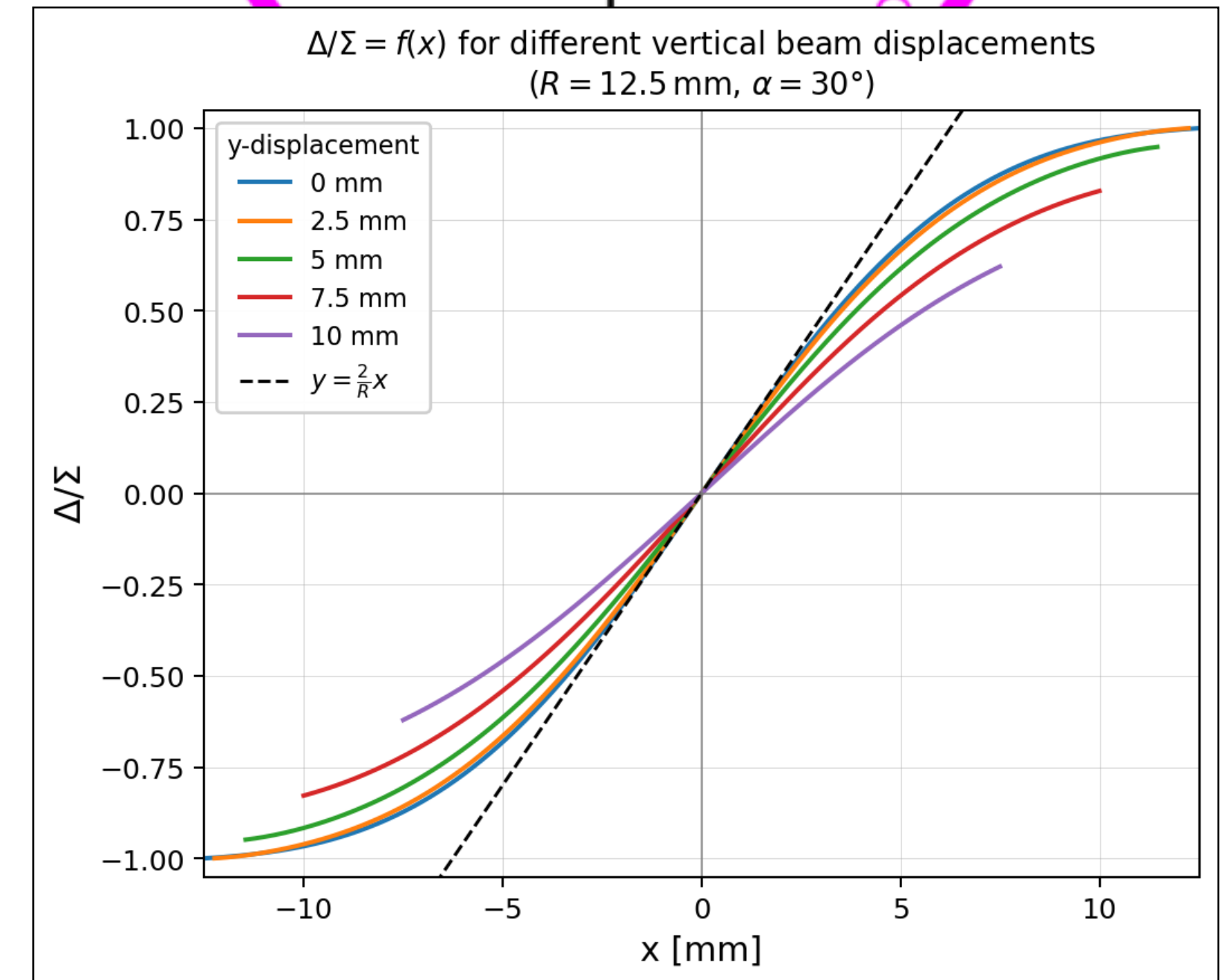
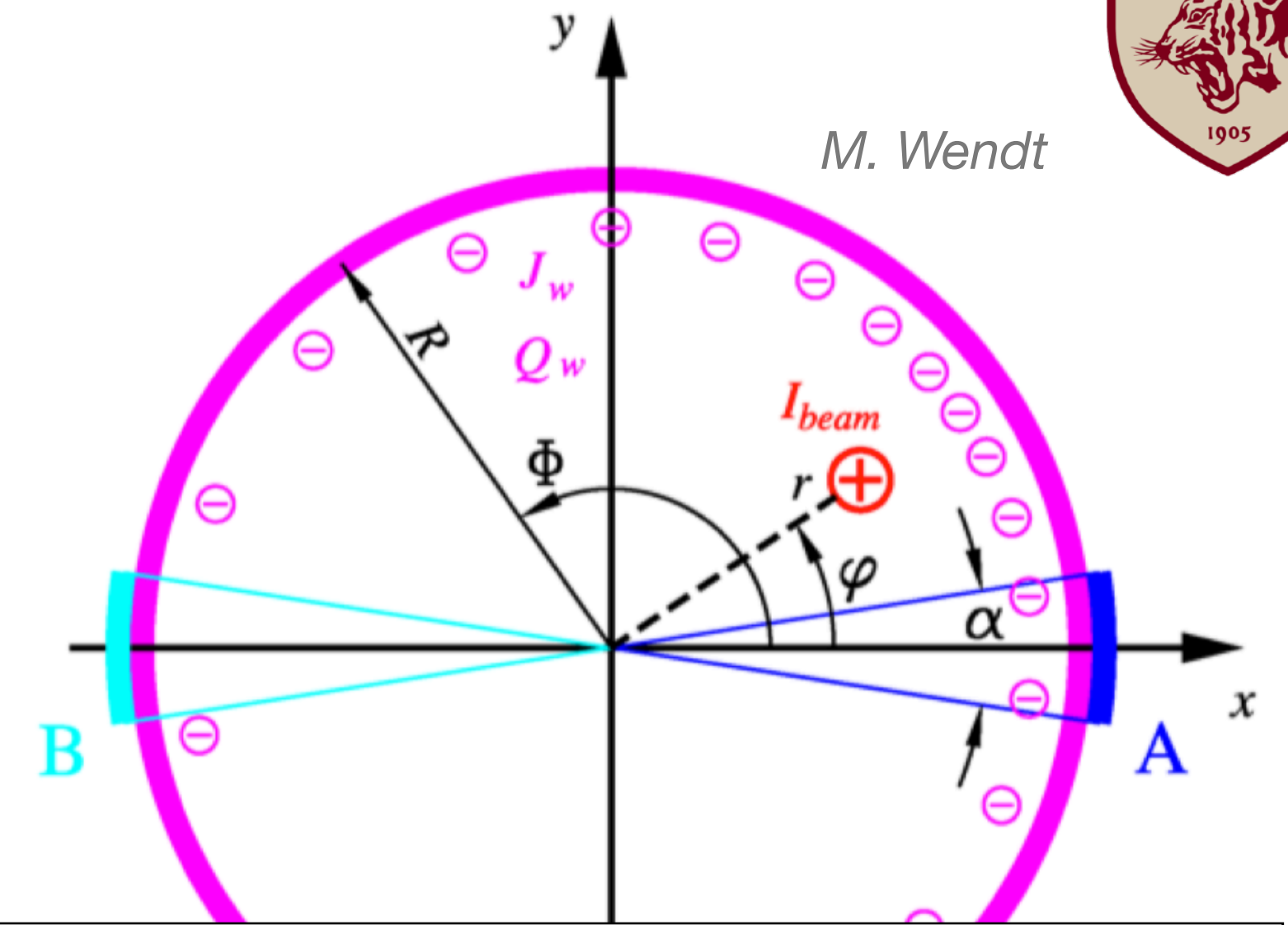
$$\frac{\Delta}{\Sigma} \simeq \frac{4 \sin(\alpha/2)}{\alpha} \frac{x}{R} + \boxed{\dots} = \text{position}$$

Normalized position

With a small narrow electrode ($\sin(\alpha/2) \simeq \alpha/2$)
and a small beam displacement $x^2 + y^2 \ll R^2$,
position $\simeq (2/R)x$

$(2/R)$ is called as a monitor constant (k_{PU})

High-order terms
Nonlinearity as $x^2 + y^2 \gg R^2$



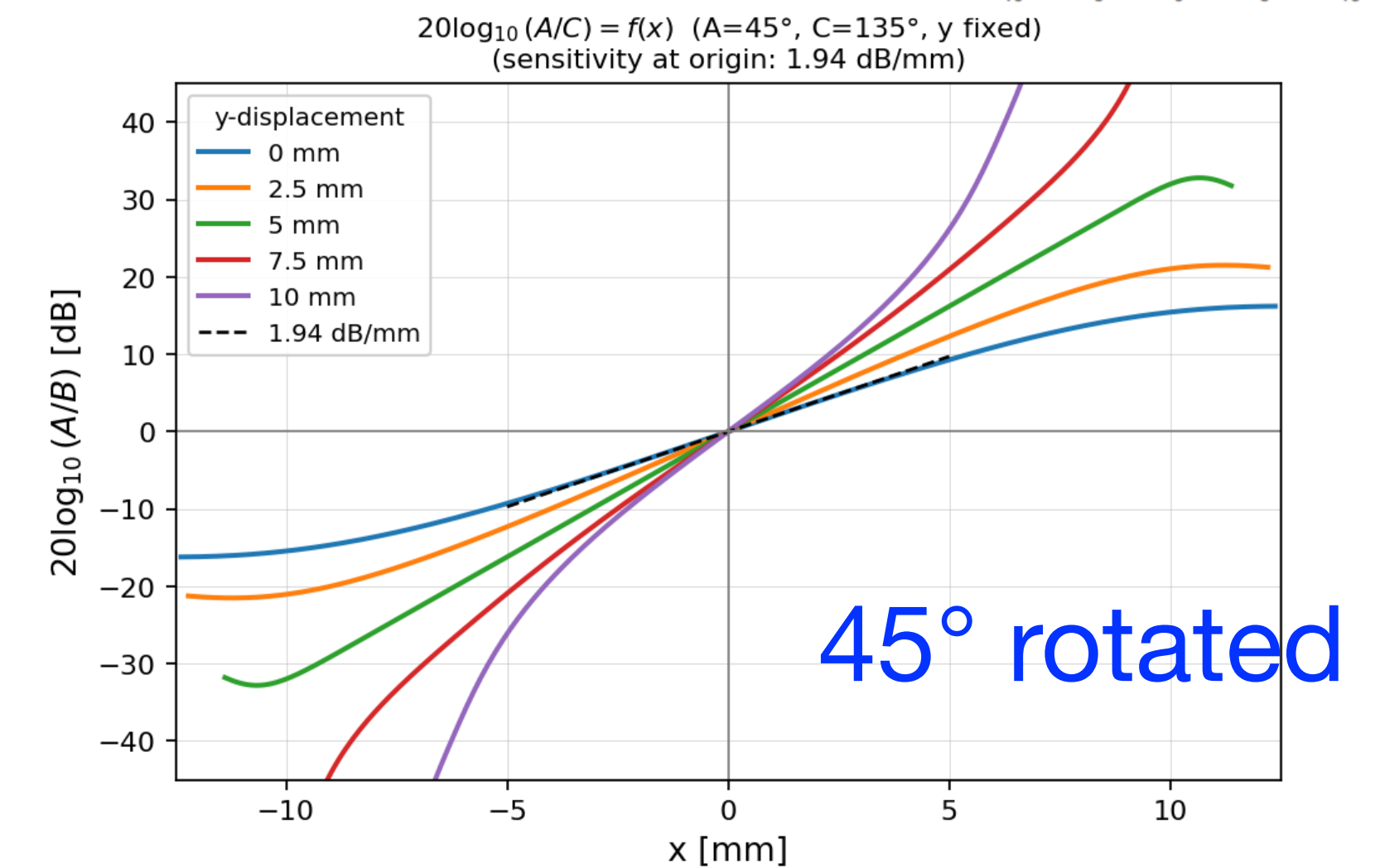
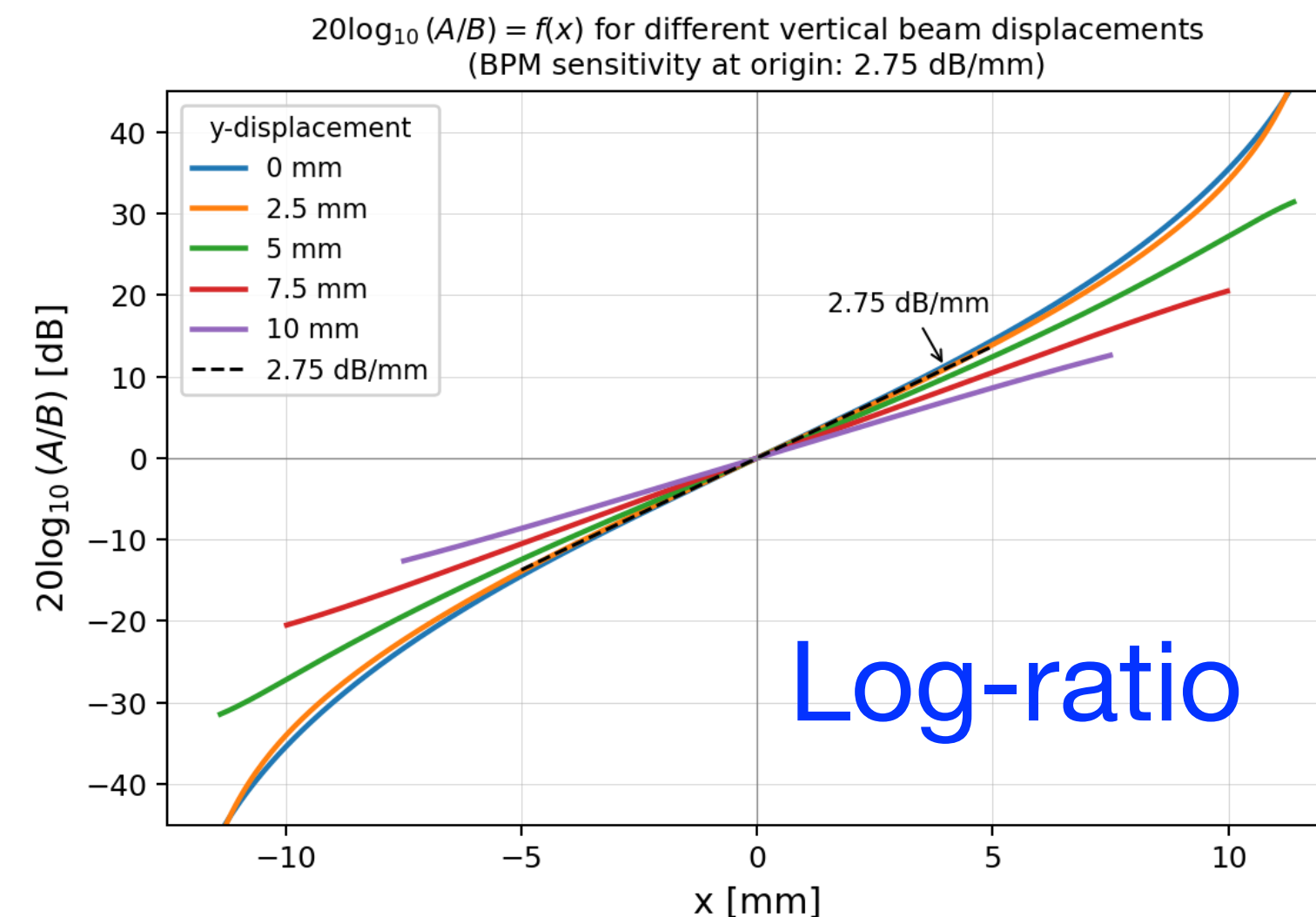
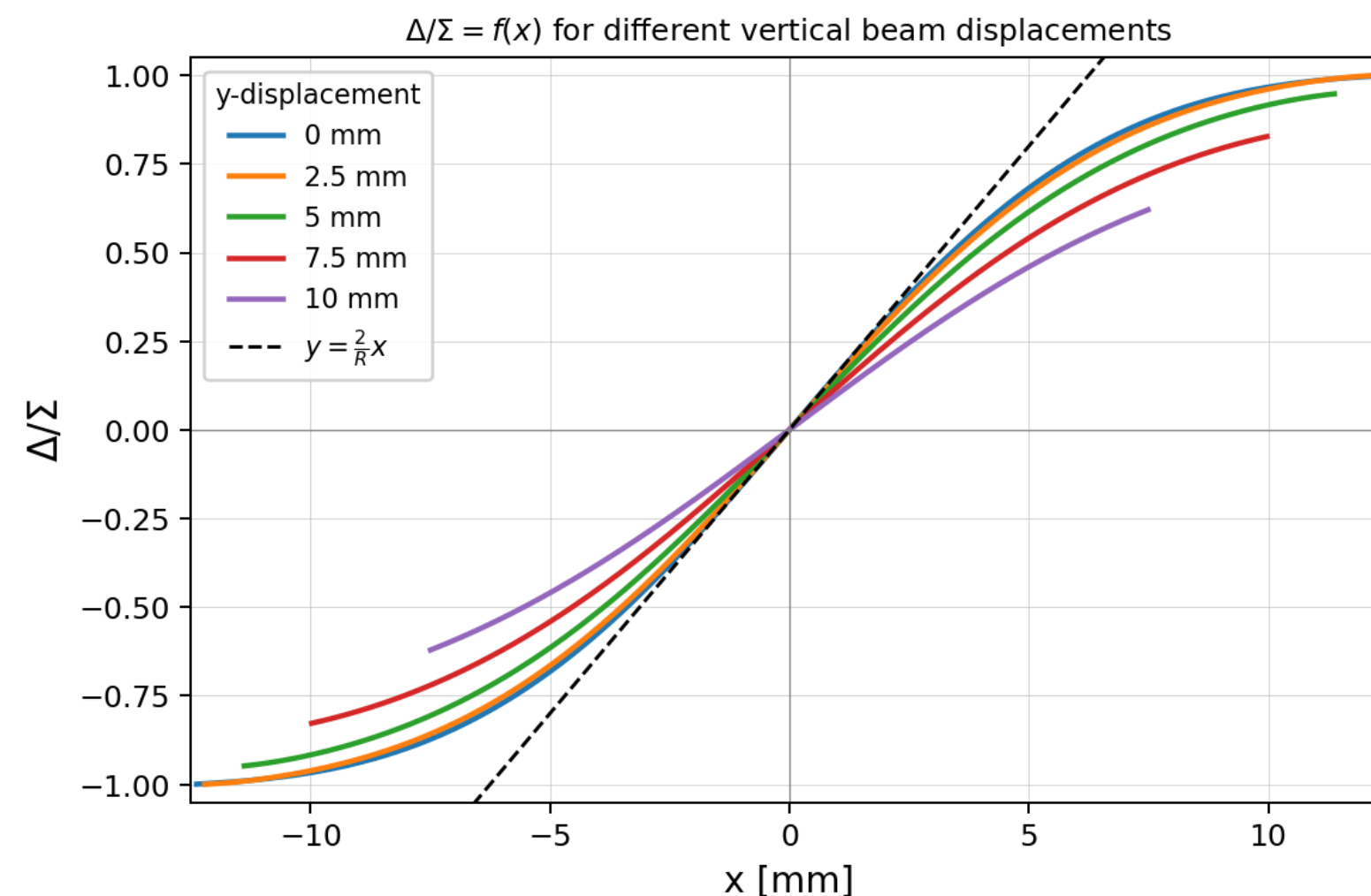
Position Characteristics

- For a large range of beam intensities, logarithmic detection would be beneficial

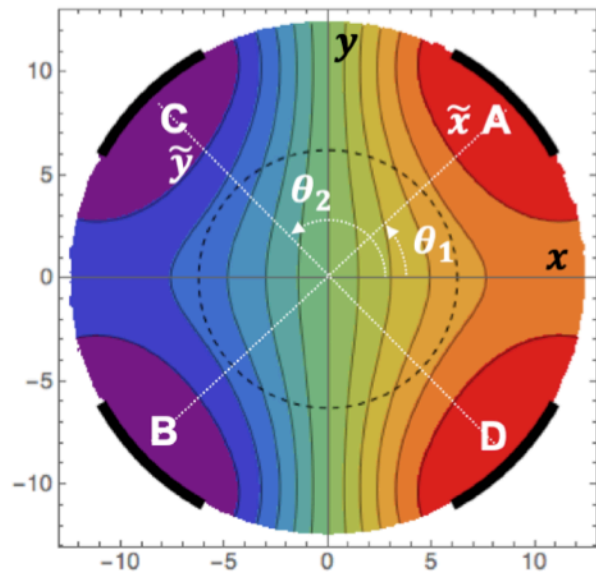
$$\text{position} \propto 20 \log_{10}(v_A) - 20 \log_{10}(v_B) = 20 \log_{10} \left(\frac{v_A}{v_B} \right) \simeq 20 \log_{10} \left(\frac{4 \sin(\alpha/2)}{\alpha} \frac{x}{R} + \dots \right)$$

- Offers more generous linearity
- “Rotated” electrodes
 - Sometimes inevitable due to limitations of spatial configurations
 - Usually more non-linearities

BPM position characteristic ($R = 12.5 \text{ mm}$, $\alpha = 30^\circ$)



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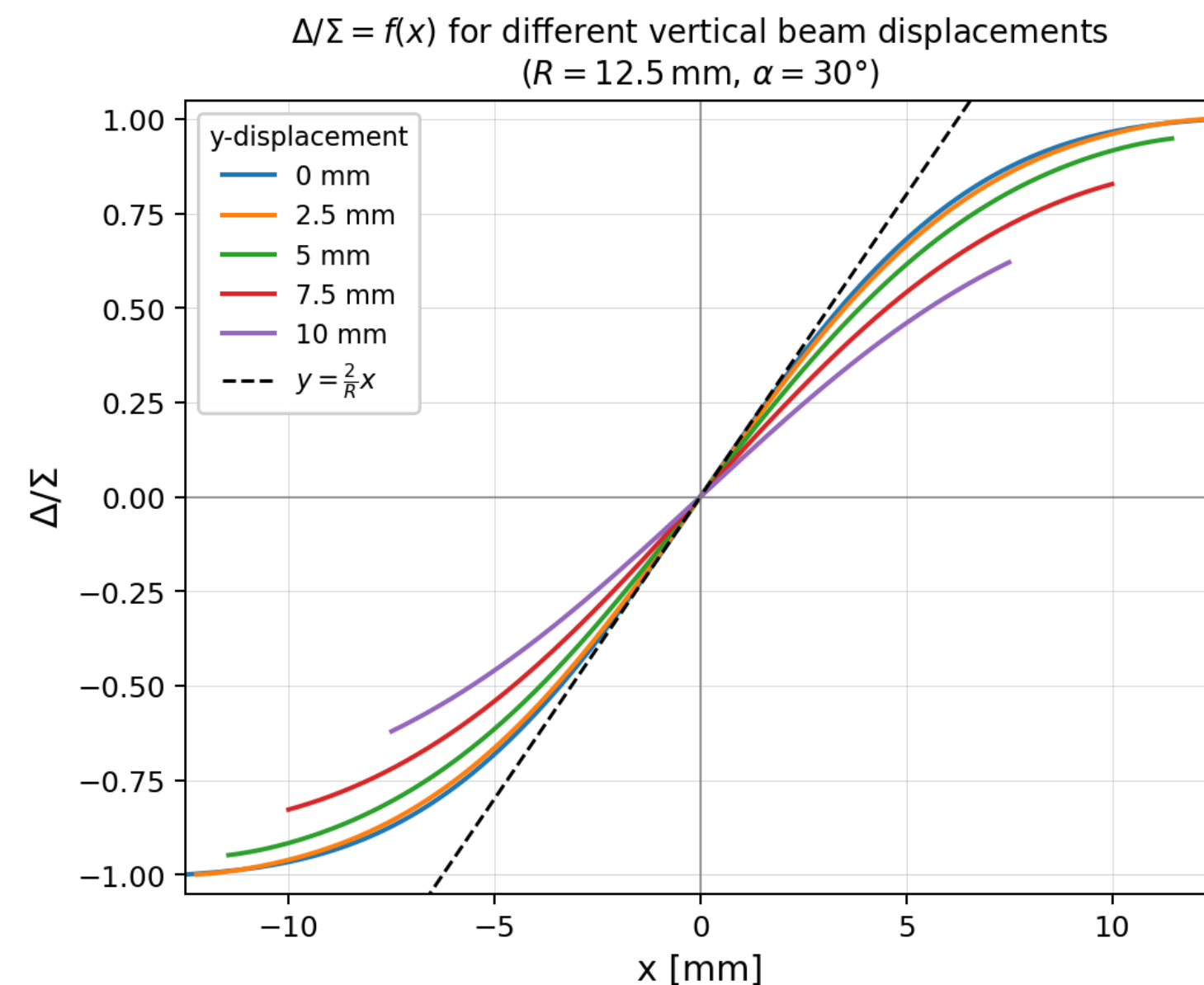


Non-linearity Corrections

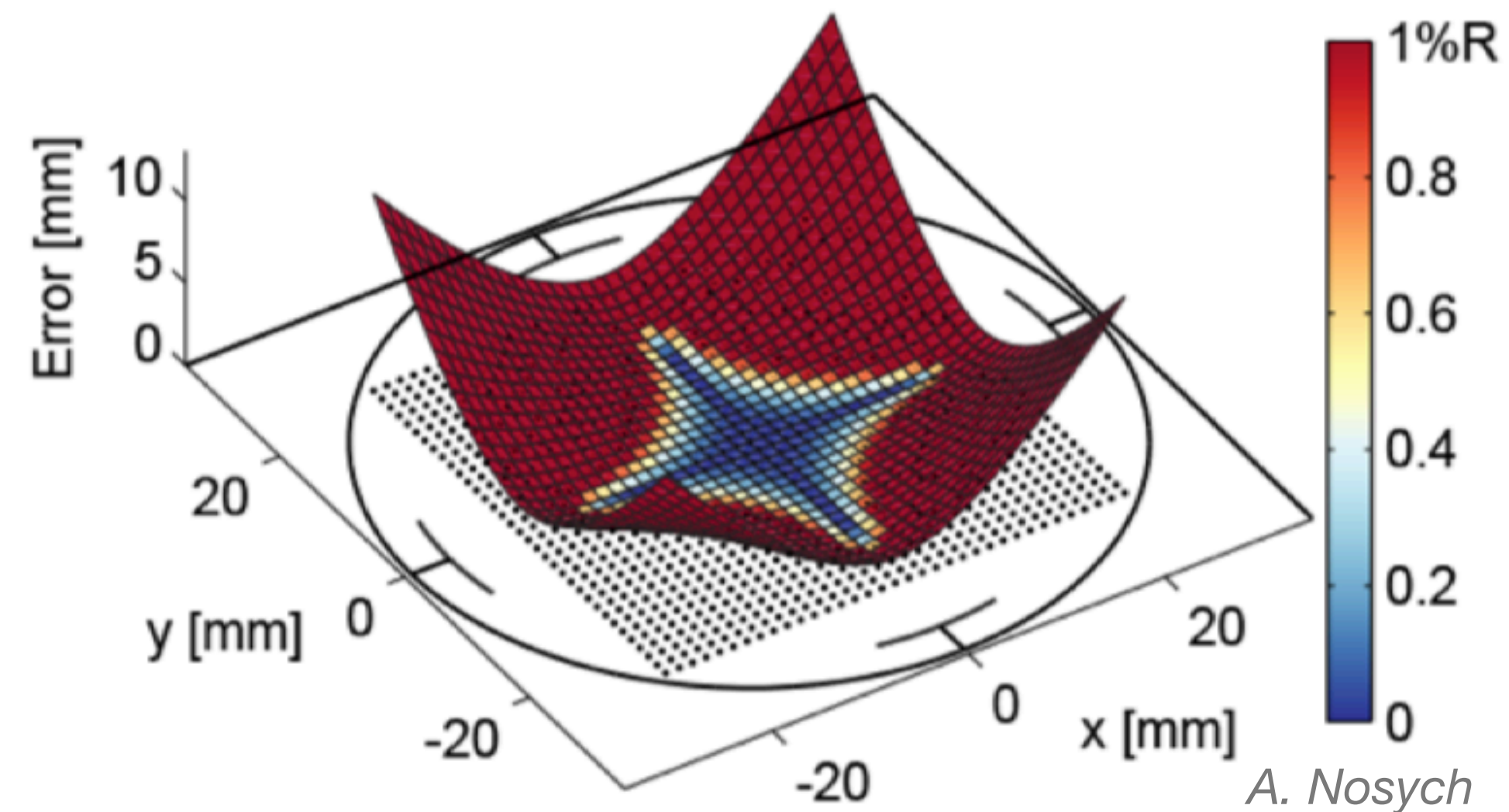
- To find a precise position, the non-linearity has to be corrected
- 1-dimensional correction with a polynomial of power p

$$x_{\text{bpm}}^{\text{1D}} = \sum_{i=0}^p c_i x_{\text{raw}}^i = U_p(x_{\text{raw}}) \simeq x$$

- Due to the symmetry, even terms ($i = 0, 2, 4, \dots$) don't come into play (i.e. $c_{i=0,2,4,\dots} = 0$)
- c_0 may come into play due to an electronics error



Corrected with $U_5(x)$, $U_5(y)$

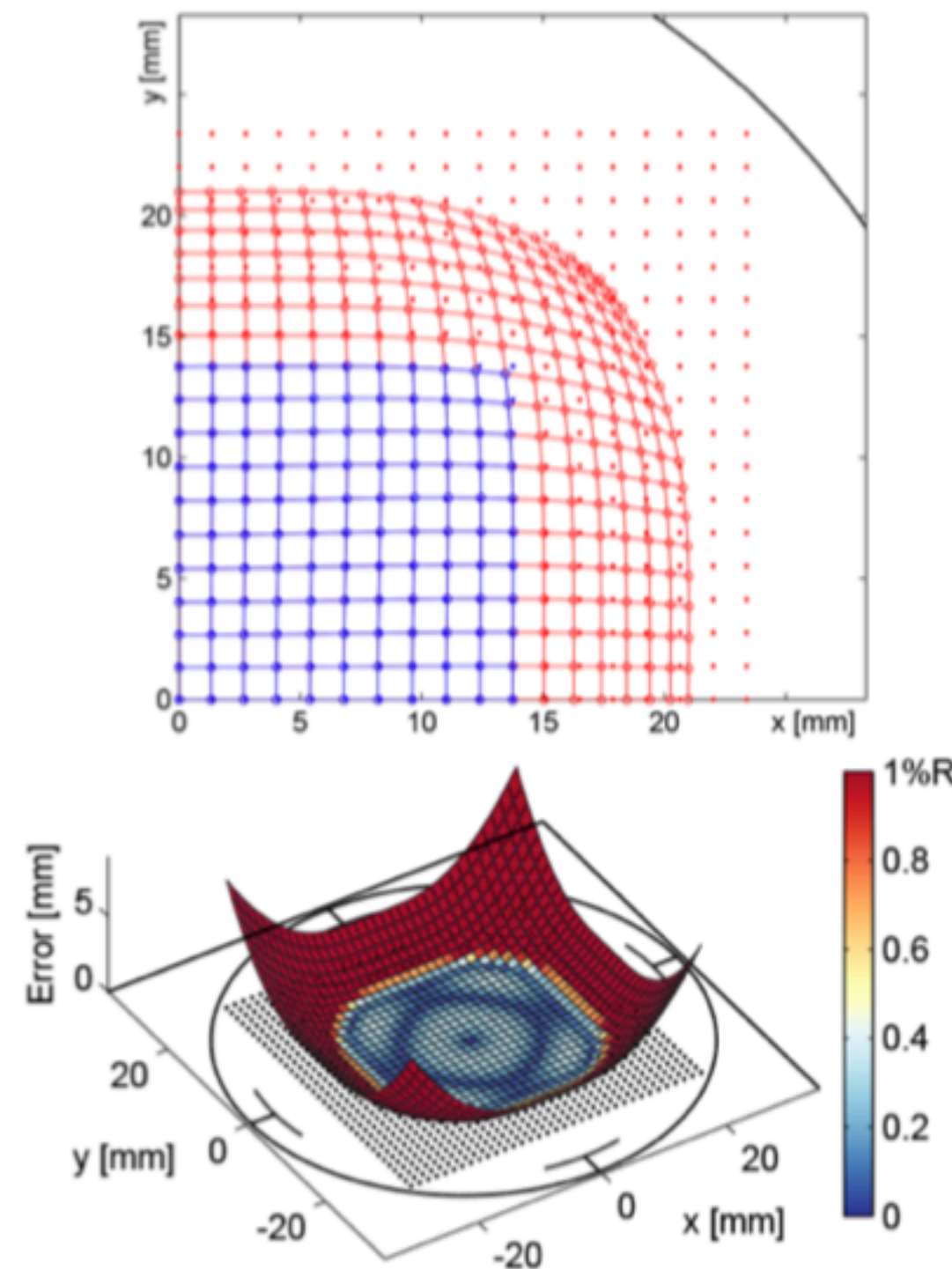


Non-linearity Corrections

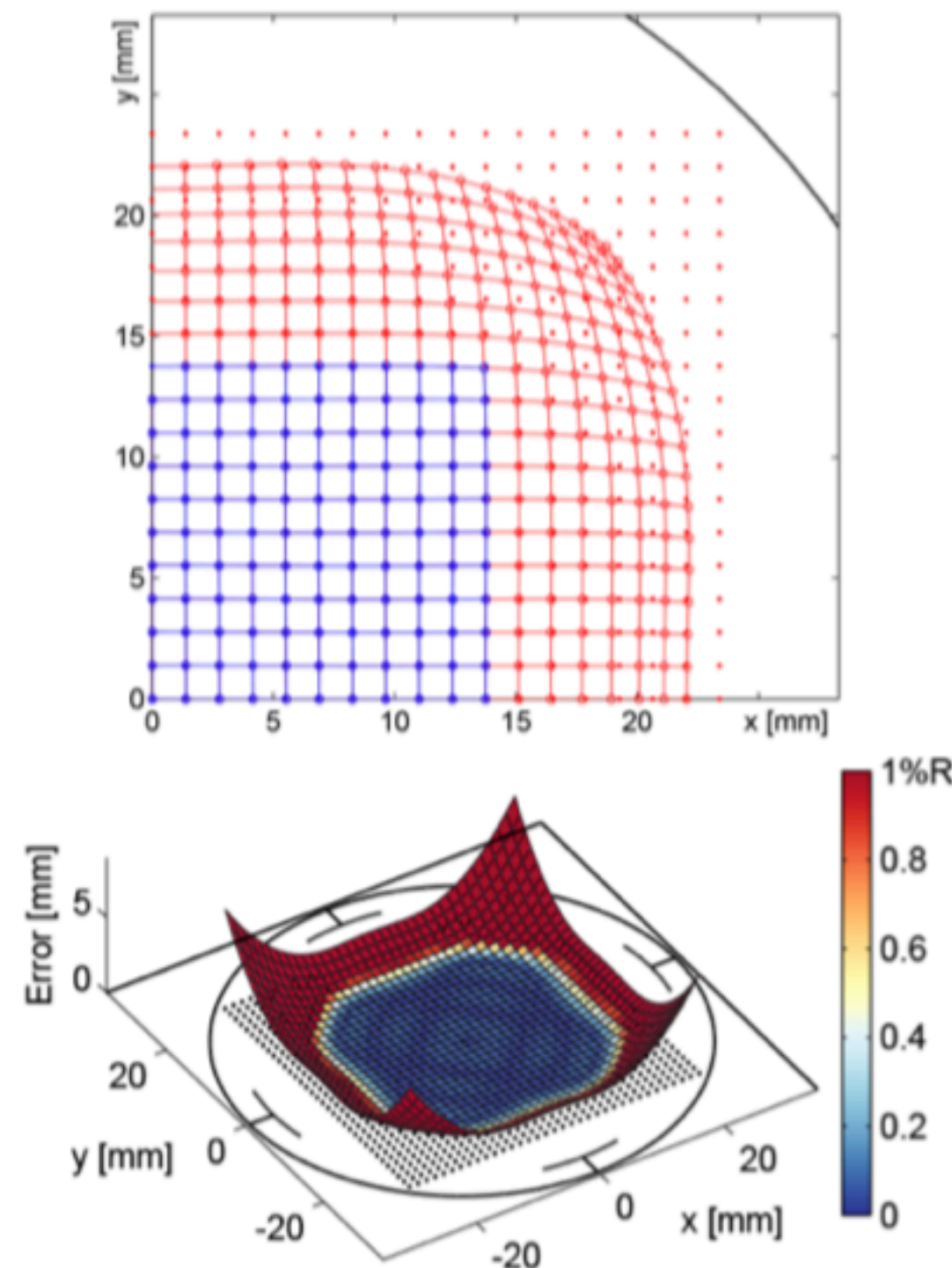
- 2-dimensional correction with a surface polynomial of power p and q

$$x_{\text{bpm}}^{2\text{D}} = \sum_{i,j=0}^{p,q} (c_{ij} x_{\text{raw}}^i y_{\text{raw}}^j) = Q_{p,q}(x_{\text{raw}}, y_{\text{raw}}) \simeq x$$

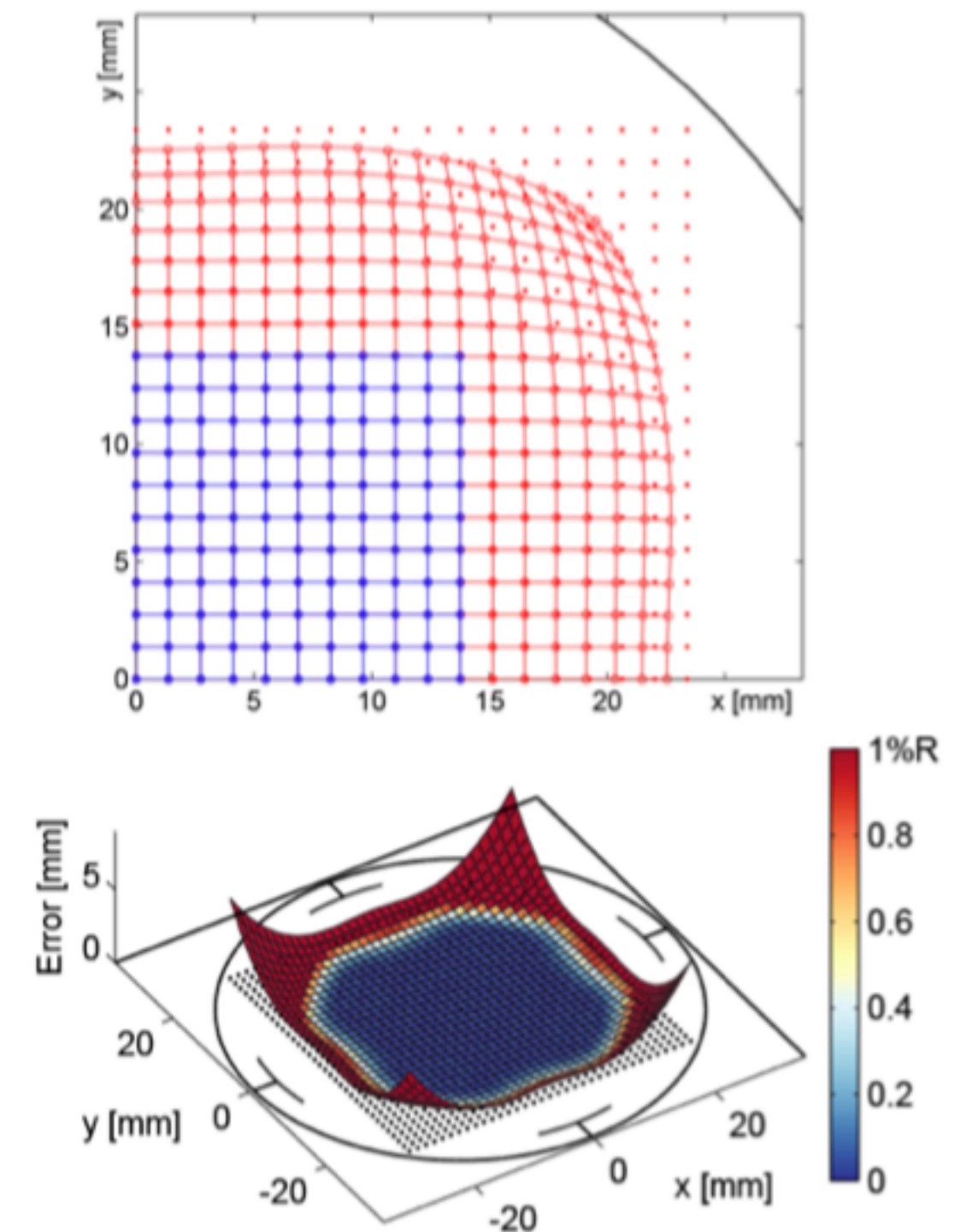
$$y_{\text{bpm}}^{2\text{D}} = \sum_{i,j=0}^{p,q} (c_{ij} y_{\text{raw}}^i x_{\text{raw}}^j) = Q_{p,q}(y_{\text{raw}}, x_{\text{raw}}) \simeq y$$



Corrected with $Q_{3,2}$



Corrected with $Q_{5,4}$



Corrected with $Q_{7,6}$

A. Nosych

Higher-order Moments

- Since a beam is a collection of point charges, we can apply the superposition principle to evaluate a beam effect
 - The beam position is defined as the center-of-charge of the beam

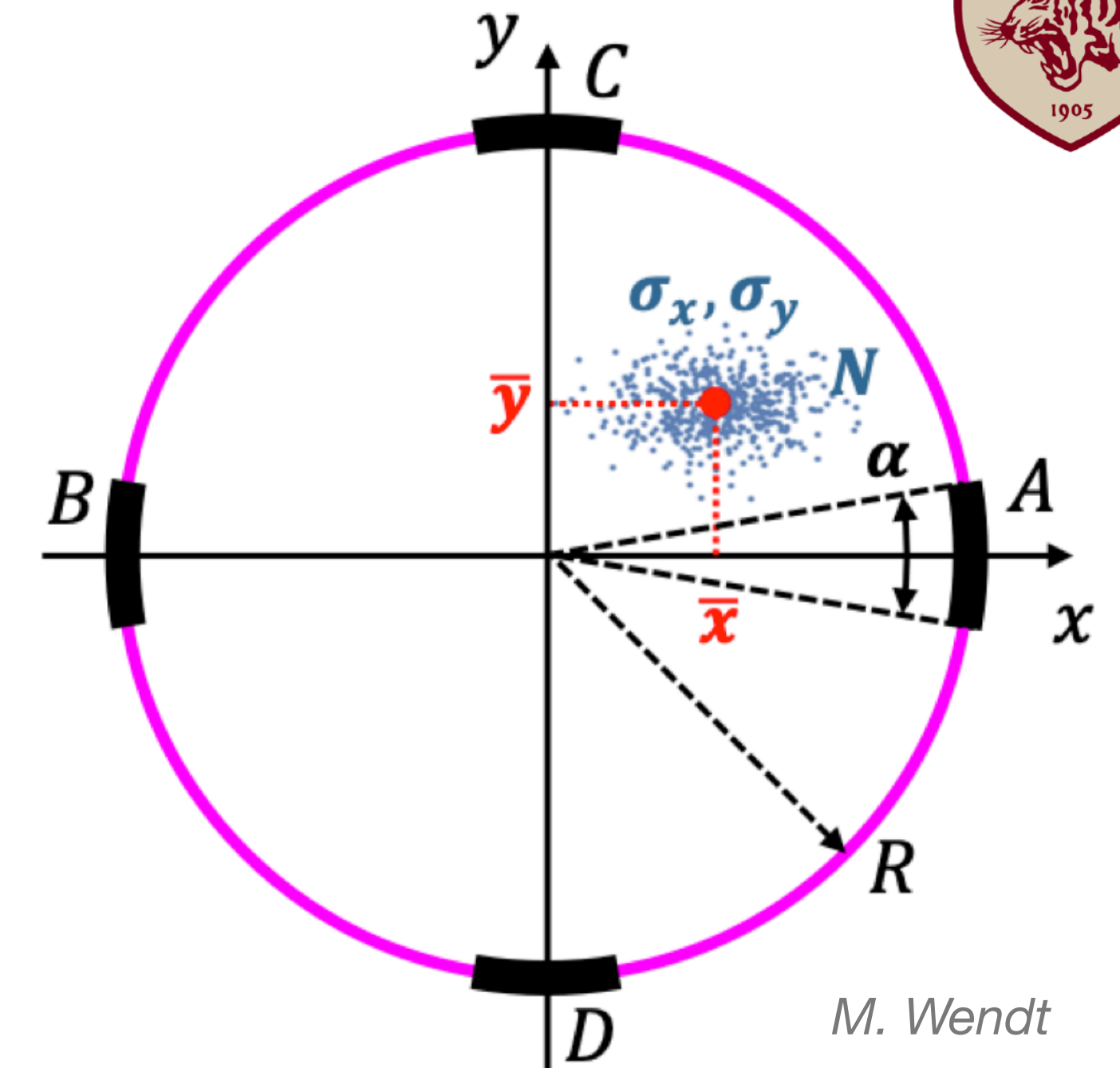
$$\vec{r} = \frac{1}{eN} \sum_{i=1}^N q_i \vec{\rho}_i$$

- Signal on the electrode A with the superposition

$$I_A = -\frac{I_{\text{beam}}}{2\pi} \left[\alpha + 4 \sum_{i=1}^N \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{r_i}{R} \right)^n \cos(n\phi_i) \sin\left(\frac{n\alpha}{2}\right) \right]$$

which can be rewritten as

$$I_A = -\frac{I_{\text{beam}}}{\pi} \left[\frac{\alpha}{2} + \frac{2}{R} \sin\left(\frac{\alpha}{2}\right) \bar{x} + \frac{1}{R^2} \sin(\alpha) \left(\sigma_x^2 - \sigma_y^2 + \bar{x}^2 - \bar{y}^2 \right) + \dots \right] \quad \text{with } \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$



Higher-order Moments

$$I_A = -\frac{I_{\text{beam}}}{\pi} \left[\boxed{\frac{\alpha}{2}} + \boxed{\frac{2}{R} \sin\left(\frac{\alpha}{2}\right) \bar{x}} + \boxed{\frac{1}{R^2} \sin(\alpha) (\sigma_x^2 - \sigma_y^2 + \bar{x}^2 - \bar{y}^2)} + \dots \right]$$

Monopole moment
Dipole moment
Quadrupole moment

- **Monopole moment**

- Only proportional to the beam intensity (I_{beam})
- Appears in all BPM electrodes
- Also called the “common mode”

- **Dipole moment**

- Interest for the beam position measurement
- $\propto (\text{position}) \times (\text{intensity})$

- **Quadrupole moment**

- Due to the non-linearity
- Transverse expansion of the beam (contains σ_x and σ_y)
- $\propto (\Delta\text{size} + \Delta\text{position}) \times (\text{intensity})$

Bunched Beam Signals

- Consider n bunches at equidistant times

$$V_{\text{elec}}(x, y, \omega) = s(x, y)Z(\omega)I_b(\omega)$$

Beam (or bunch) current

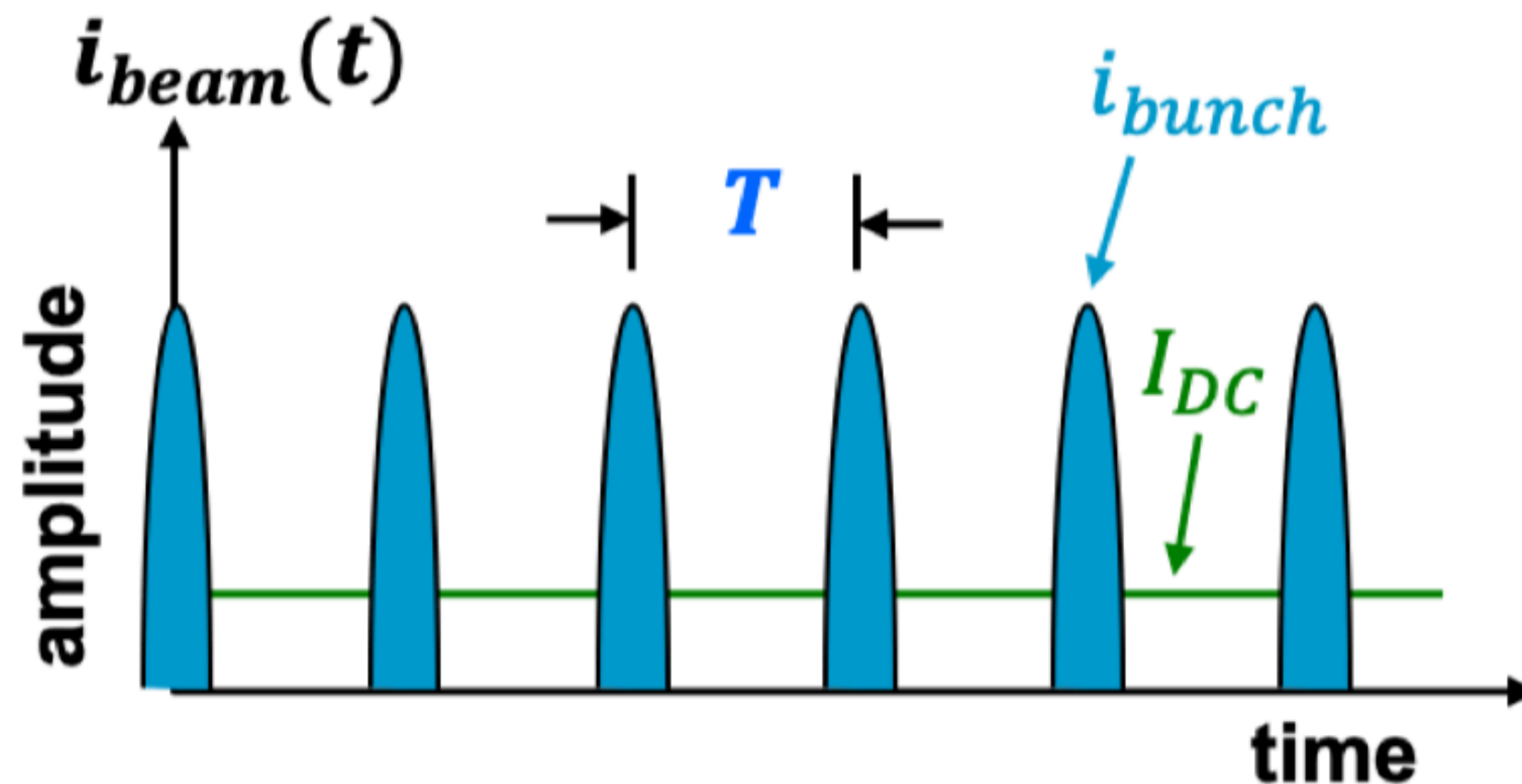
$$i_{\text{beam}}(t) = \sum_{k=-\infty}^{+\infty} i_{\text{bunch}}(t - kT) = \langle I_{\text{DC}} \rangle + 2\langle I_{\text{DC}} \rangle \sum_{m=1}^{\infty} A_m \cos(m\omega t)$$

DC component
→ Average beam current

Harmonic amplitude factor

$$\left(= \frac{I_{\text{bunch}}(m\omega)}{I_{\text{bunch}}(0)} \right)$$

For $\omega \rightarrow 0$, $A_m \rightarrow 1$



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bunch shape	harmonic amplitude factor A_m	comments
δ -function	1	for all harmonics
Gaussian	$\exp \left[-\frac{(m\omega\sigma)^2}{2} \right]$	$\sigma = \text{RMS bunch length}$
parabolic	$3 \left(\frac{\sin \alpha}{\alpha^3} - \frac{\cos \alpha}{\alpha^2} \right)$	$\alpha = m\pi t_b/T$
$(\cos)^2$	$\frac{\sin(\alpha-2)\frac{\pi}{2}}{(\alpha-2)\pi} + \frac{\sin \frac{\alpha\pi}{2}}{\frac{\alpha\pi}{2}} + \frac{\cos(\alpha+2)\frac{\pi}{2}}{(\alpha+2)\pi}$	$\alpha = 2mt_b/T$
triangular	$\frac{2(1-\cos \alpha)}{\alpha^2}$	$\alpha = m\pi t_b/T$
square	$\frac{\sin \alpha}{\alpha}$	$\alpha = m\pi t_b/T$

Button BPM

- Most popular type: simple, robust, compact
- Capacitive (C_b) and works like a high-pass filter

$$v_{\text{beam}}(t) \simeq -\frac{1}{C_b} i_{\text{beam}}(t) \underbrace{\frac{r}{2D}}_{\text{Coverage factor } \phi} \underbrace{\frac{2r}{c_0}}_{\text{Transit time}} = -i_{\text{beam}}(t) \frac{r^2}{D c_0 C_b}$$

with relativistic beams and $\sigma_z \gg 2r$

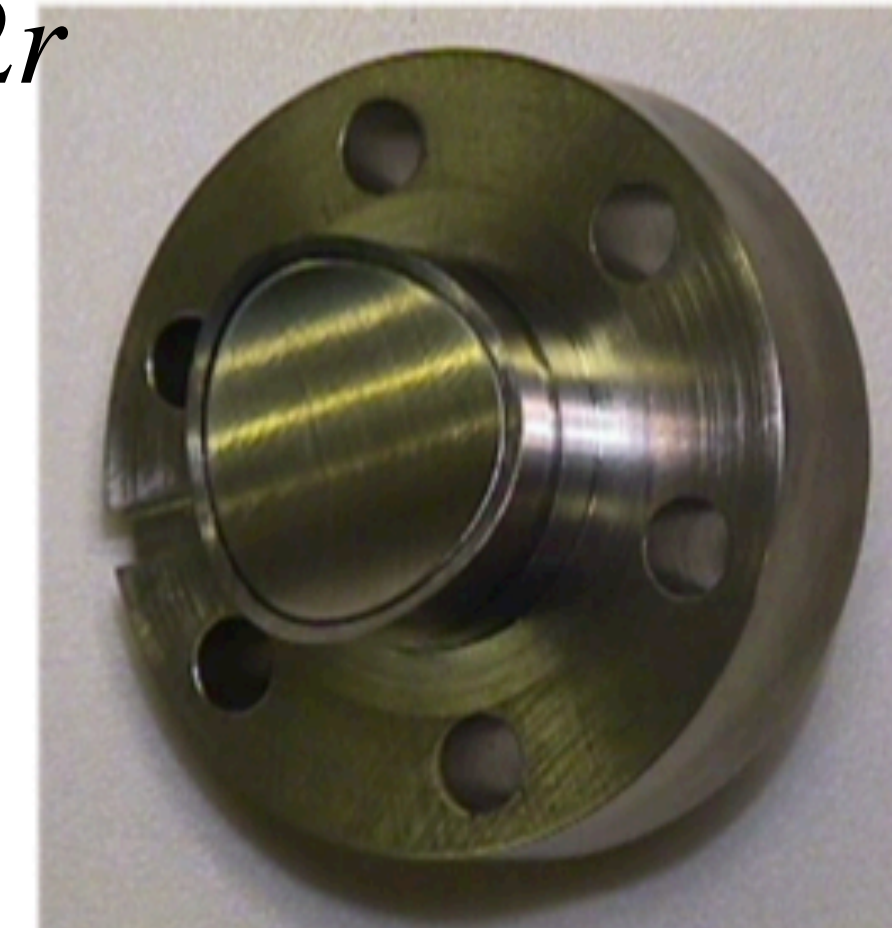
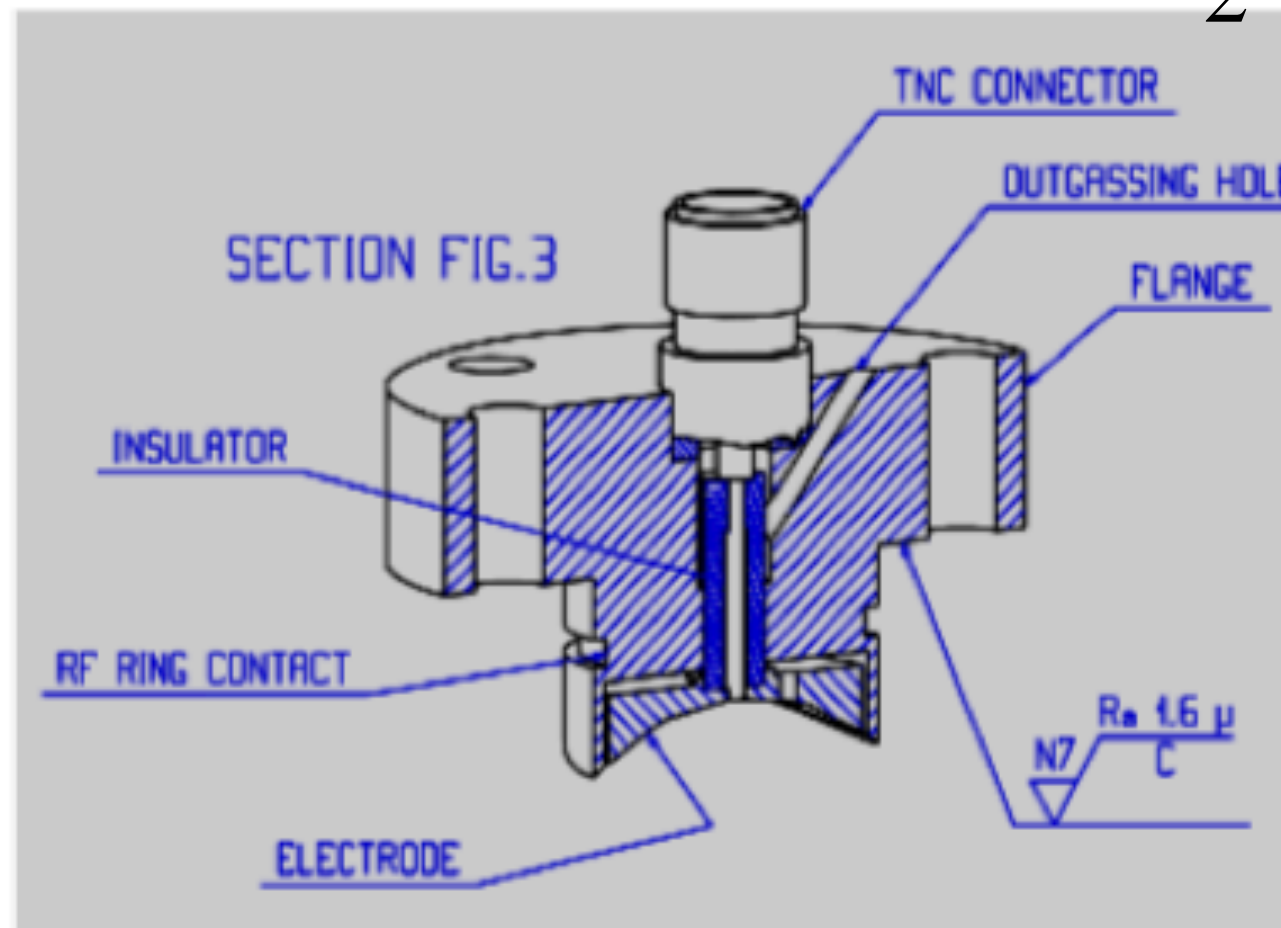
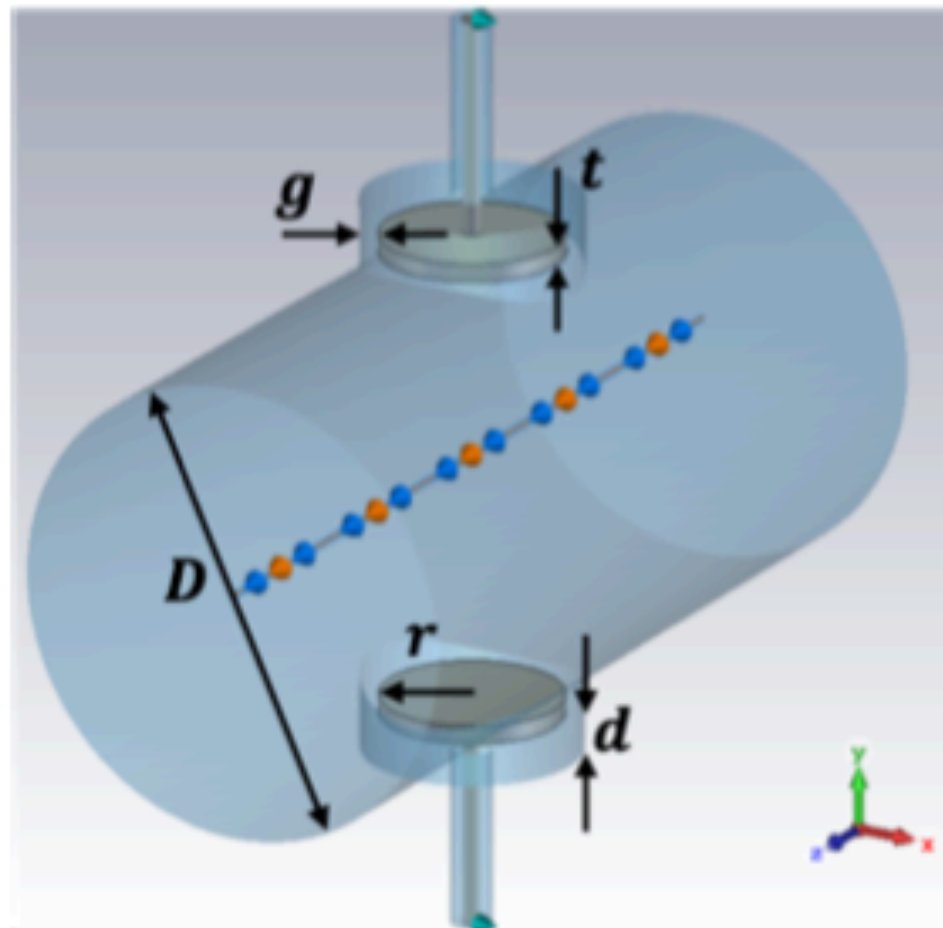
- Transfer impedance $Z_b(\omega)$

$$Z_b(\omega) = \frac{V_b(\omega)}{I_{\text{beam}}(\omega)} = \phi R_l \frac{\omega_1}{\omega_2} \frac{j\omega/\omega_1}{1 + \omega/\omega_1}$$

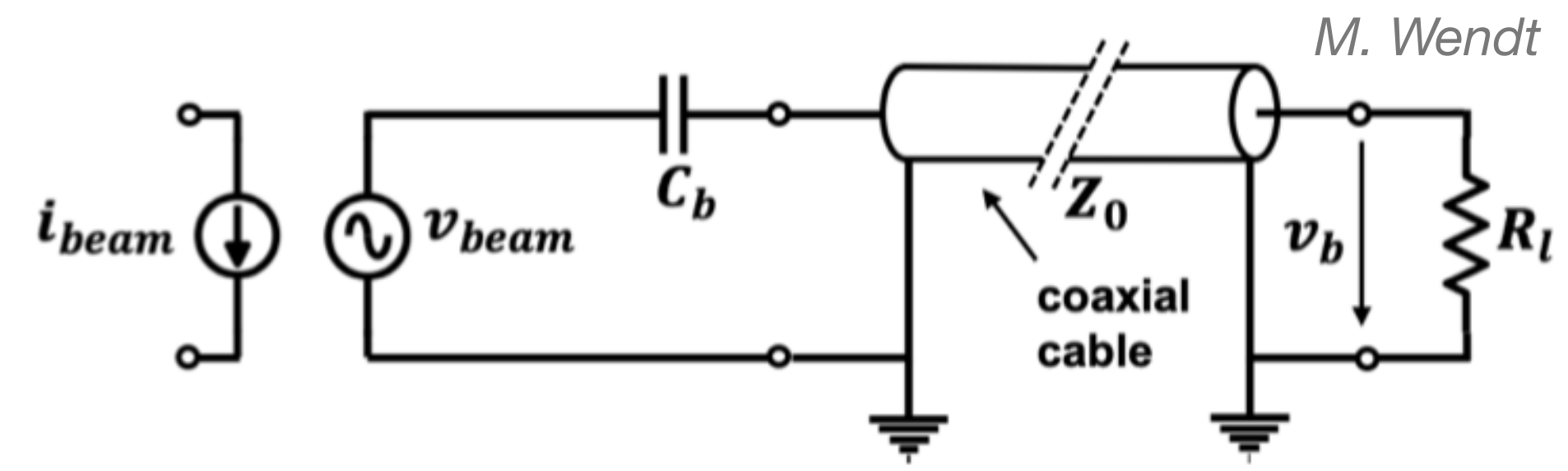
$$\omega_1 = \frac{1}{R_l C_b}$$

cutoff frequency: $f_{3\text{dB}} = \frac{\omega_1}{2\pi}$

$$\omega_2 = \frac{c_0}{2r}$$



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Button BPM

$$Z_b(\omega) = \frac{V_b(\omega)}{I_{\text{beam}}(\omega)} = \phi R_l \frac{\omega_1}{\omega_2} \frac{j\omega/\omega_1}{1 + \omega/\omega_1}$$

$$\omega_1 = \frac{1}{R_l C_b}$$

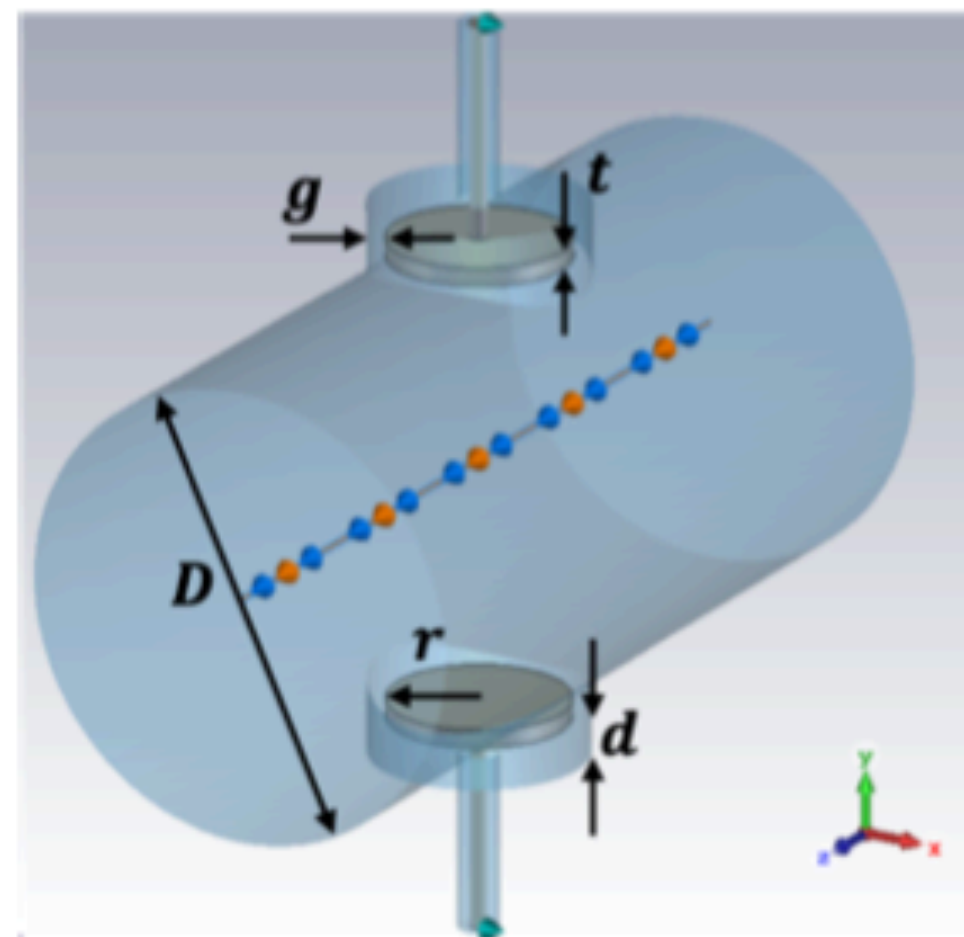
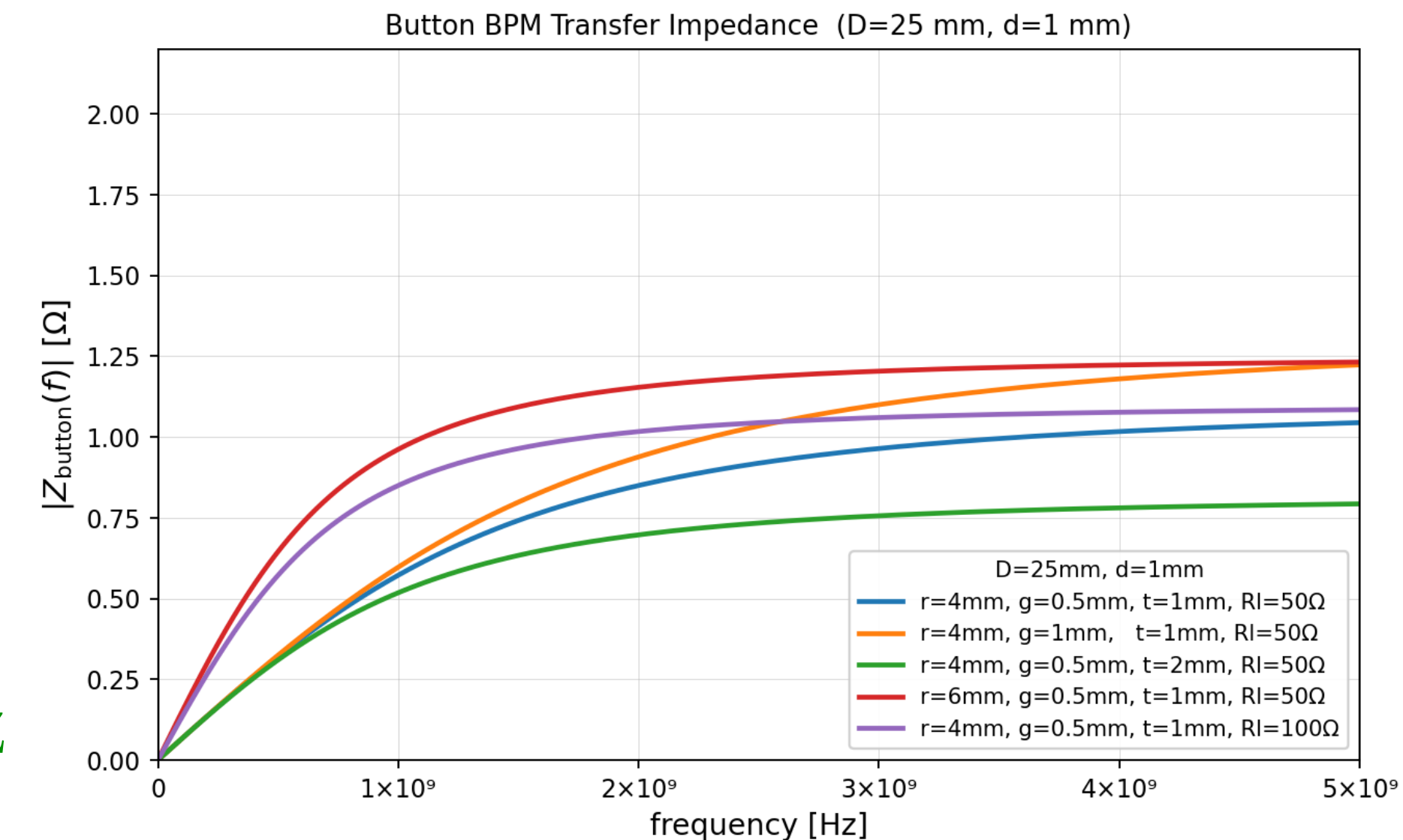
$$\omega_2 = \frac{c_0}{2r}$$

cutoff frequency: $f_{3\text{dB}} = \frac{\omega_1}{2\pi}$

- Button-to-ground capacitance

$$C_b \approx \underbrace{\frac{\pi \epsilon t_b}{\ln\left(\frac{r+g}{r}\right)}}_{\text{side}} + \underbrace{\frac{\pi \epsilon r^2}{d}}_{\text{back}} + C_{\text{fringe}}$$

Typical $Z_b \approx 1 \Omega$ at $f \geq 1 \text{ GHz}$



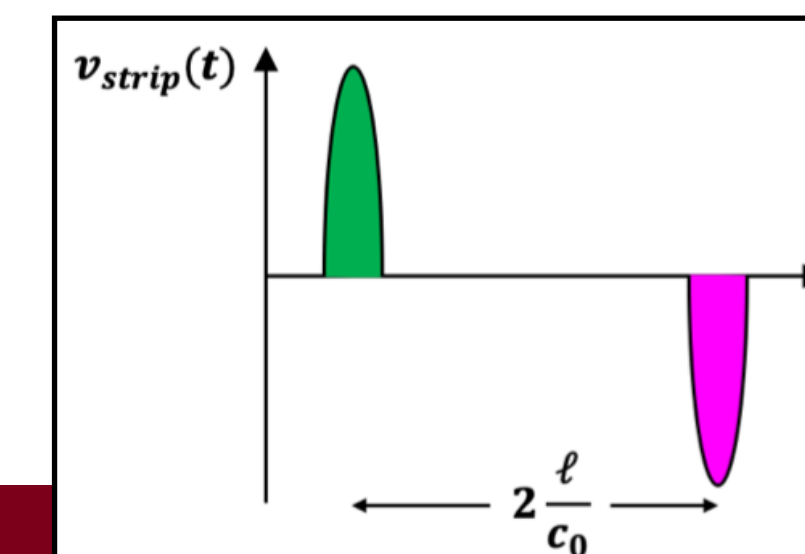
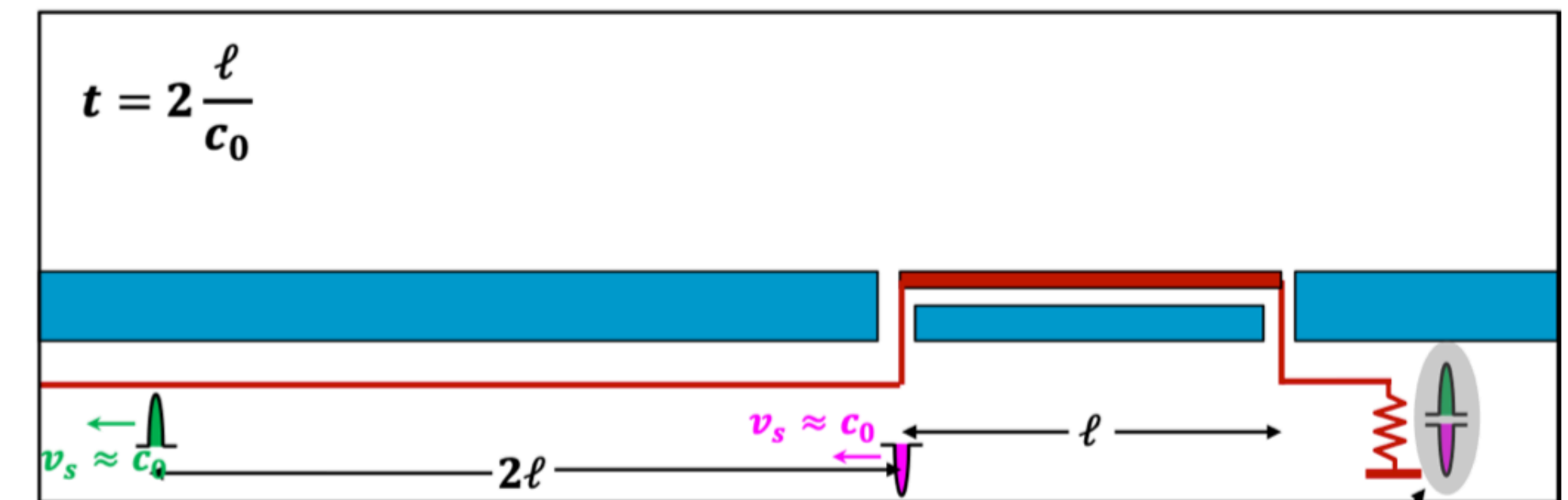
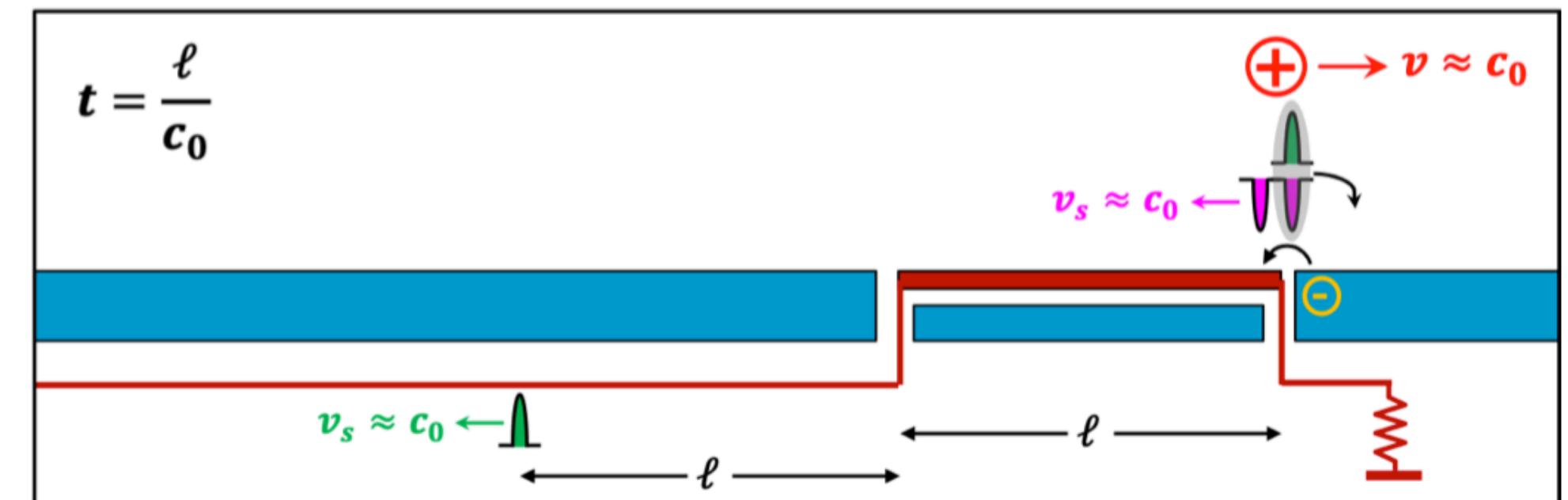
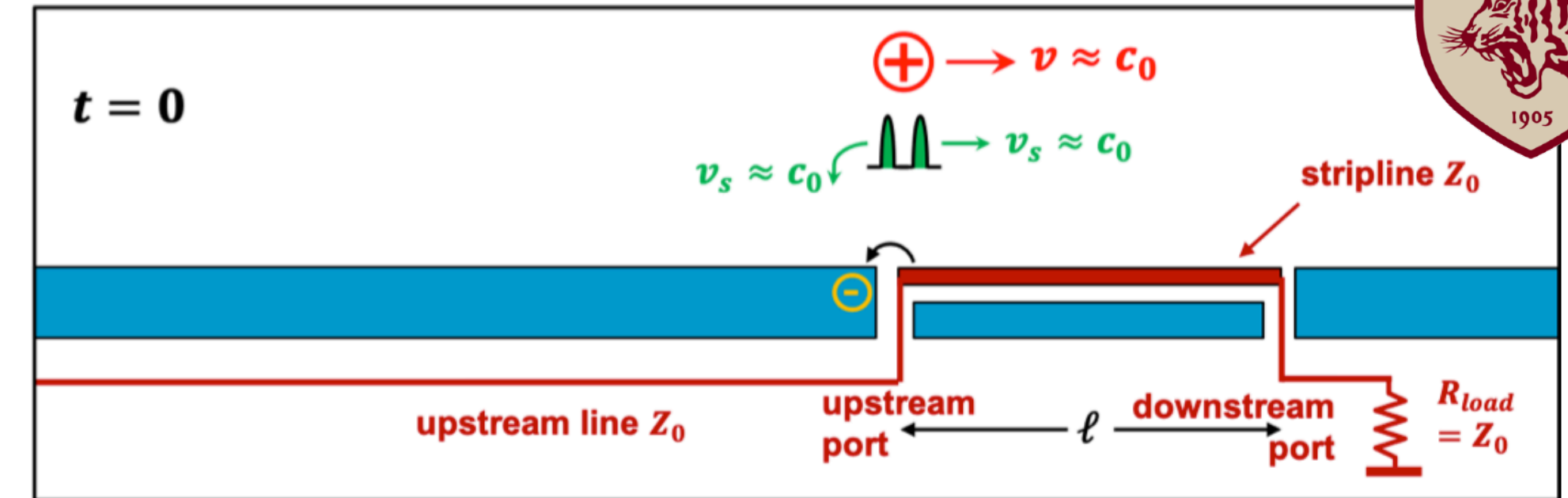
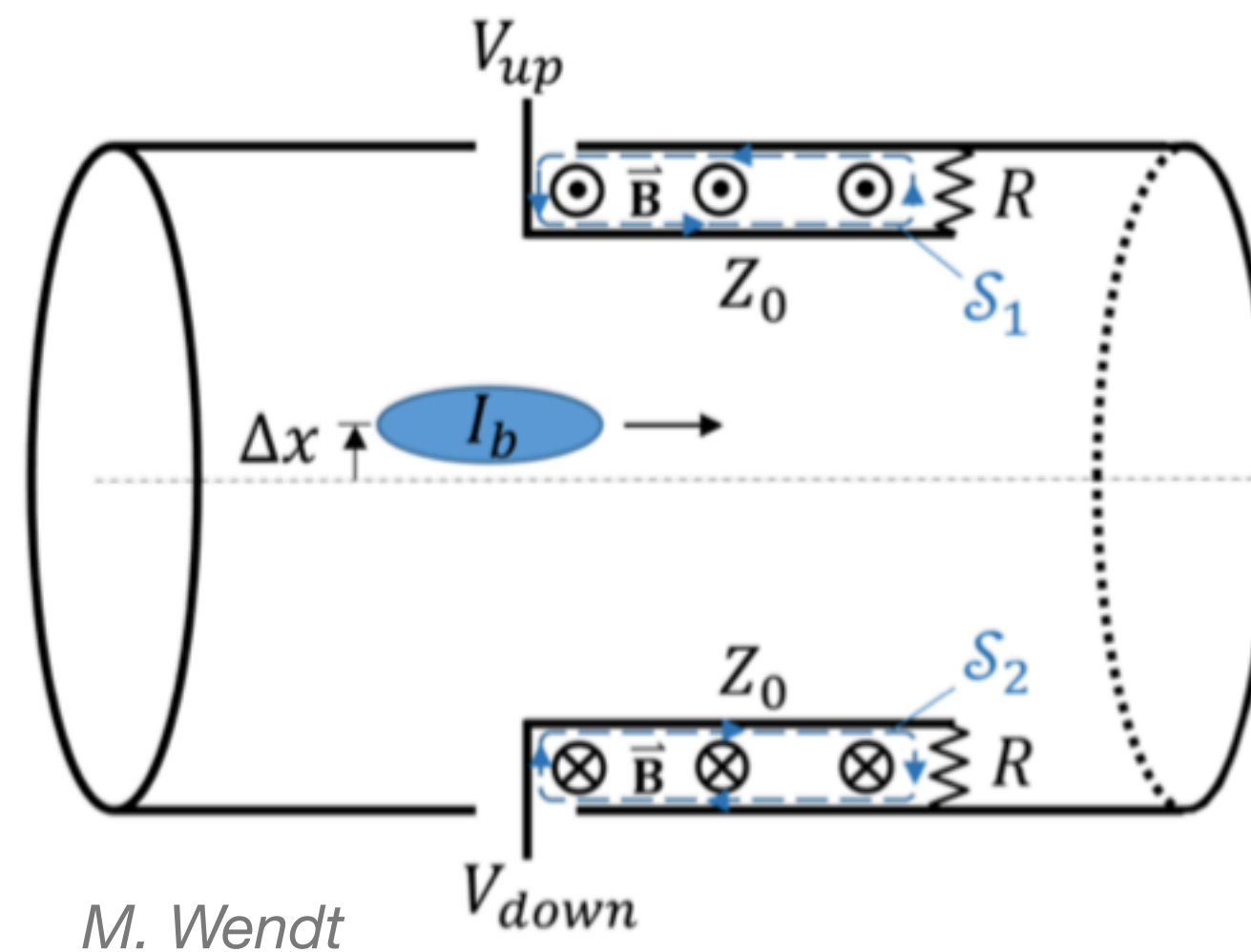
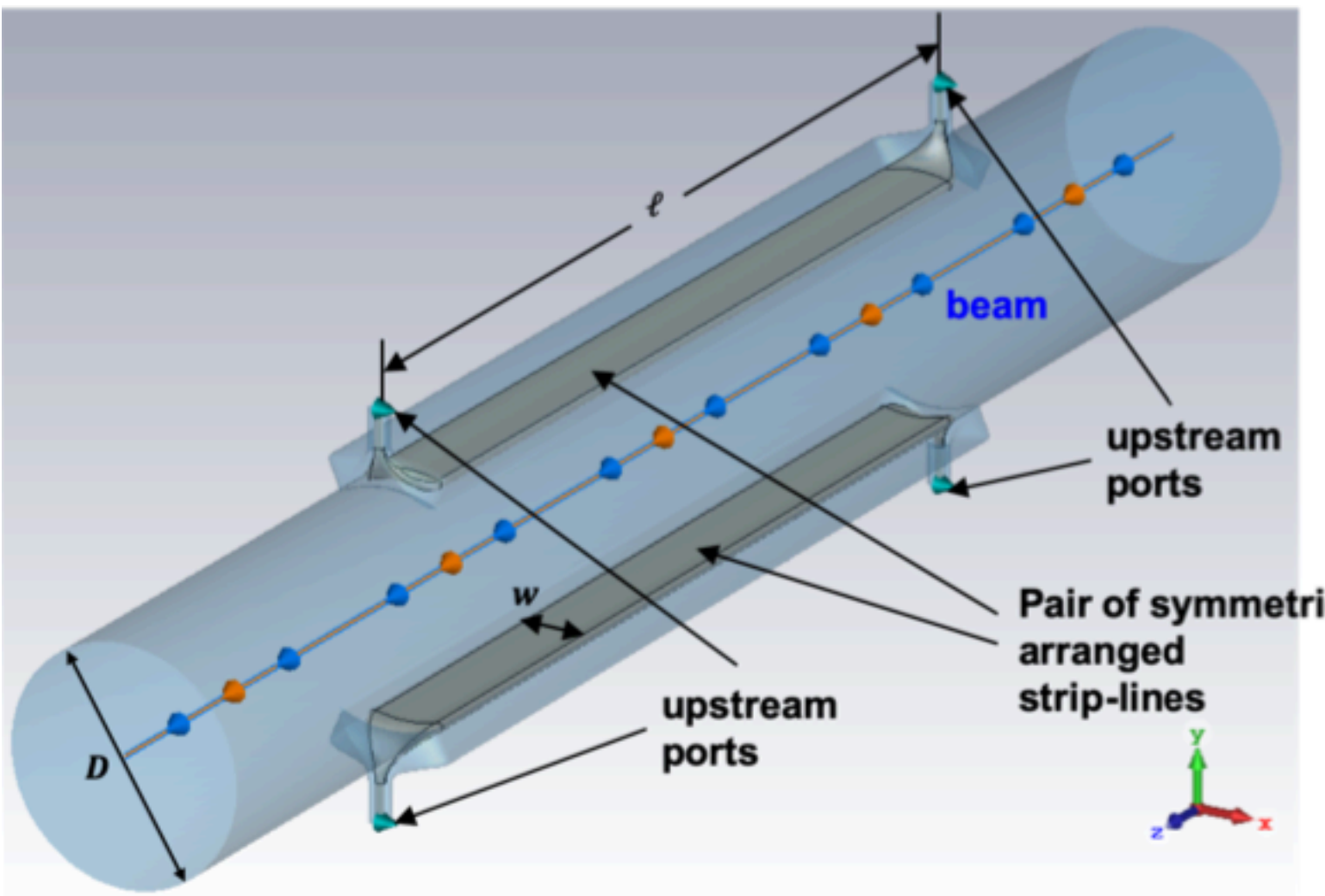
M. Wendt

To get higher coupling to the beam:

- $R_l \uparrow$: typically fixed as 50Ω
- $r \uparrow$ (bigger electrode) $\rightarrow C_b \uparrow$
 - $\rightarrow v_{\text{beam}} \downarrow$: lower SNR
 - $\rightarrow f_{3\text{dB}} \downarrow$: unwanted wakefields
- $g \downarrow \rightarrow C_b \uparrow$ but beneficial for beam coupling impedance

Stripline BPM

- Electromagnetic coupling, works as directional coupler
 - Directivity: signal extraction at only upstream port (downstream signal cancels)
 - Two pulses with opposite sign come along with a time interval depending on the stripline length



M. Wendt

annihilation

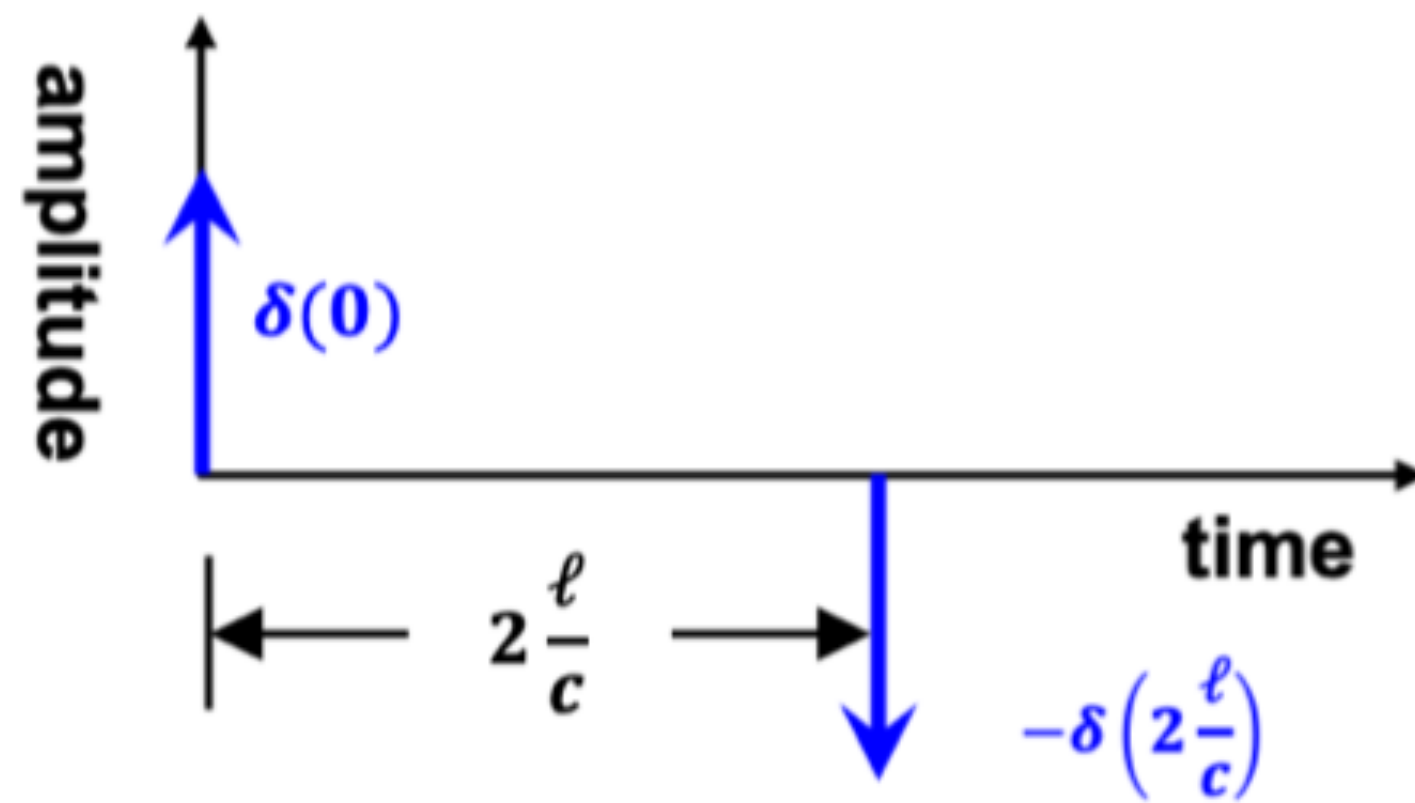
Stripline BPM

- Transfer impedance of an ideal, lossless stripline electrode

$$z(t) = \phi \frac{Z_0}{2} \left[\delta(t) - \delta\left(t - 2\frac{l}{c_0}\right) \right] \quad (\text{time-domain}) \quad \phi = \frac{1}{\pi} \arcsin(\omega/D) : \text{geometric coverage factor}$$

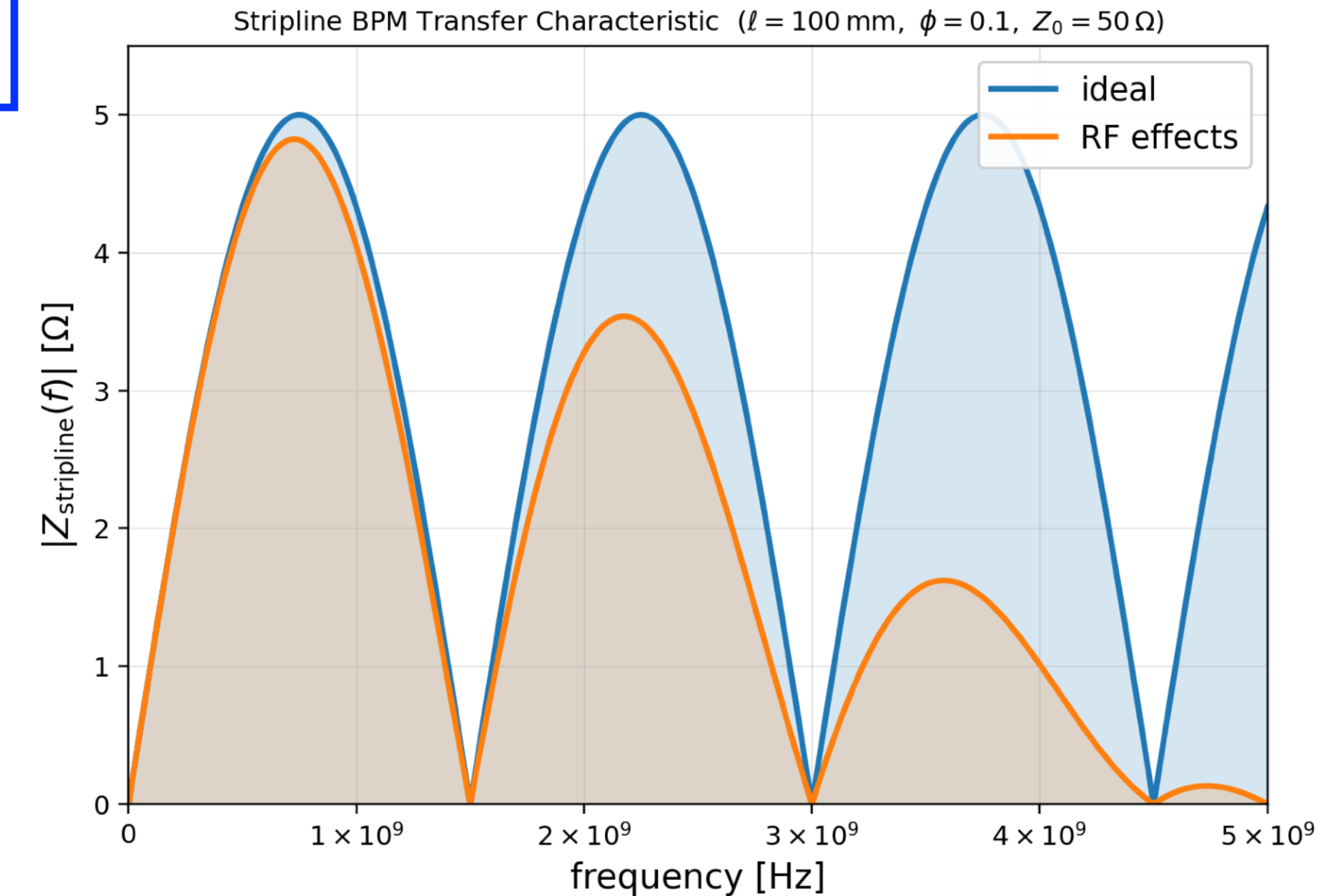
$$Z(\omega) = \phi \frac{Z_0}{2} \left(1 - e^{-j2\omega \frac{l}{c_0}} \right) = j\phi Z_0 e^{-j\omega \frac{l}{c_0}} \sin\left(\omega \frac{l}{c_0}\right)$$

(frequency-domain)



M. Wendt

By the length l , optimal operating frequency can be adjustable



Position Resolution

- The smallest detectable change of the beam position
- The most important performance parameter of BPM

$$\text{position} = k_{\text{PU}} \frac{\Delta_{\text{meas}}}{\Sigma_{\text{meas}}} = k_{\text{PU}} \frac{\Delta_{\text{PU}} + \Delta_{\text{noise}}}{\Sigma_{\text{PU}} + \Sigma_{\text{noise}}} \simeq k_{\text{PU}} \frac{\Delta_{\text{PU}}}{\Sigma_{\text{PU}}} + \boxed{k_{\text{PU}} \frac{\Delta_{\text{noise}}}{\Sigma_{\text{PU}}}}$$

Describing a resolution of a BPM

$$\boxed{\Delta_{\text{meas}}} = \boxed{\Delta_{\text{PU}}} + \boxed{\Delta_{\text{noise}}}$$

Signal we measured Actual signal read by electrodes Noise we don't want

For a beam near the center (small displacement):

$$A_{\text{PU}} \simeq B_{\text{PU}} = S$$

$$|A_{\text{noise}}|_{\text{rms}} \simeq |B_{\text{noise}}|_{\text{rms}} = N$$

$$k_{\text{PU}} \frac{|A_{\text{noise}} - B_{\text{noise}}|_{\text{rms}}}{A_{\text{PU}} + B_{\text{PU}}} = k_{\text{PU}} \frac{\sqrt{2}N}{2S} = \frac{k_{\text{PU}}}{\sqrt{2}} \left(\frac{S}{N} \right)^{-1}$$

Resolution $\propto$ SNR

- Minimum level of noise N limited by a thermal noise

$$v_{\text{noise}} = \sqrt{4k_B \boxed{T} \boxed{R} \boxed{\Delta f}}$$

Usually 300 K
Low-temperature may be beneficial

Broadband BPM $\rightarrow$ High thermal noise

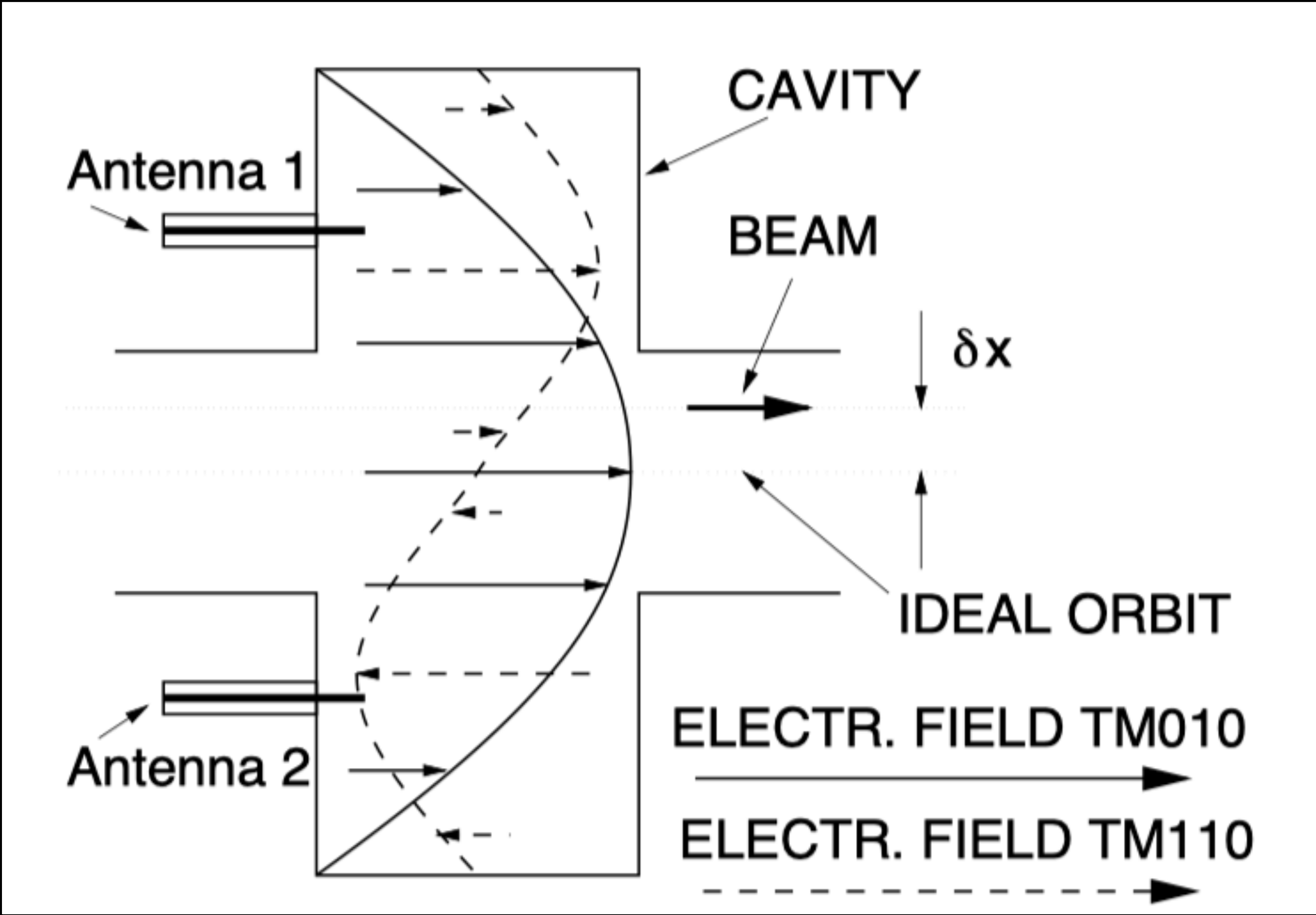
Button vs Stripline BPMs

	Button BPM	Stripline BPM	Cavity BPM
Coupling to the beam	Capacitive	Electromagnetic	Resonant
Signal shape	Bipolar differentiated pulse	Doublet pulse pair	Decaying sinusoid (ring-down)
Transfer impedance	$\sim 0.5 - 2 \Omega$	$\sim 1 - 5 \Omega$	$R_s \sim \text{k}\Omega/\text{mm}$
Frequency response	Broadband	Broadband	Narrowband
Typical resolution	10 - 50 μm (single bunch) $\sim 1 \mu\text{m}$ (multi-turn average)	1 - 10 μm ($< 1 \mu\text{m}$ with optimization)	100 nm - 1 μm (typical) Best: 8.7 nm (KEK ATF2)
Beam coupling impedance	Low	Medium	High
Electrode size	Small	Medium	Large
Optimal bunch length	Short bunches	Long bunches	Short bunches
Beam type	Electron beam	Proton and Ion beams	Electron beam
DC/debunched beam	No	No	No
Primary use cases	Storage rings, damping rings, collider rings	Proton linacs, Ion accelerators, hadron synchrotrons	FEL undulator sections, high-precision linac sections, IP

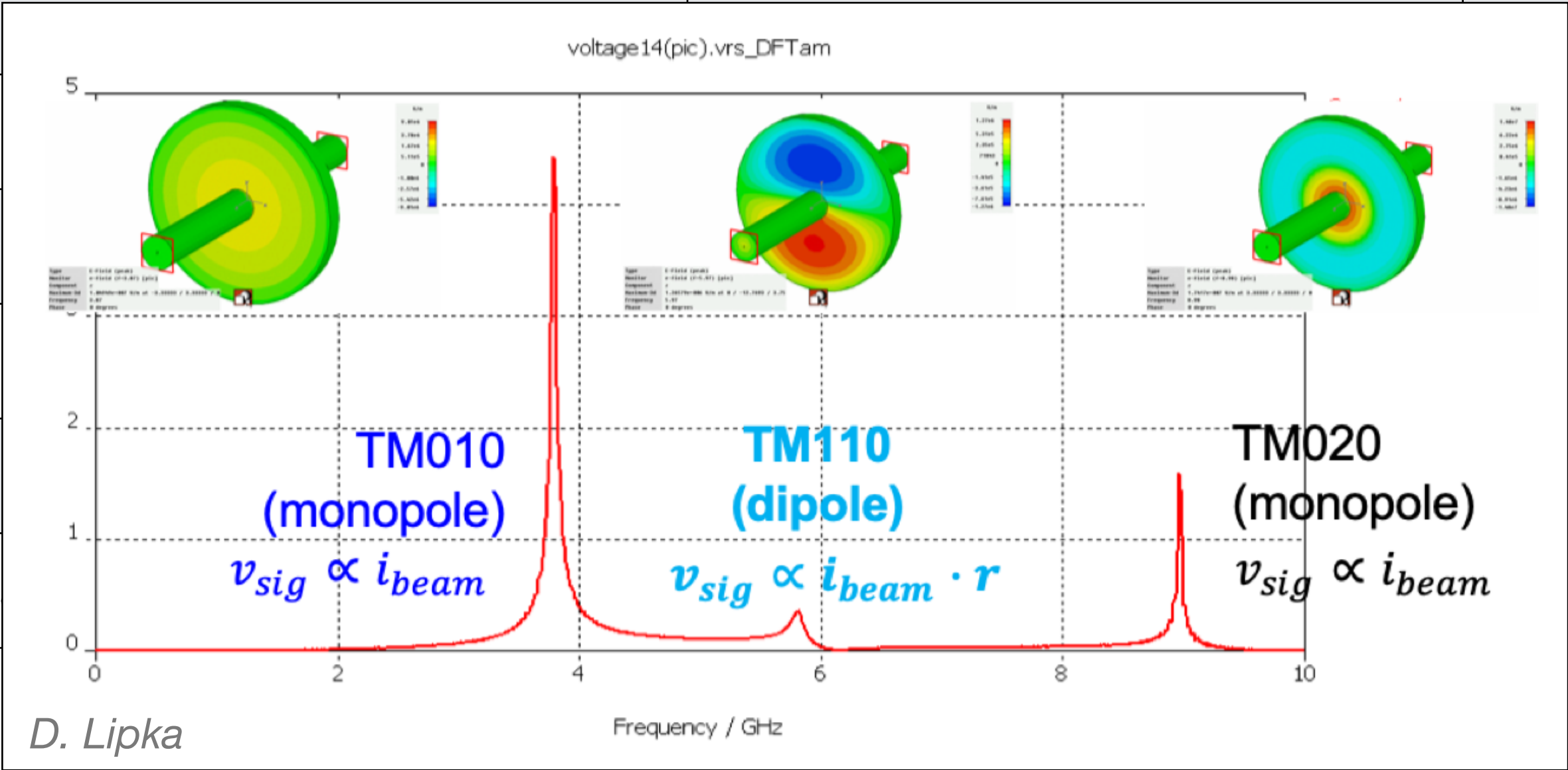
Cavity BPM



	Button BPM	Stripline BPM	Cavity BPM
Coupling to the beam	Capacitive	Electromagnetic	Resonant
Signal shape	Bipolar differentiated pulse	Doublet pulse pair	Decaying sinusoid (ring-down)
Transfer impedance	$\sim 0.5 - 2 \, \Omega$	$\sim 1 - 5 \, \Omega$	$R_s \sim \text{k}\Omega/\text{mm}$
Frequency response	Broadband	Broadband	Narrowband
Typical resolution	10 - 50 μm (single bunch) $\sim 1 \, \mu\text{m}$ (multi-turn average)	1 - 10 μm ($< 1 \, \mu\text{m}$ with optimization)	100 nm - 1 μm (typical) Best: 8.7 nm (KEK ATF2)
Beam coupling impedance			High



R. Lorenz

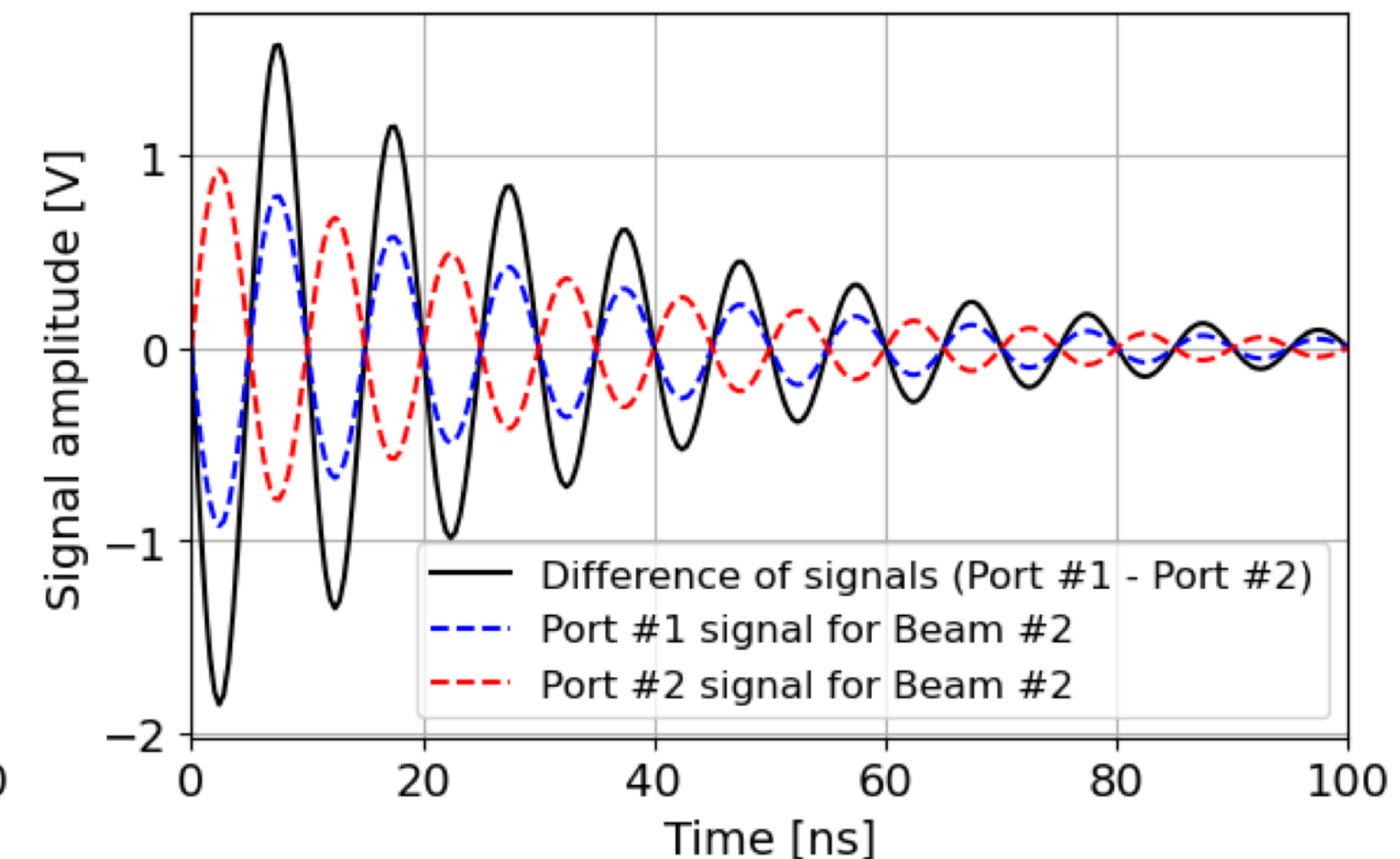
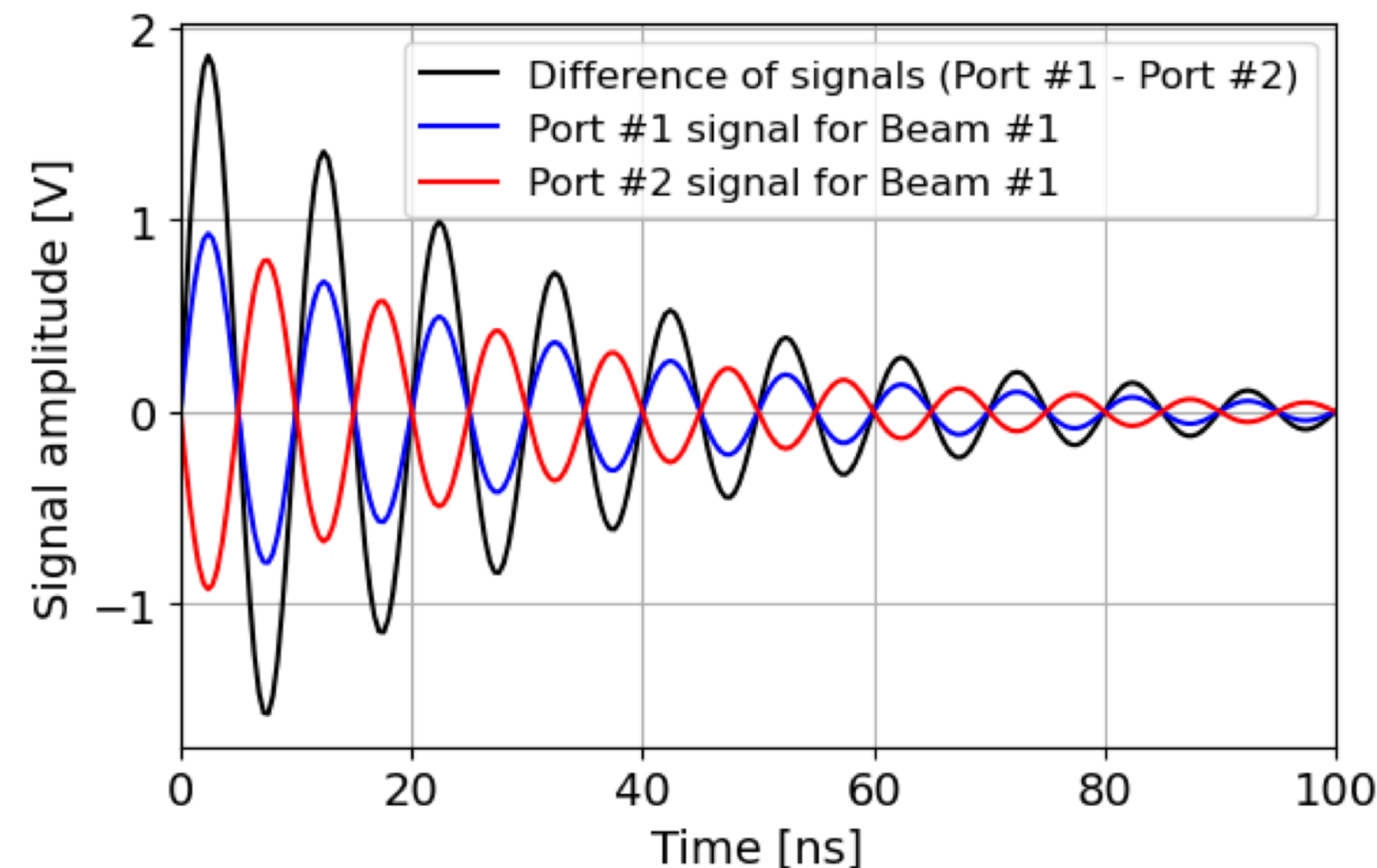
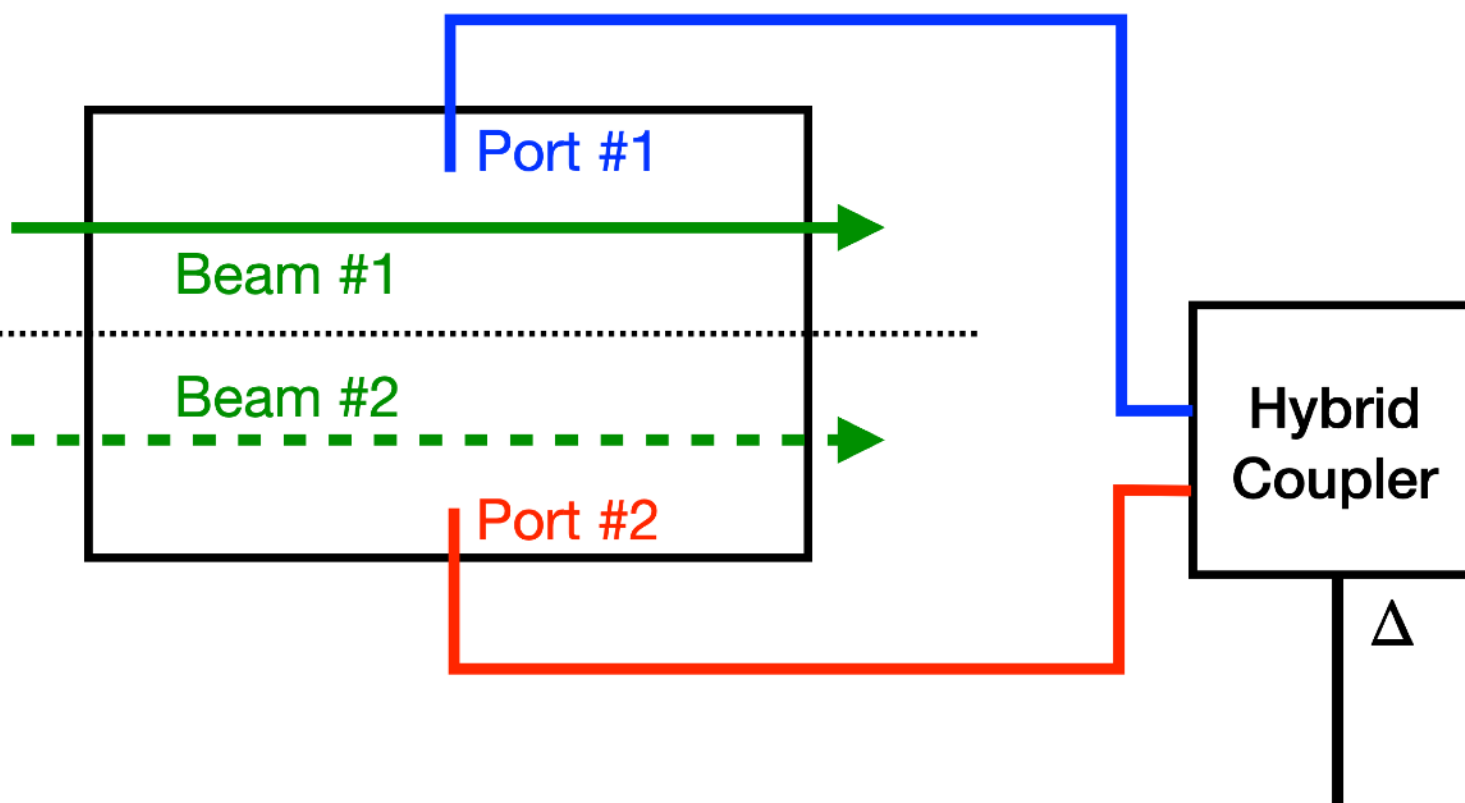
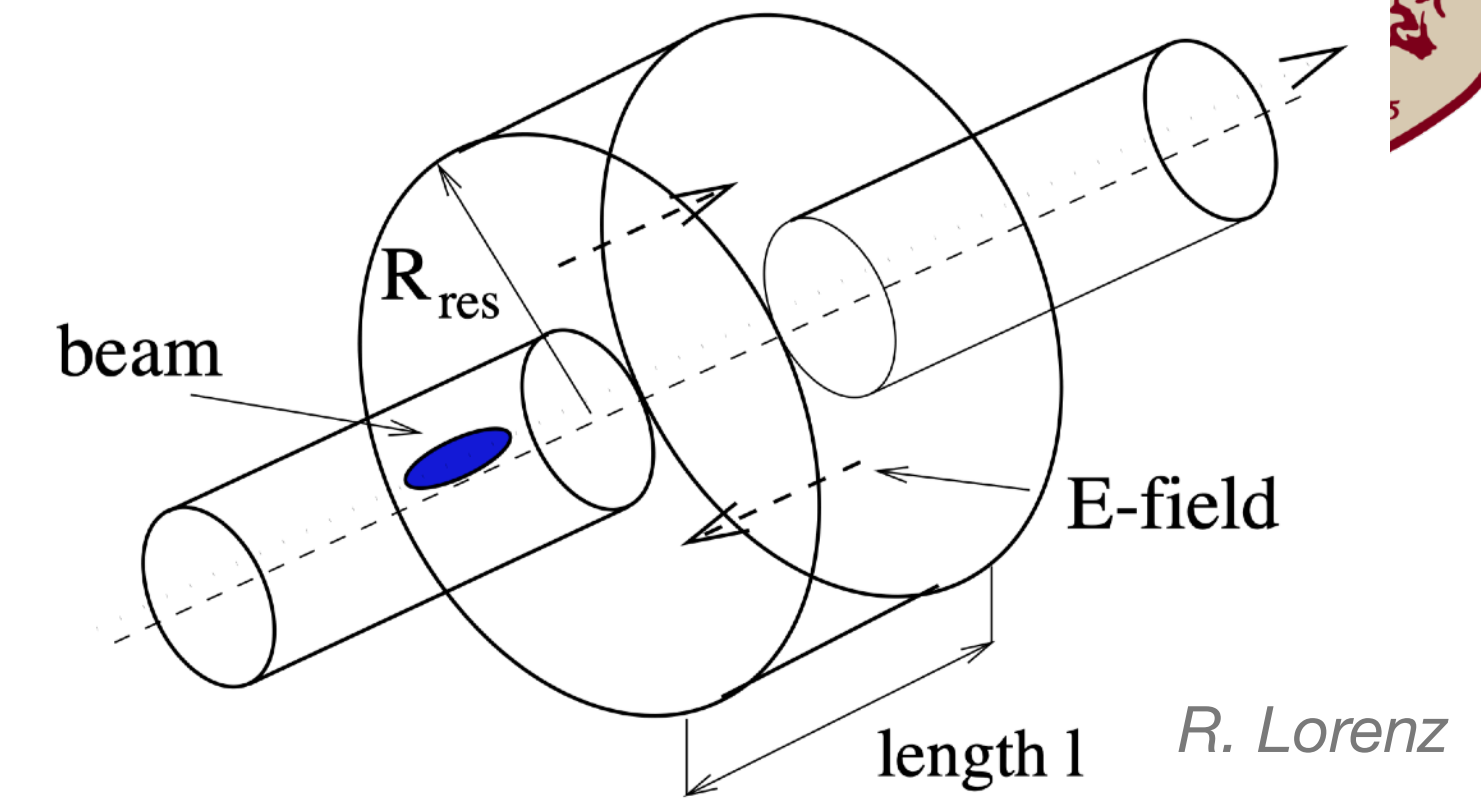


D. Lipka

High
Large
Short bunches
Electron beam
No
undulator sections, high-precision linac sections, IP

Cavity BPM Basics

- Dipole mode (TM_{110}) is used since the dipole mode signal is proportional to the offset of the beam from the cavity center
 - Same offset, opposite direction $\rightarrow$ same amplitude, opposite phase
 - If beam passes through the center of the cavity, no signal appears
 - Single cavity BPM cannot determine the absolute position, but an offset from the center
 - To determine the absolute position, another cavity BPM (reference cavity) is necessary

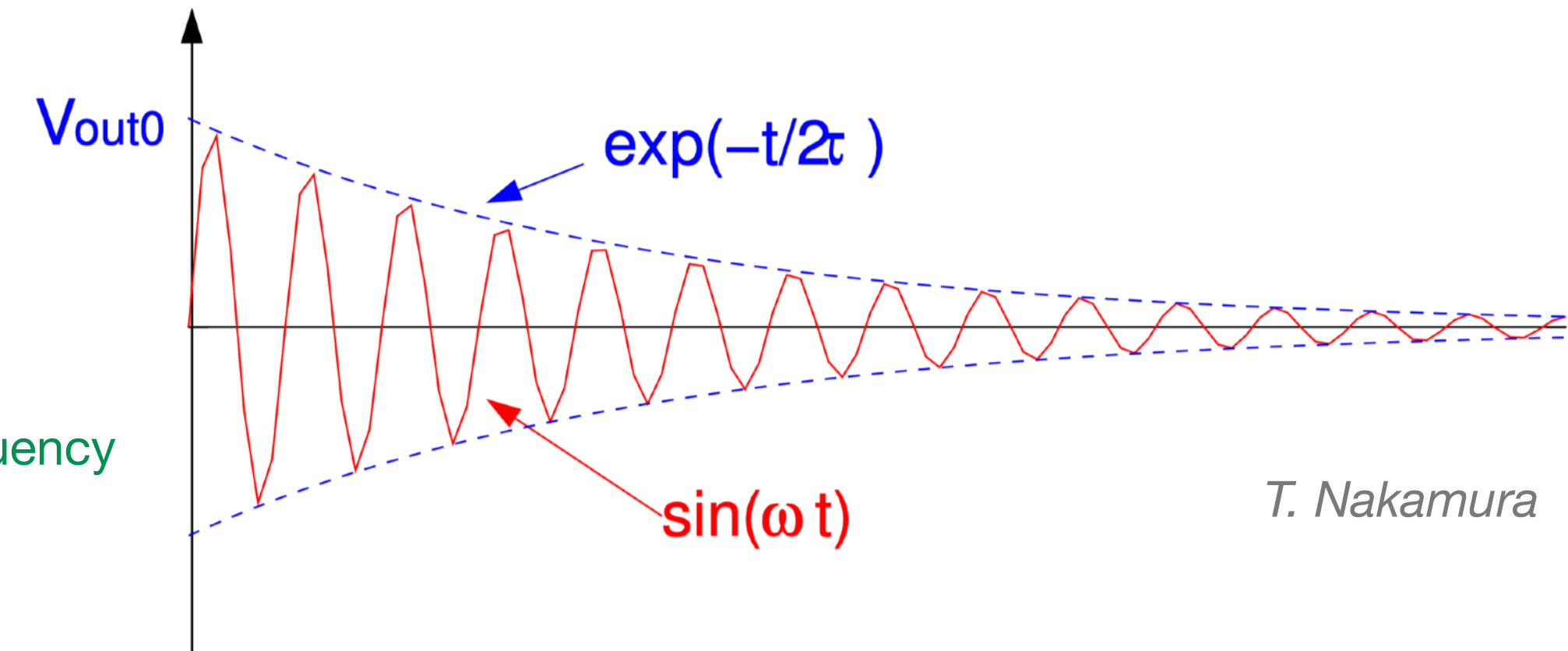


Output Signals

- Pickup signal from a cavity BPM

$$V_{\text{out}} = V_0 e^{-\frac{t}{2\tau}} \sin(\omega t + \phi)$$

ω : Cavity TM₁₁₀ resonant frequency
 ϕ : Phase of BPM signal



T. Nakamura

$$V_0 = \frac{\omega q}{2} \sqrt{\frac{Z}{Q_{\text{ext}}}} \left(\frac{R}{Q} \right) \exp\left(-\frac{\omega^2 \sigma_z^2}{2c^2}\right)$$

From the Gaussian beam shape
 σ_z : Beam bunch length

q : Charge of the beam
 Z : Load impedance (usually 50 Ω)

$$Q_{\text{ext}} = Q_0 / \beta$$

Q_0 : Unloaded quality factor of the cavity TM₁₁₀ mode
 β : Coupling coefficient

$$\frac{R}{Q} \simeq 50.3 \times \left(\frac{\omega}{c} \right)^3 L T^2 x^2$$

L : Cavity length
 c : Speed of light
 x : Beam offset from the cavity center
 T : Transit time factor (phase transition during the beam passing time)

or,

$$Q_0 \equiv \frac{\omega U}{P_{\text{wall}}} \text{ and } Q_{\text{ext}} \equiv \frac{\omega U}{P_{\text{out}}}$$

U : Stored energy in a cavity eigenmode
 P_{wall} : Dissipated power at the cavity wall
 P_{out} : Output power to the circuit

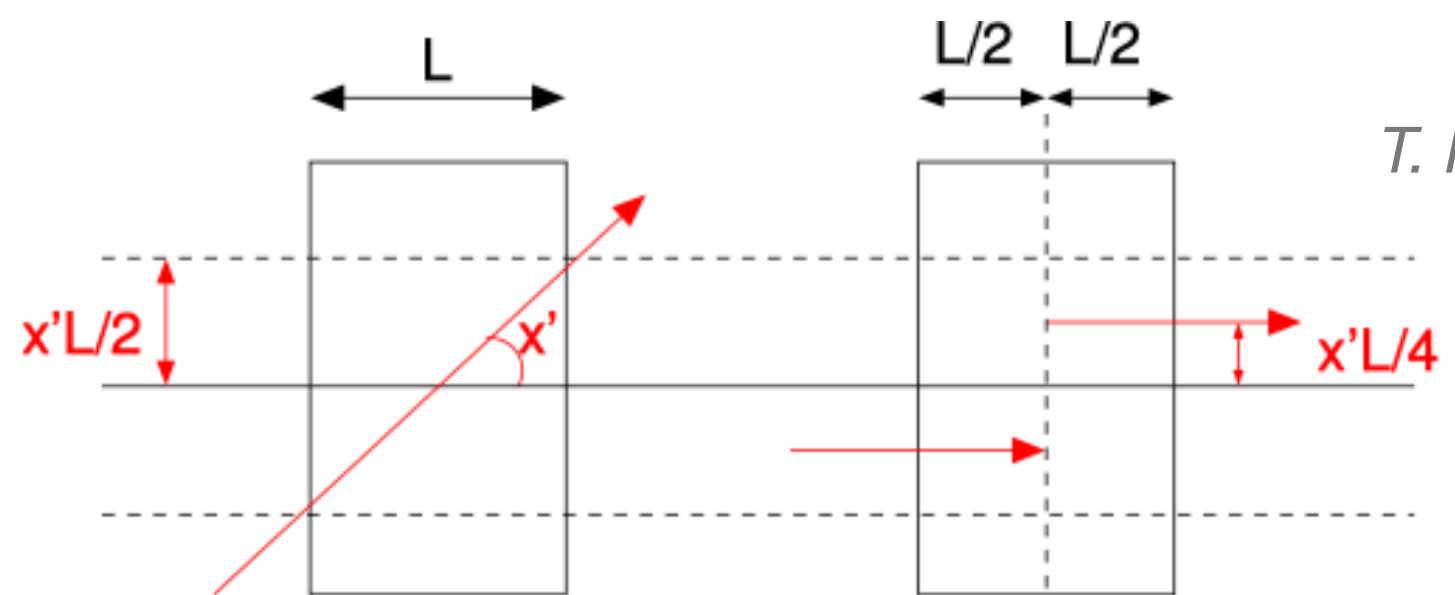
Noise Components

- Other contributions in the signal we detect:

$$V = V_{\text{out}} + i(V_{\text{angle}} + V_{\text{tilt}} + V_{\text{common}}) + (\text{others} \dots)$$

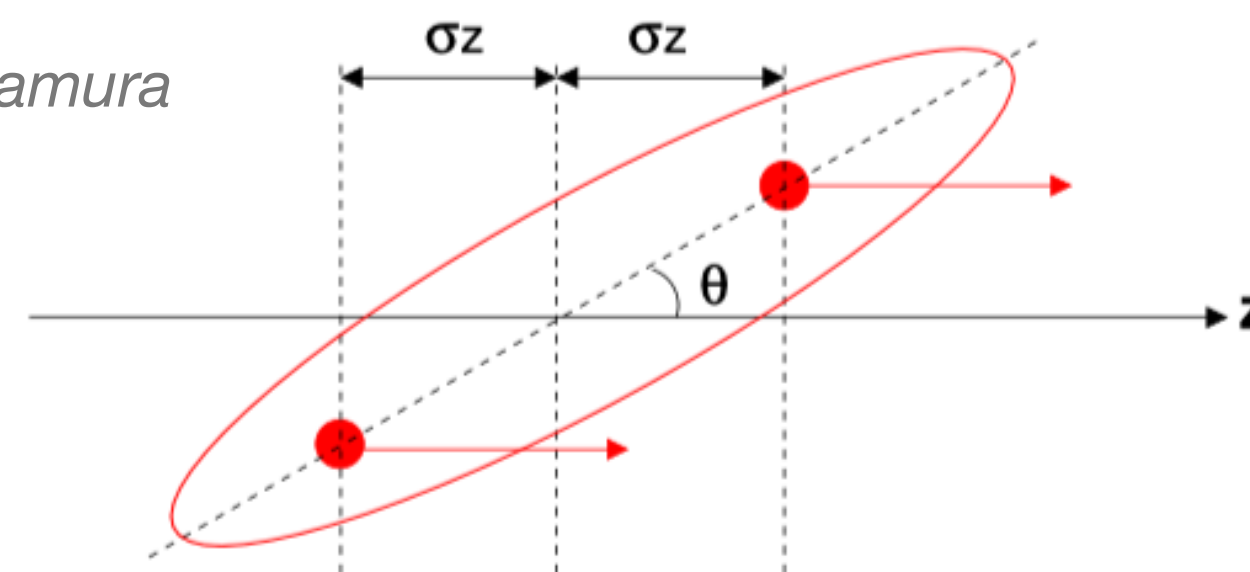
 "signal" we desire 90° off-phase "noise" we don't want 

- V_{angle} : Effect of the beam NOT passing through the BPM straight
- V_{tilt} : Effect of tilted beam bunch
- V_{common} : Contribution from the common mode tail
- Others: Electronics noise, and so on

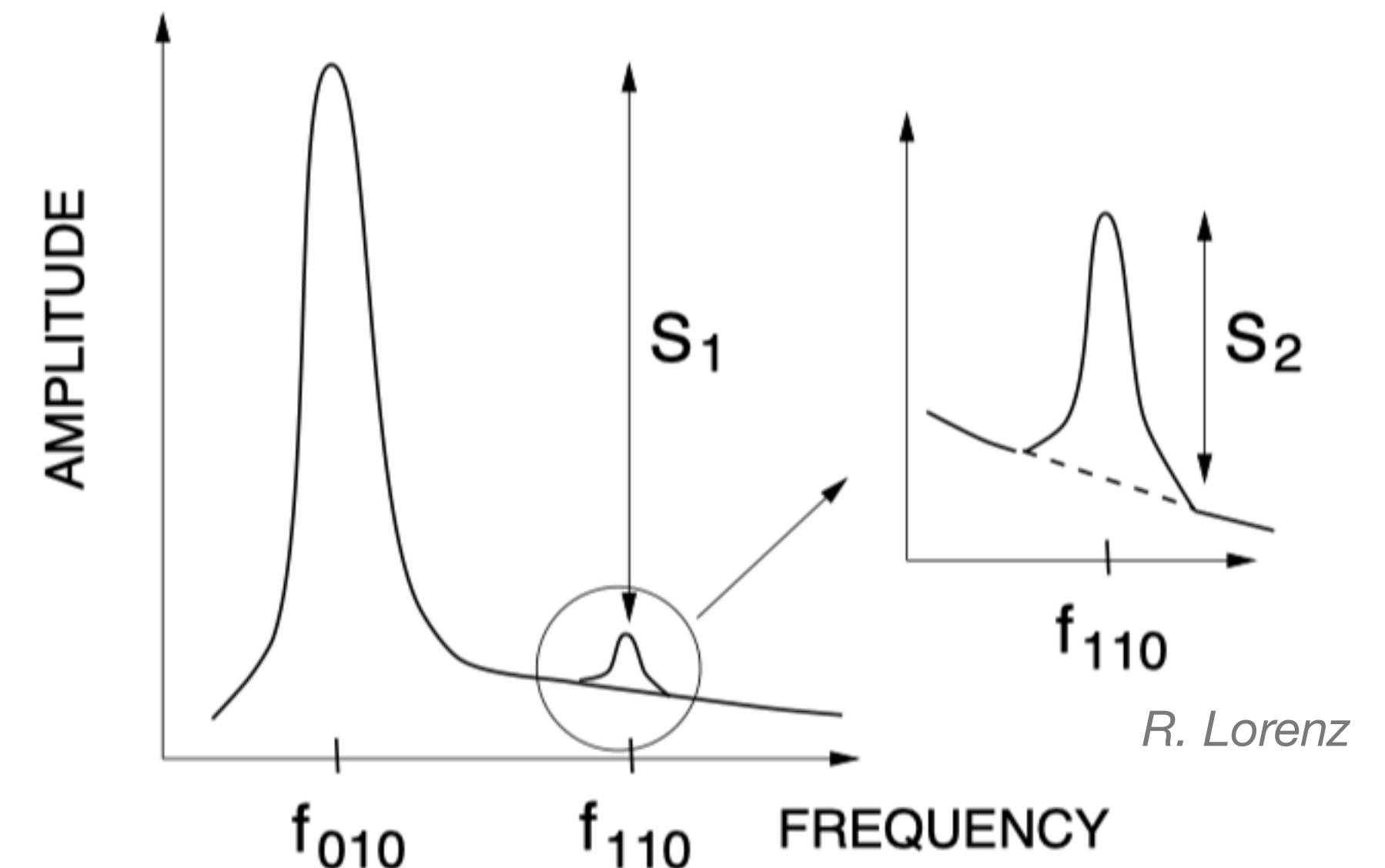


Beam with an angle x'

T. Nakamura



Tilted beam by θ



Common mode tail

Noise Components

- Beam angle signals (V_{angle})

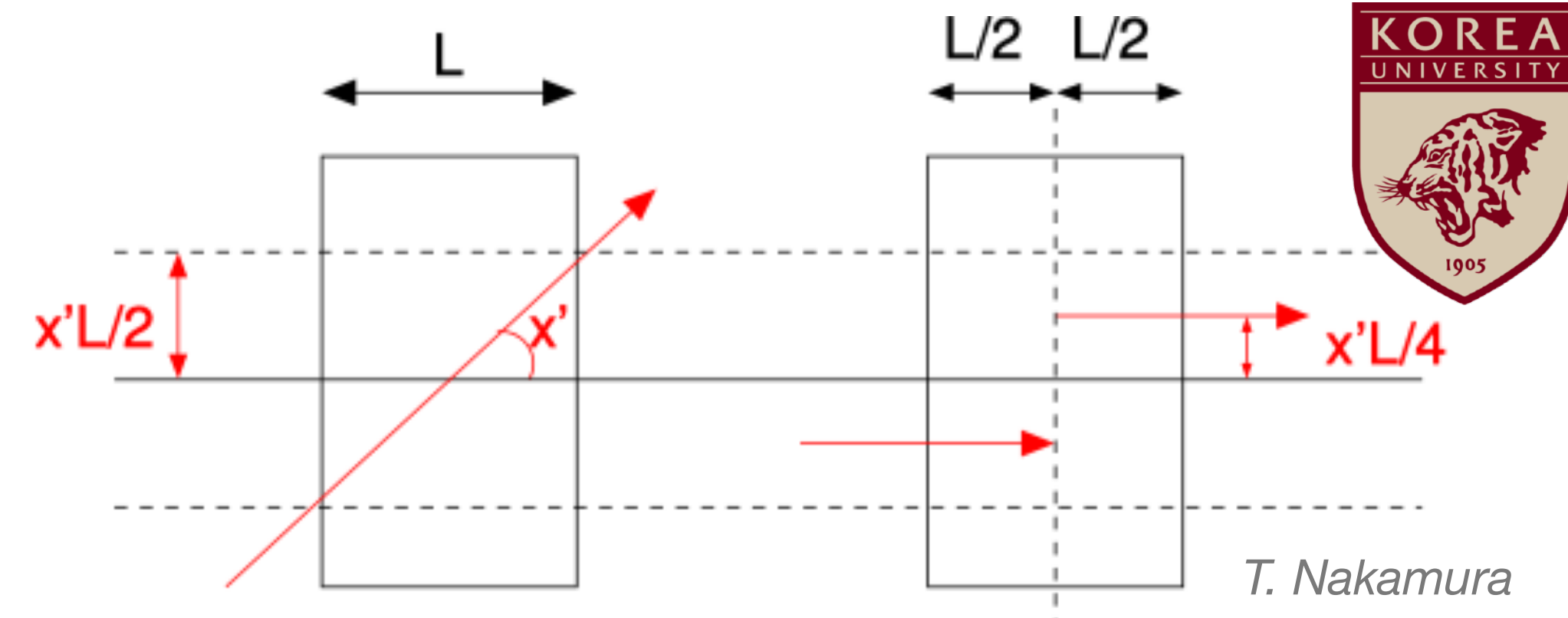
- Straight beam position signal at x : $V_{\text{out}} = Ax\sqrt{L} \sin(\omega t)$
- Angled beam can be modeled with two separate beam positions (with $\tan x' \simeq x'$) at:
 $x = -x' \frac{L}{4}$ for $-\frac{L}{2} < z < 0$ and $x = +x' \frac{L}{4}$ for $0 < z < +\frac{L}{2}$

$$V_{\text{angle}} = -A \frac{x'L}{4} \sqrt{\frac{L}{2}} \sin \left[\omega \left(t + \frac{L}{4c} \right) \right] + A \frac{x'L}{4} \sqrt{\frac{L}{2}} \sin \left[\omega \left(t - \frac{L}{4c} \right) \right]$$

$$= Ax' \frac{L}{2} \sqrt{\frac{L}{2}} \sin \left(\frac{\omega L}{4c} \right) \boxed{\cos(\omega t)}$$

90° off-phase

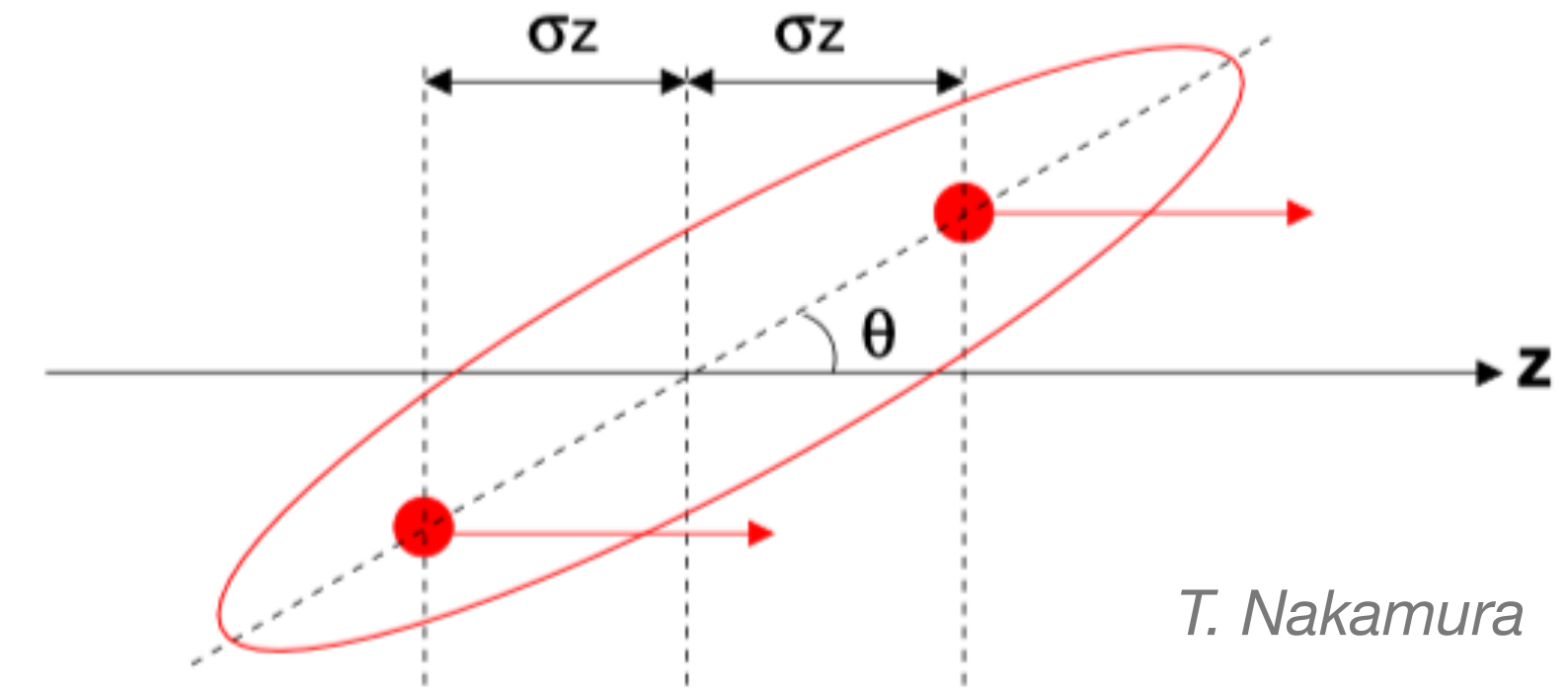
$$\frac{|V_{\text{angle}}|}{|V_{\text{out}}|} = \frac{L}{2\sqrt{2}} \sin \left(\frac{\omega L}{4c} \right) \frac{x'}{x} \simeq \frac{\omega L^2}{8\sqrt{2}c} \frac{x'}{x} \rightarrow \frac{|V_{\text{angle}}|}{|V_{\text{out}}|} \propto L^2 x'$$



Noise Components

- Bunch tilt signal (V_{tilt})

- For non-tilted beam at x : $V_{\text{out}} = Aqx \sin(\omega t)$
- Tilted bunch can be modeled with two point charges at:
 $x = -\sigma_z \theta$ and $x = +\sigma_z \theta$ with $q \rightarrow q/2$ for each



$$V_+ = A \frac{q}{2} (\sigma_z \theta) \sin \left[\omega \left(t + \frac{\sigma_z}{c} \right) \right] \quad V_- = A \frac{q}{2} (-\sigma_z \theta) \sin \left[\omega \left(t - \frac{\sigma_z}{c} \right) \right]$$

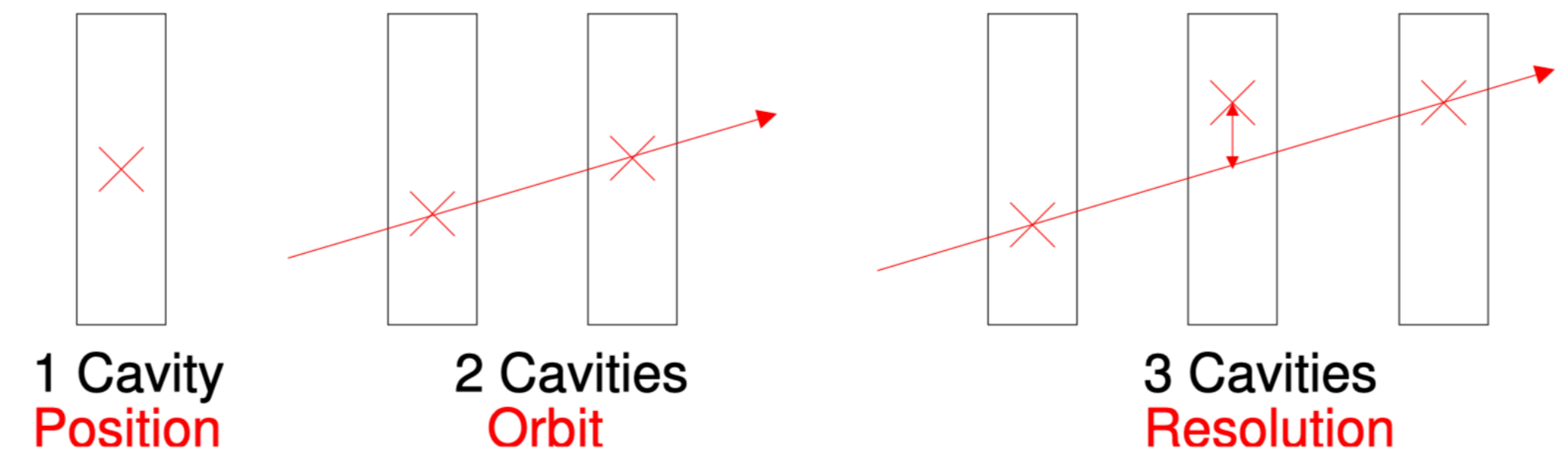
$$V_{\text{tilt}} = V_+ + V_- = Aq\sigma_x\theta \sin \left(\frac{\omega\sigma_z}{c} \right) \cos(\omega t) \simeq \frac{Aq\theta\omega\sigma_z^2}{c} \boxed{\cos(\omega t)}$$

90° off-phase

$$\frac{|V_{\text{tilt}}|}{|V_{\text{out}}|} \simeq \frac{\omega\sigma_z^2}{c} \frac{\theta}{x} \rightarrow \frac{|V_{\text{angle}}|}{|V_{\text{out}}|} \propto \sigma_z^2 \theta$$

Position Resolution of BPM

- The position resolution is a critical factor of a BPM
 - Precision of the position measurement should be comparable to beam size
 - e.g. $\sigma_y = 5.9$ nm at ILC IP
- Cavity BPM is the only easy option to achieve <1 μm position resolution
 - Button BPM: $\mathcal{O}(10)$ μm
 - Stripline BPM: $\mathcal{O}(1) - \mathcal{O}(10)$ μm
- Position resolution measurements → “3 BPM method”
 - Calibration run: Obtaining a translation factor from arbitrary FFT value to real world quantity
 - Resolution run: Data at a beam position to measure the resolution

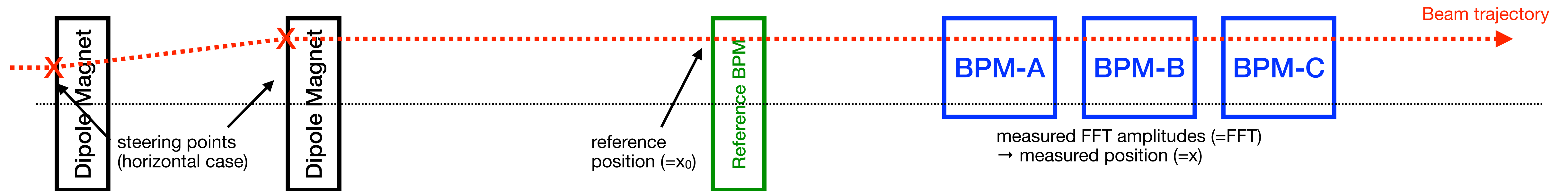


T. Nakamura

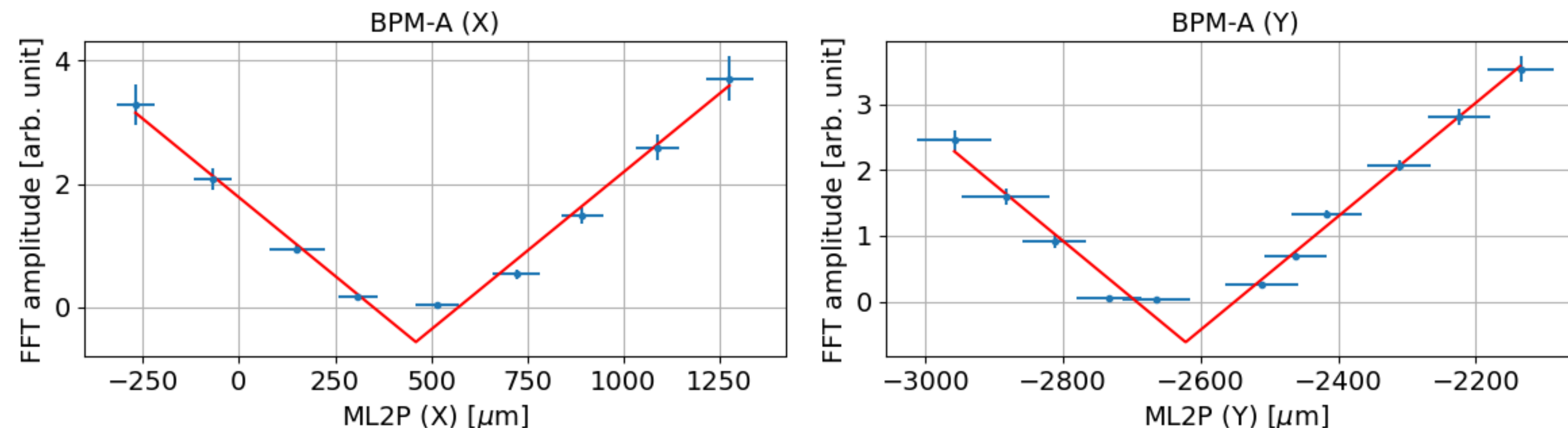
Resolution Measurements of Cavity BPMs

- Calibrations

- Changing the beam orbit by steering magnets
- Referring a reference BPM



- Mapping FFT to x_0 : obtaining $\Delta\text{FFT}/\Delta x_0$
 - V-shape appears since the cavity BPM does not know the absolute position of the beam



Resolution Measurements of Cavity BPMs

- Resolution measurements
 - Finding a correlation between all channels and the one of interest (k -th channel):

$$\mathbf{d}_k = \mathbf{D}_k \cdot \mathbf{v}$$

$\mathbf{d}_k$: measured positions of the k -th channel

$\mathbf{D}_k$: measured positions not of the k -th channel but of the others

$\mathbf{v}$: correlation coefficients

- Once we get $\mathbf{v}$, predictions of measurements can be made by:

$$\mathbf{d}_k^{\text{pred.}} = \mathbf{D}_k \cdot \mathbf{v}$$

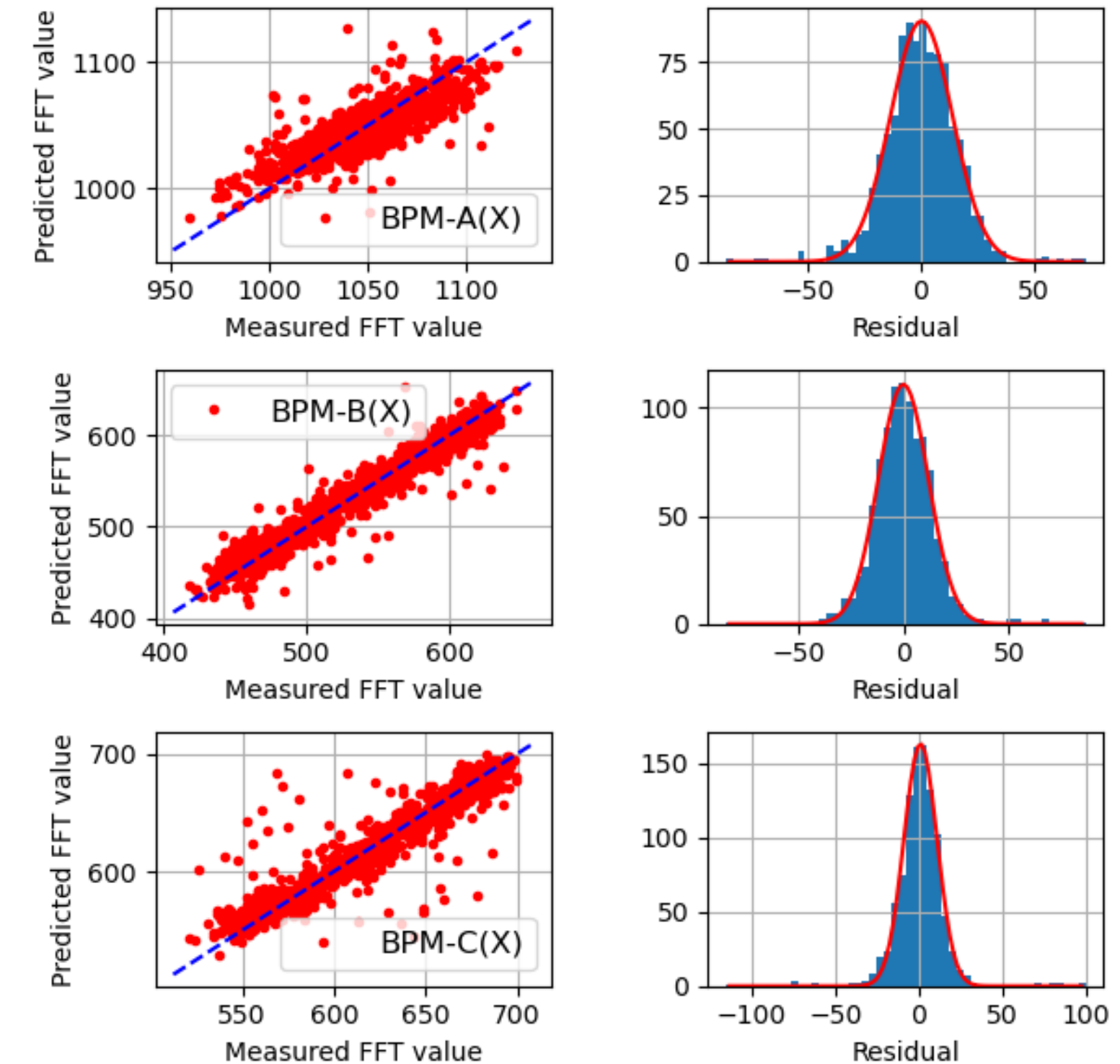
- Singular value decomposition (SVD)

$$\mathbf{D}_k = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \rightarrow \mathbf{v} = (\mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^T) \cdot \mathbf{d}_k$$

- $\mathbf{d}_k$ is still in FFT amplitude, so it needs to be translated by the calibration factors

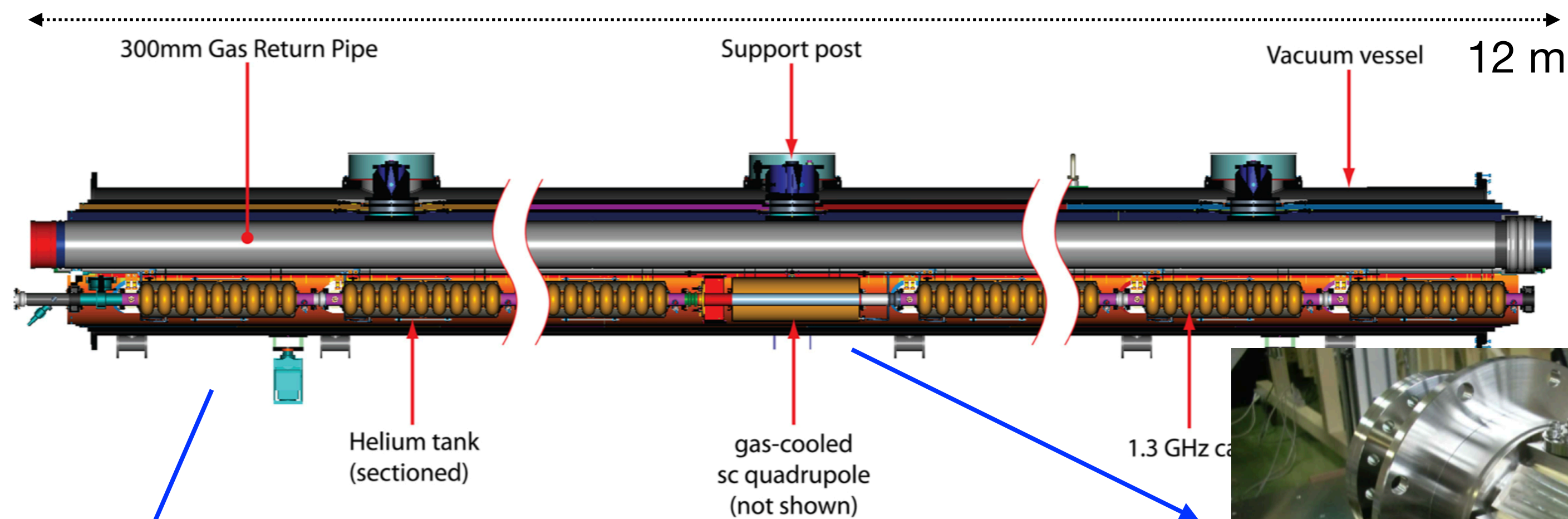
$$\mathbf{d}^{\mu\text{m}} = \mathbf{d}^{\text{FFT}} \cdot \frac{\Delta x_0}{\Delta \text{FFT}}$$

- From the residual ($\mathbf{R} \equiv \mathbf{d}_k^{\text{pred.}} - \mathbf{d}_k^{\text{meas.}}$), the resolutions (σ) are obtained



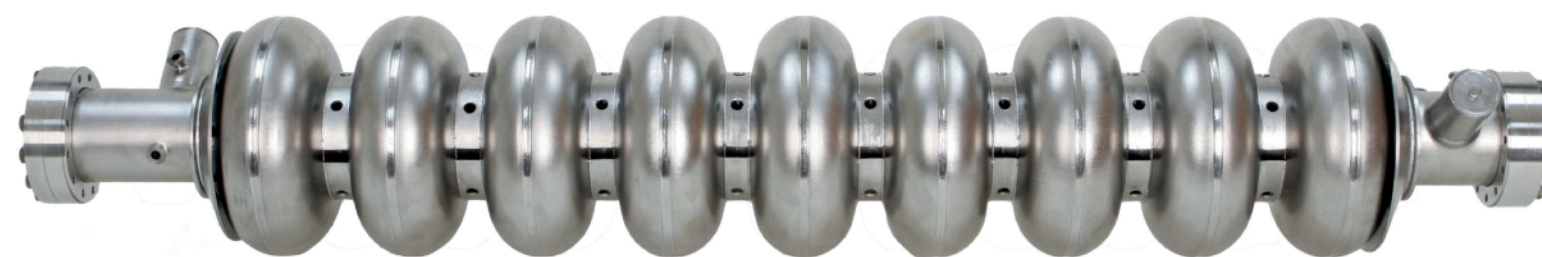
L-band Cavity BPM for ILC

- Two types of cryomodule in the main LINAC of ILC
 - Type A (~1,100 in total): 1.3 GHz 9-cell cavities * 9
 - Type B (~600 in total, at every 3 modules): 1.3 GHz 9-cell cavities * 8 + a SC quadrupole package (quadrupole/corrector/[BPM](#))
- BPM resolution requirement: $5 \mu\text{m}$ (TDR), $\mathcal{O}(100) \text{ nm}$ (goal)

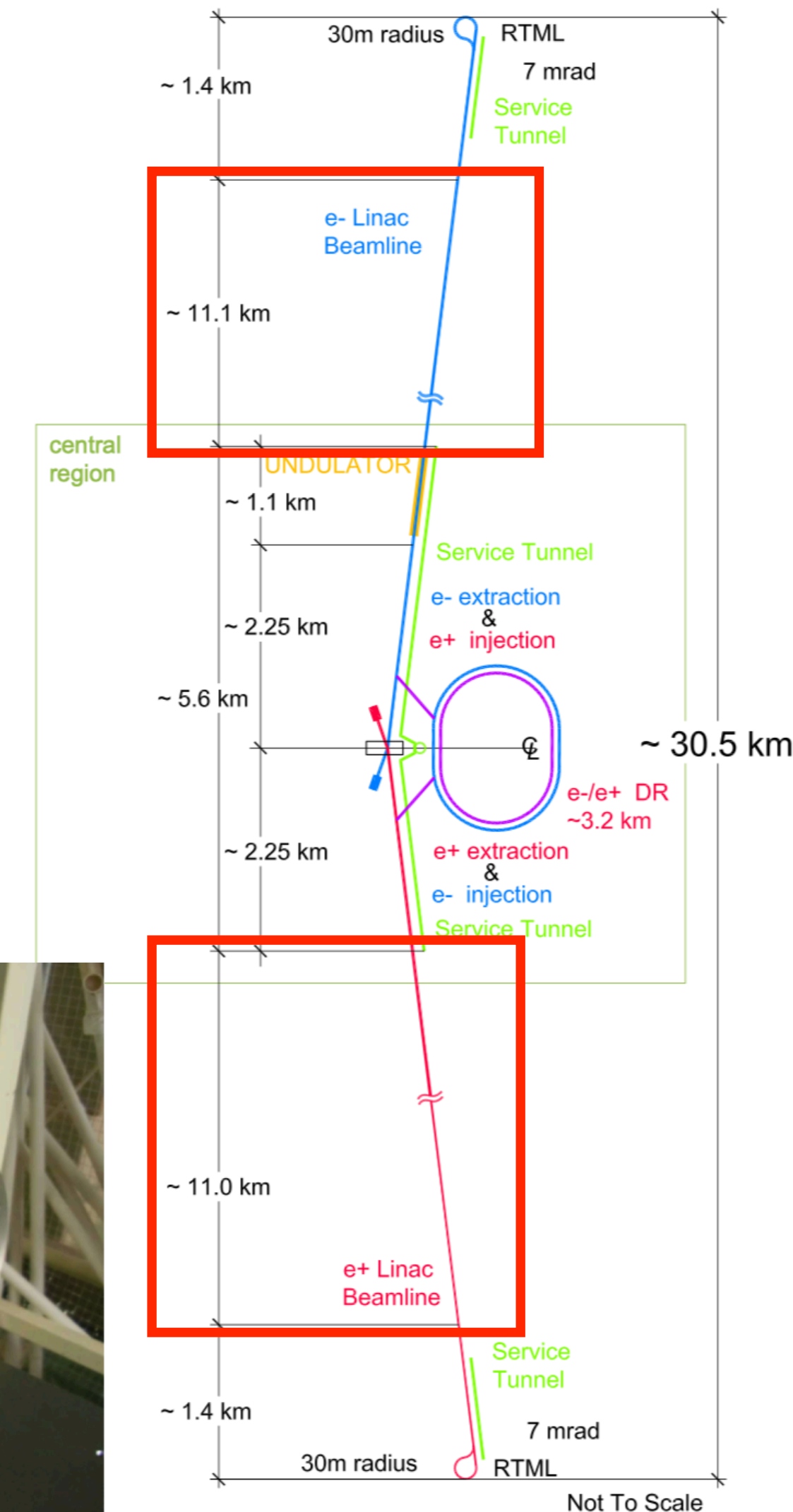
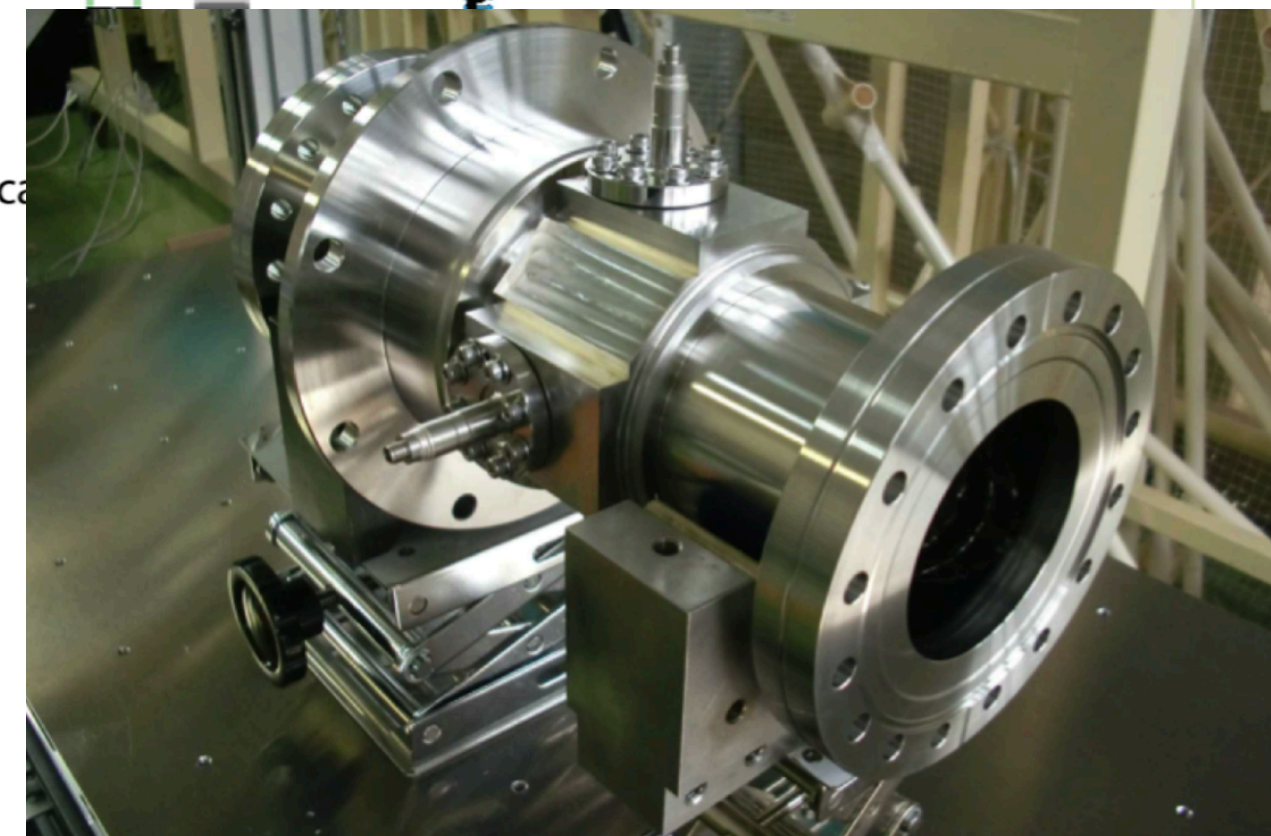


ILC TDR

L-band re-entrant cavity BPM

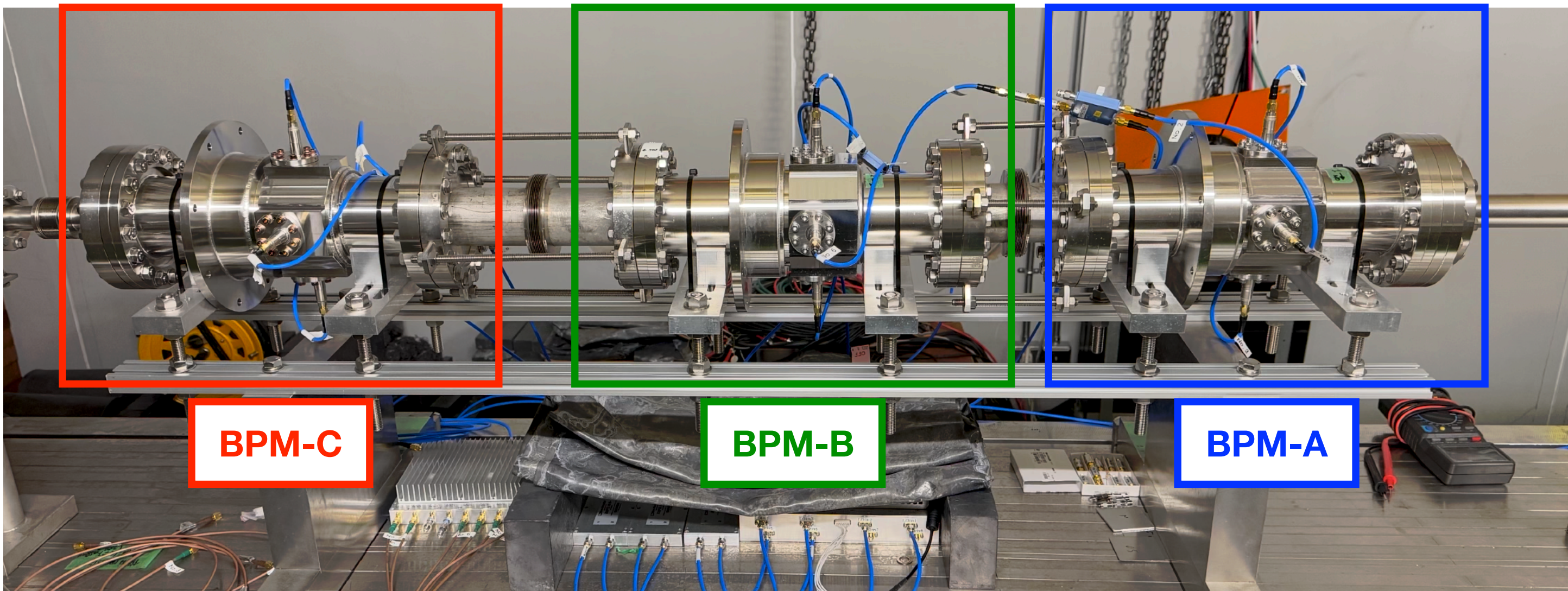
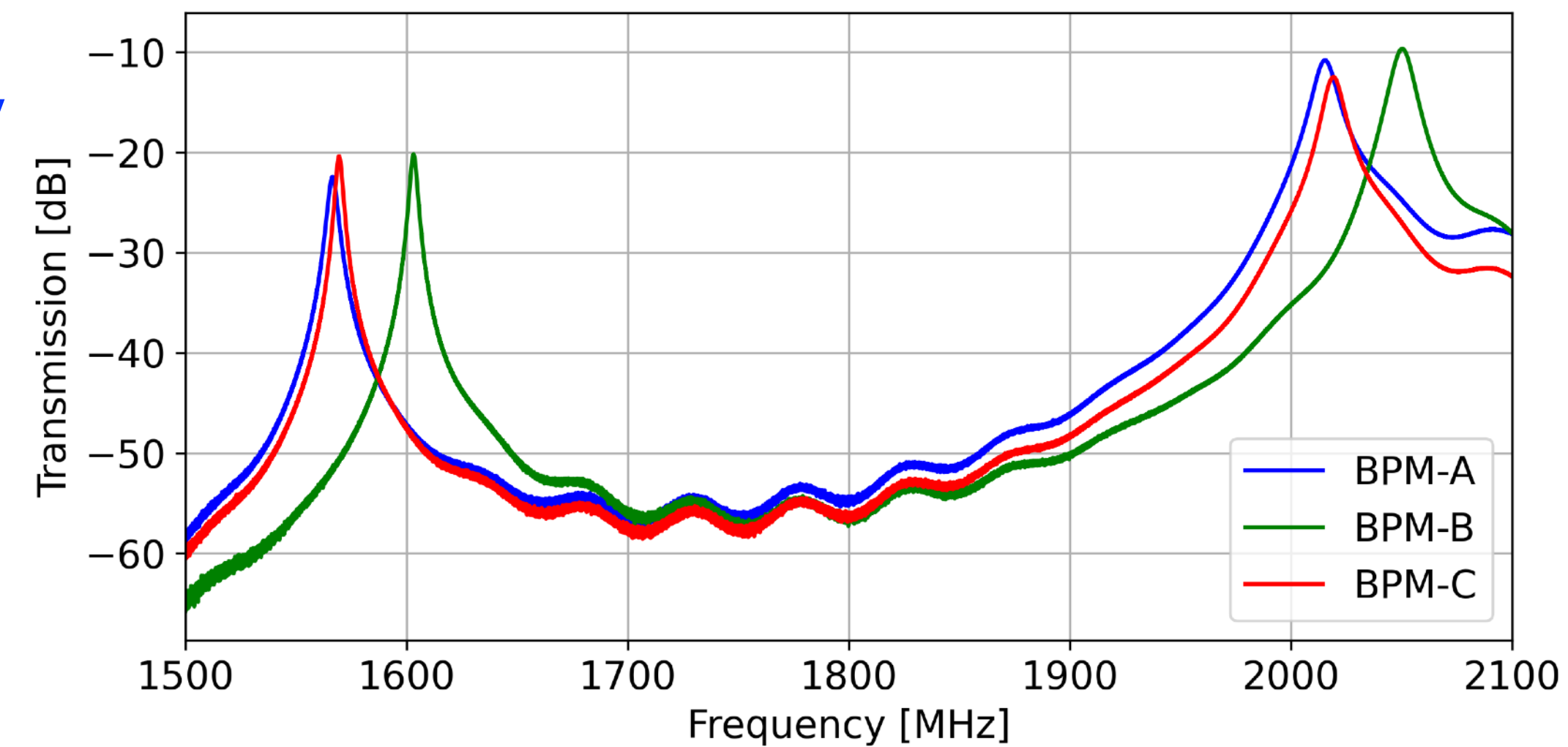


1.3 GHz 9-cell cavity (Nb)



L-band Cavity BPM Prototype

- Three prototypes were fabricated by KNU
 - $f_{110} \sim 2$ GHz with $Q_{110} \sim 200$
 - Low quality factor necessary for a reasonably low cavity decay time constant
 - Slightly different f_{110} due to the machining tolerance

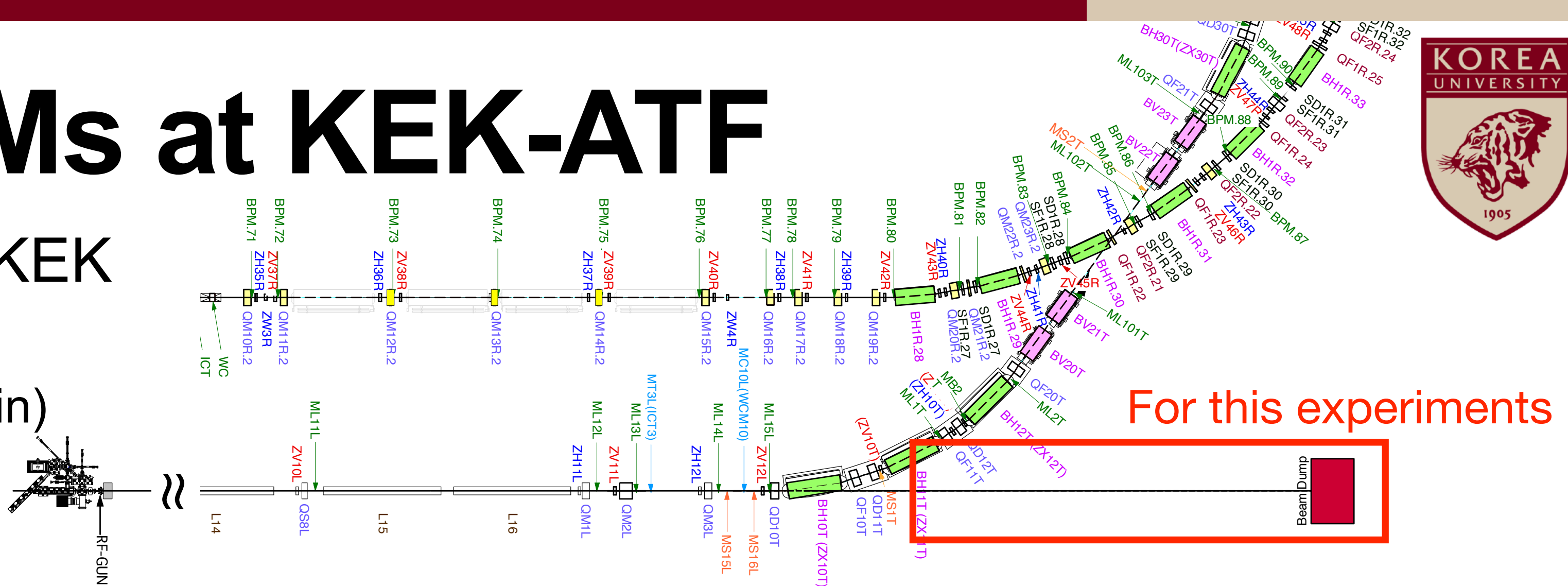


	TM ₀₁₀		TM ₁₁₀	
	f_x [MHz] f_y [MHz]	Q_x Q_y	f_x [MHz] f_y [MHz]	Q_x Q_y
BPM-A	1566.59 1566.60	320 320	2015.53 2015.48	170 187
BPM-B	1603.20 1603.18	444 607	2050.34 2050.26	186 195
BPM-C	1569.54 1569.54	441 445	2019.32 2021.60	197 219

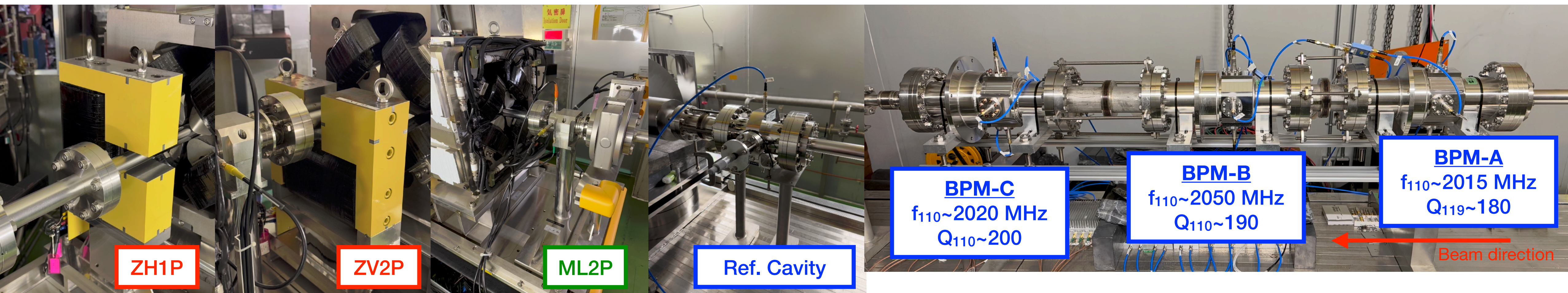
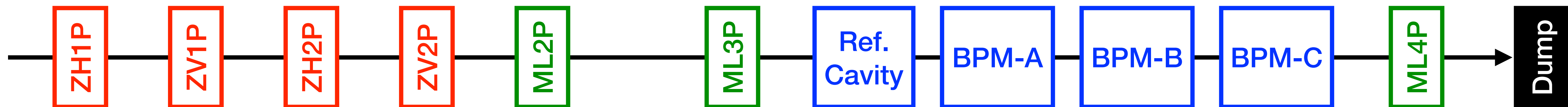
L-band Cavity BPMs at KEK-ATF

- Accelerator Test Facility (ATF) at KEK

- 1.3 GeV electron LINAC of 110 m long
- $\sim 10^{10}$ electrons/bunch (1~20 bunches/train)
- Repetition: 3.125 Hz

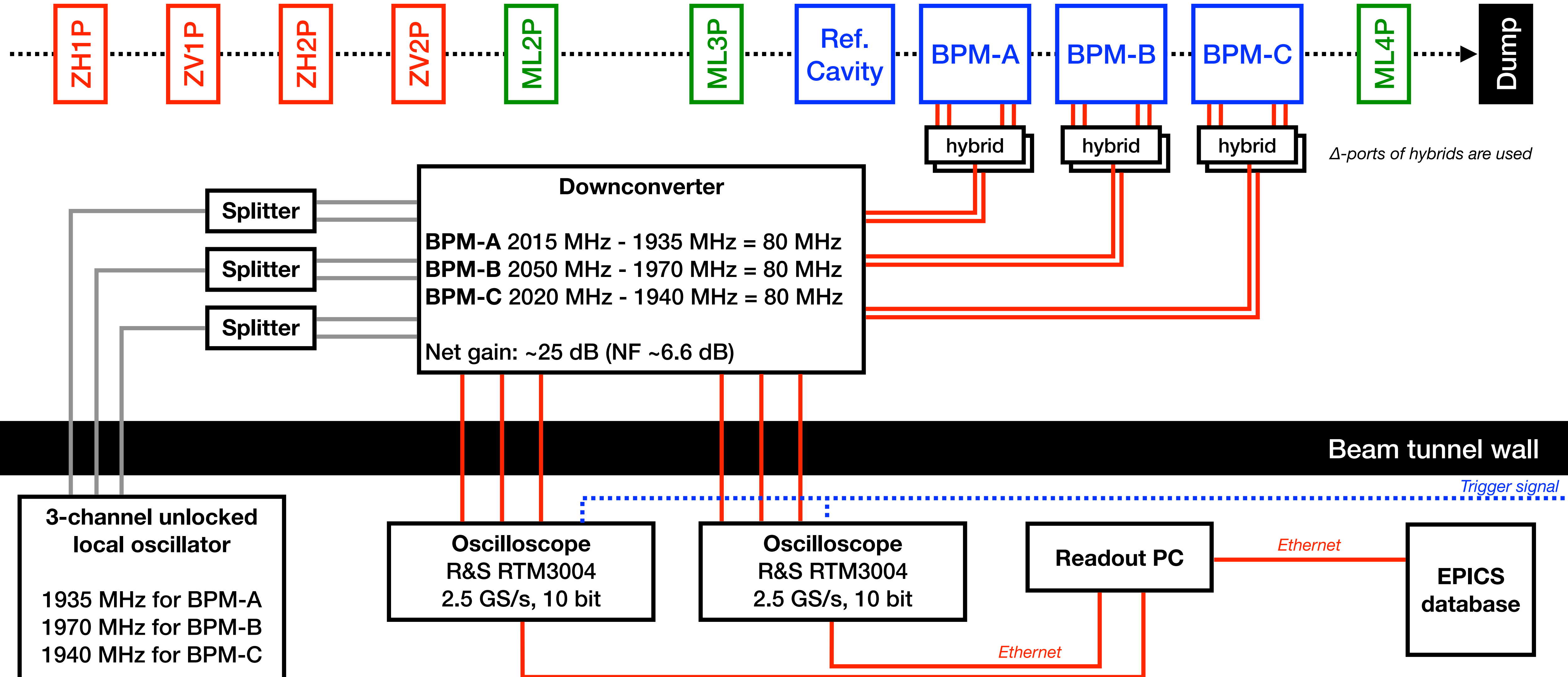


- At the end of the LINAC, 4 steering magnets, 4 stripline BPMs, and 4 cavity BPMs are located



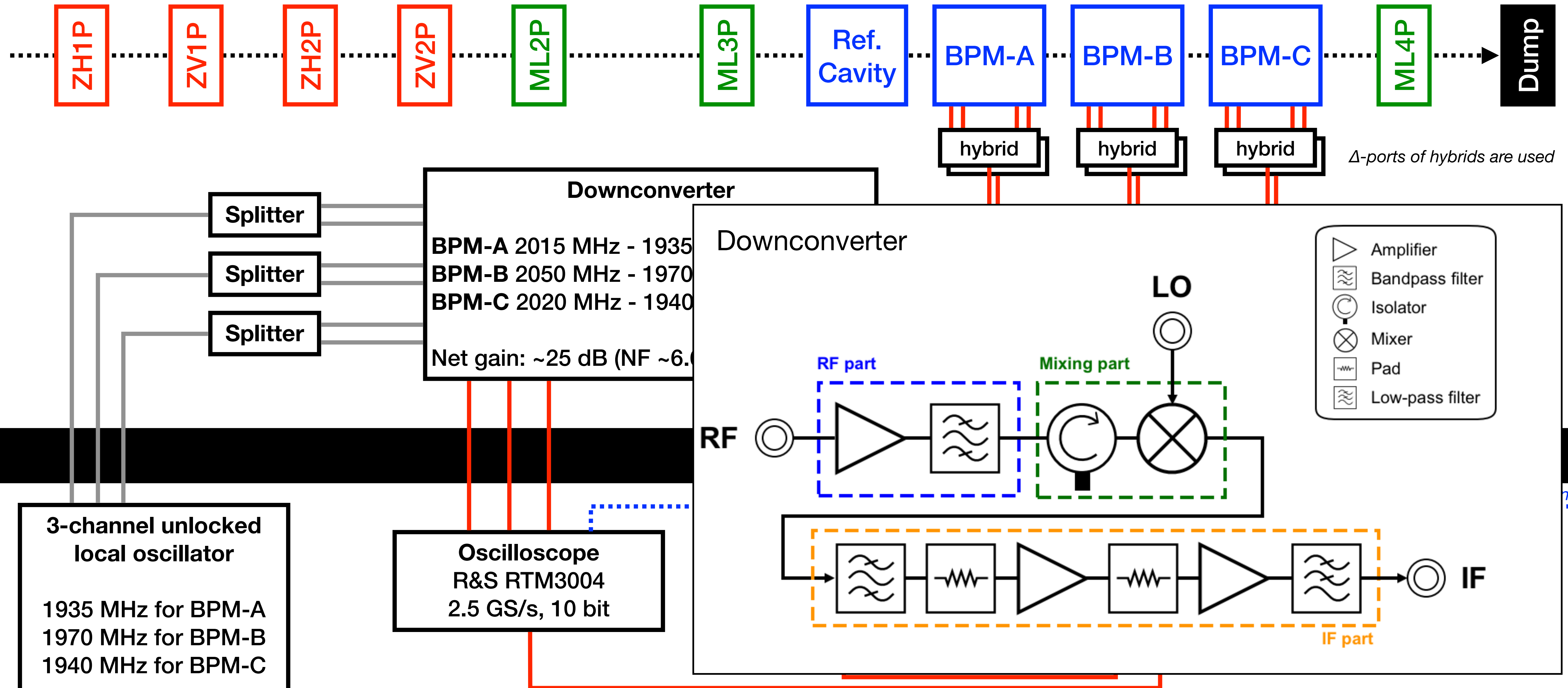
L-band Cavity BPM Readouts

- Signals are merged by hybrid couplers and pass through a downconverter

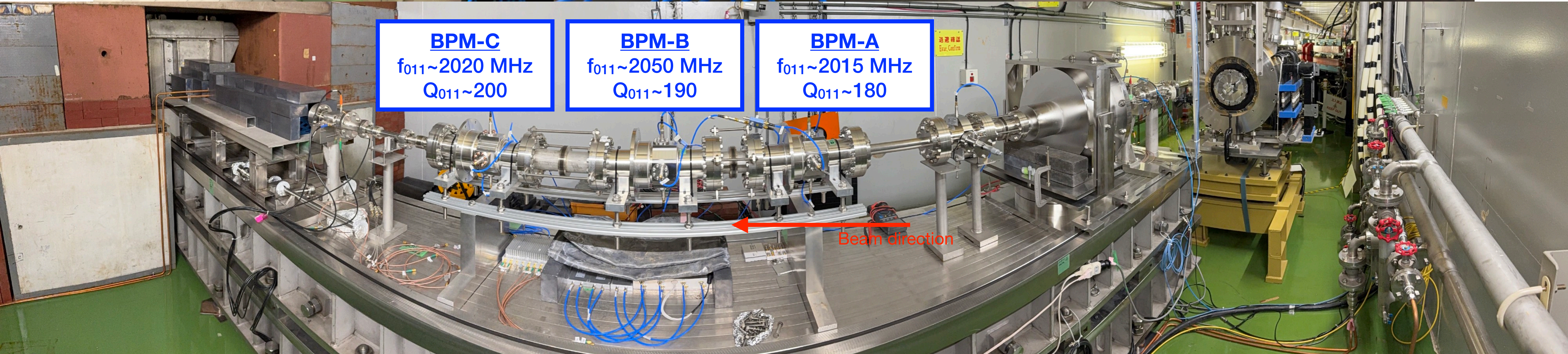
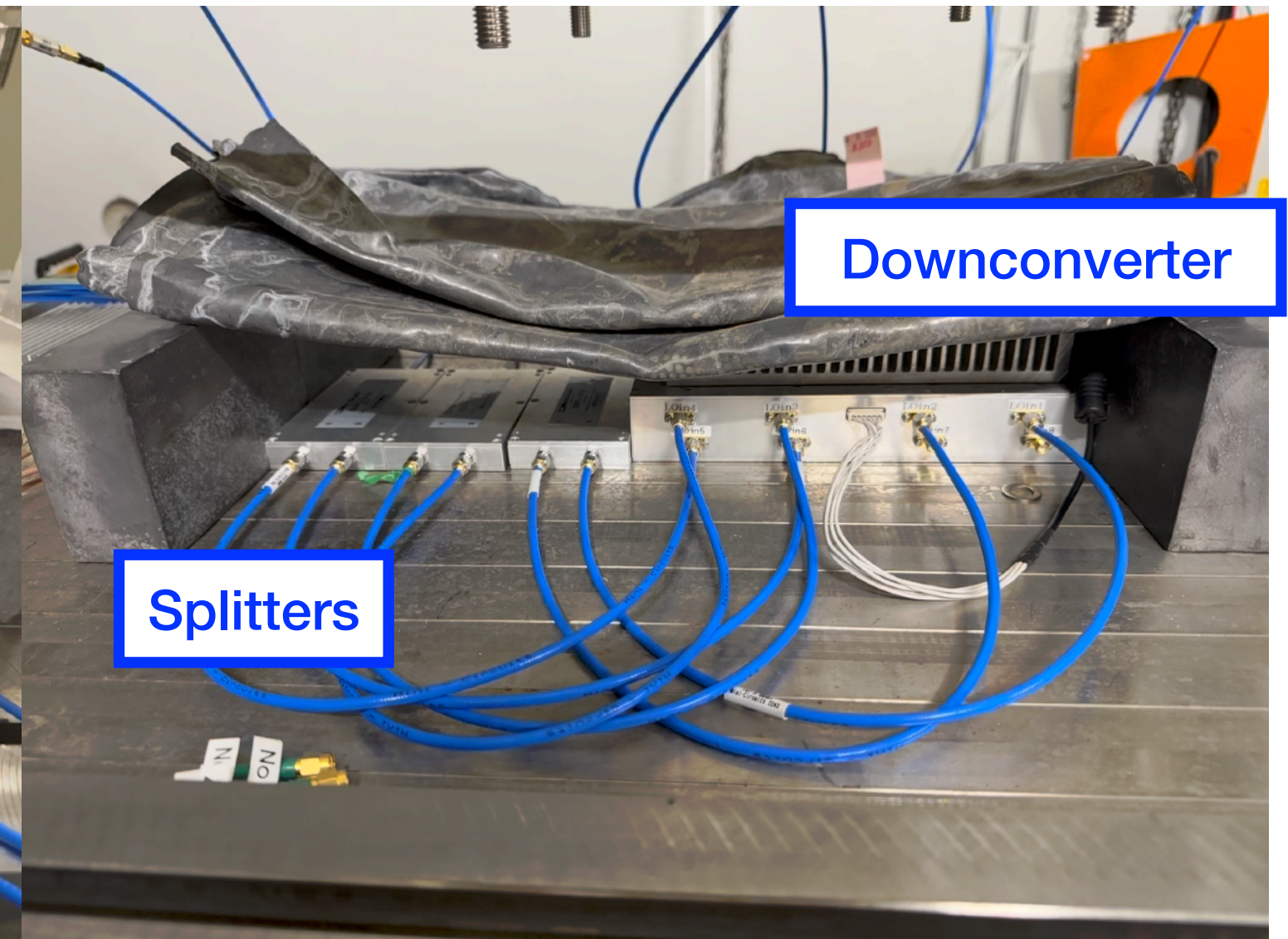
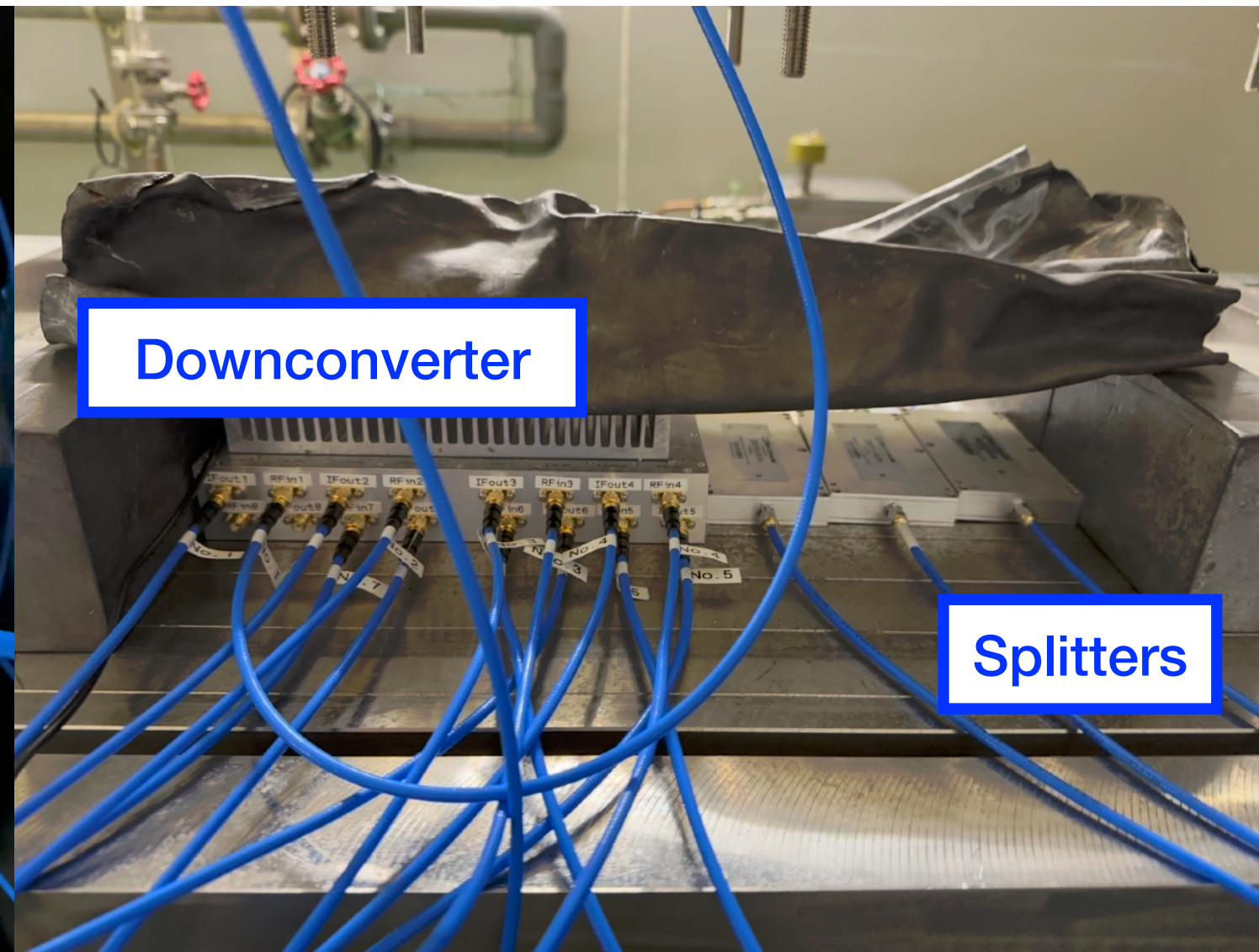
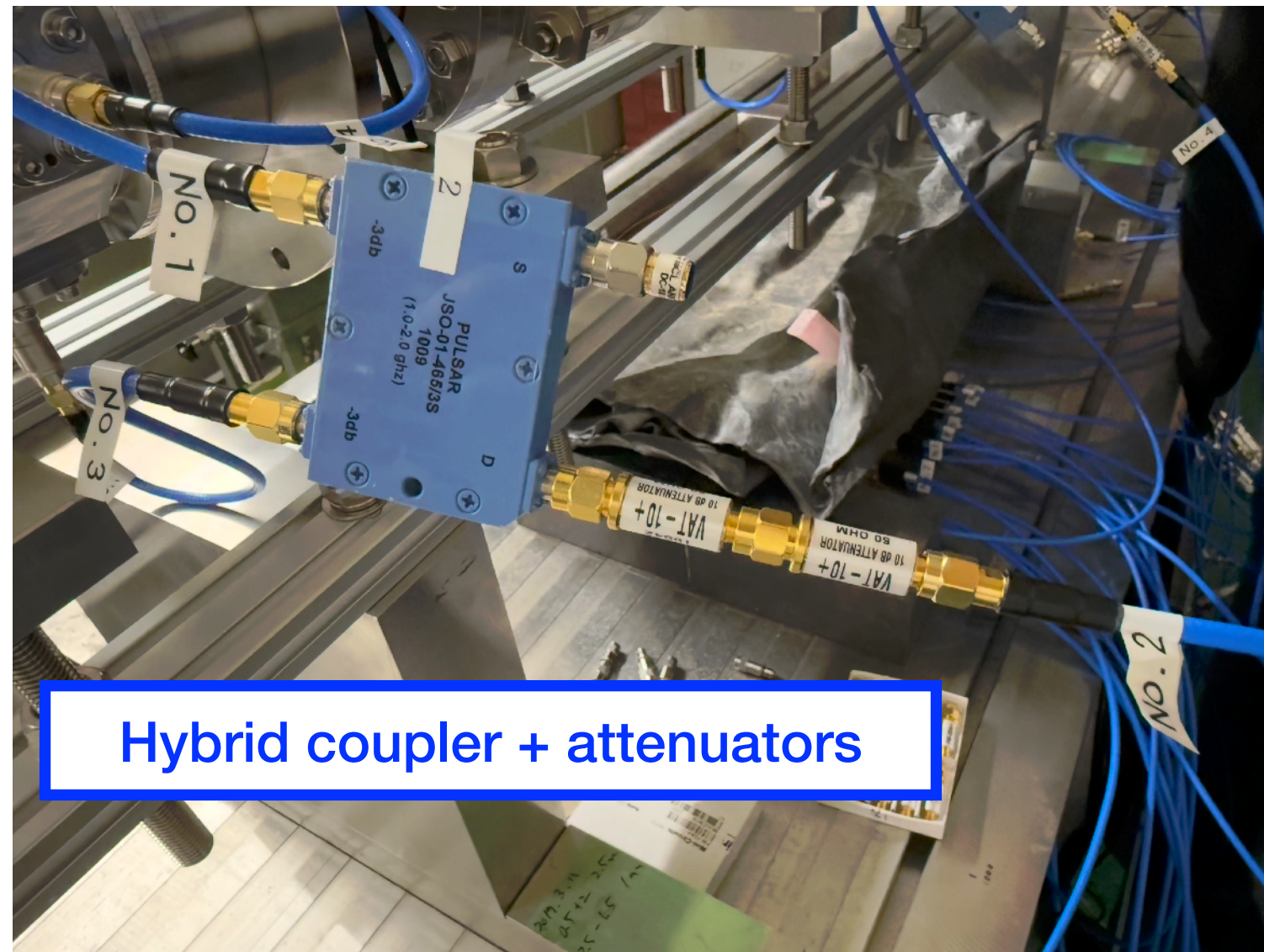


L-band Cavity BPM Readouts

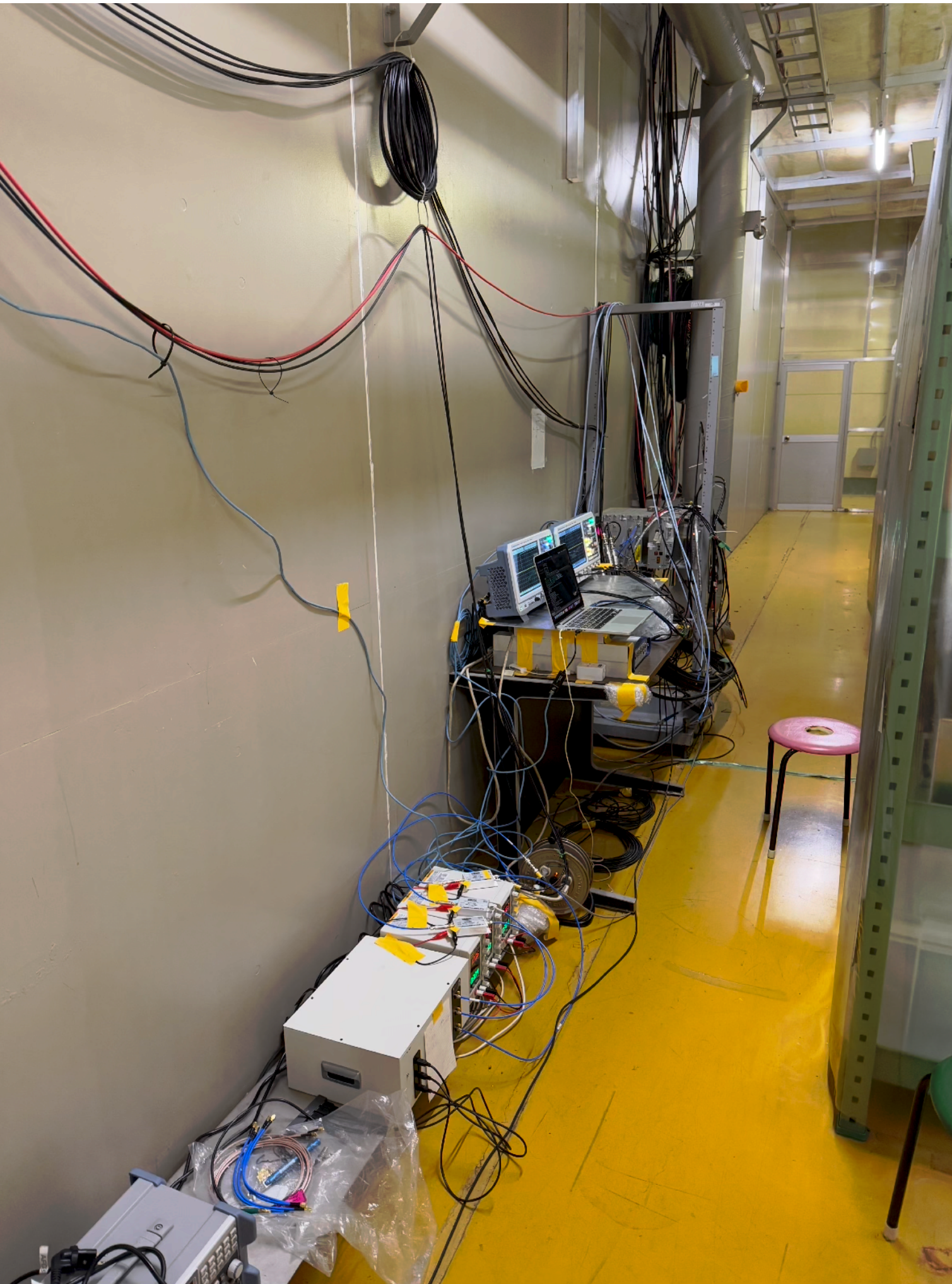
- Signals are merged by hybrid couplers and pass through a downconverter



L-band Cavity BPM Readouts (inside the tunnel)



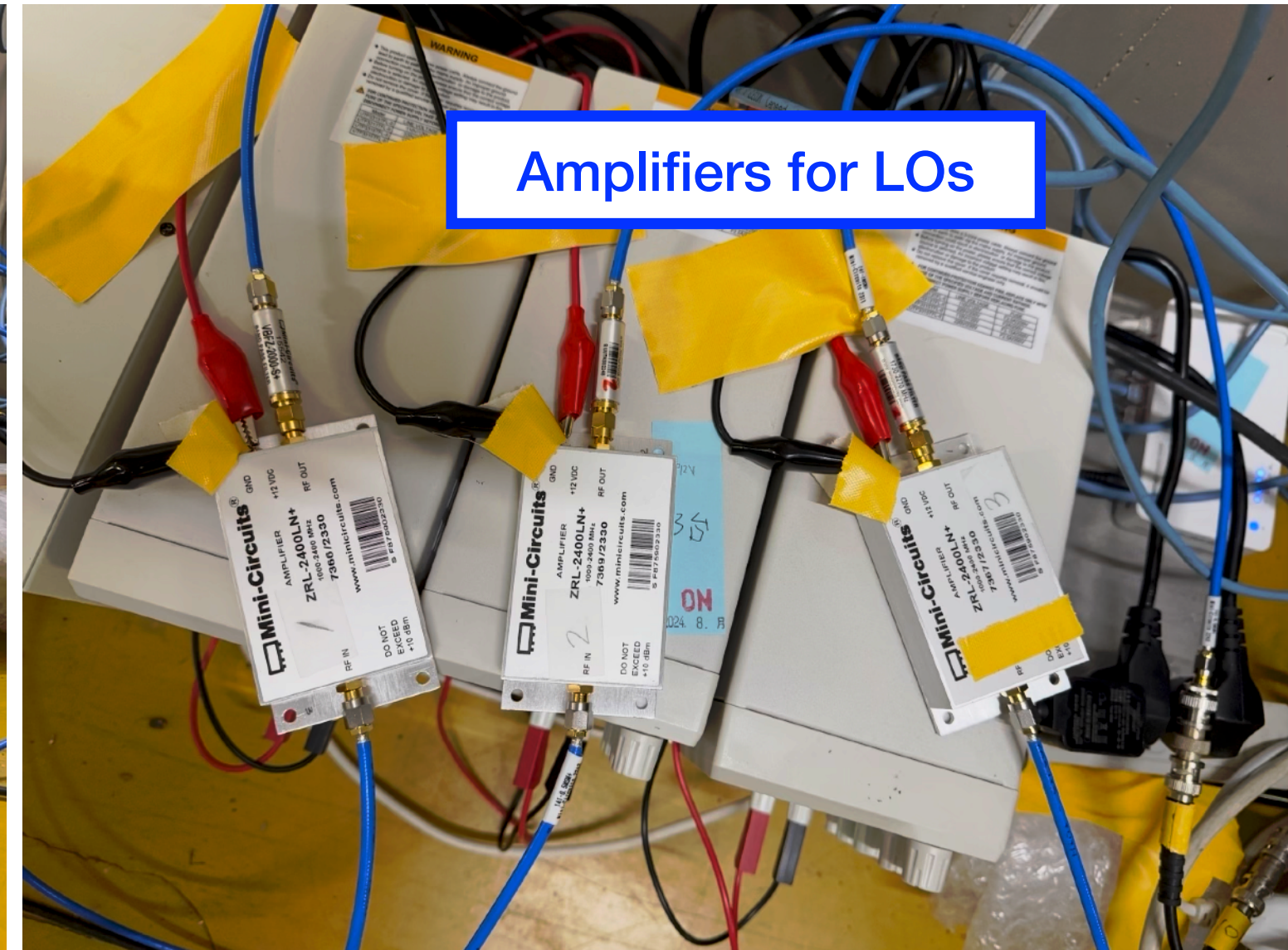
L-band Cavity BPM Readouts (outside the tunnel)



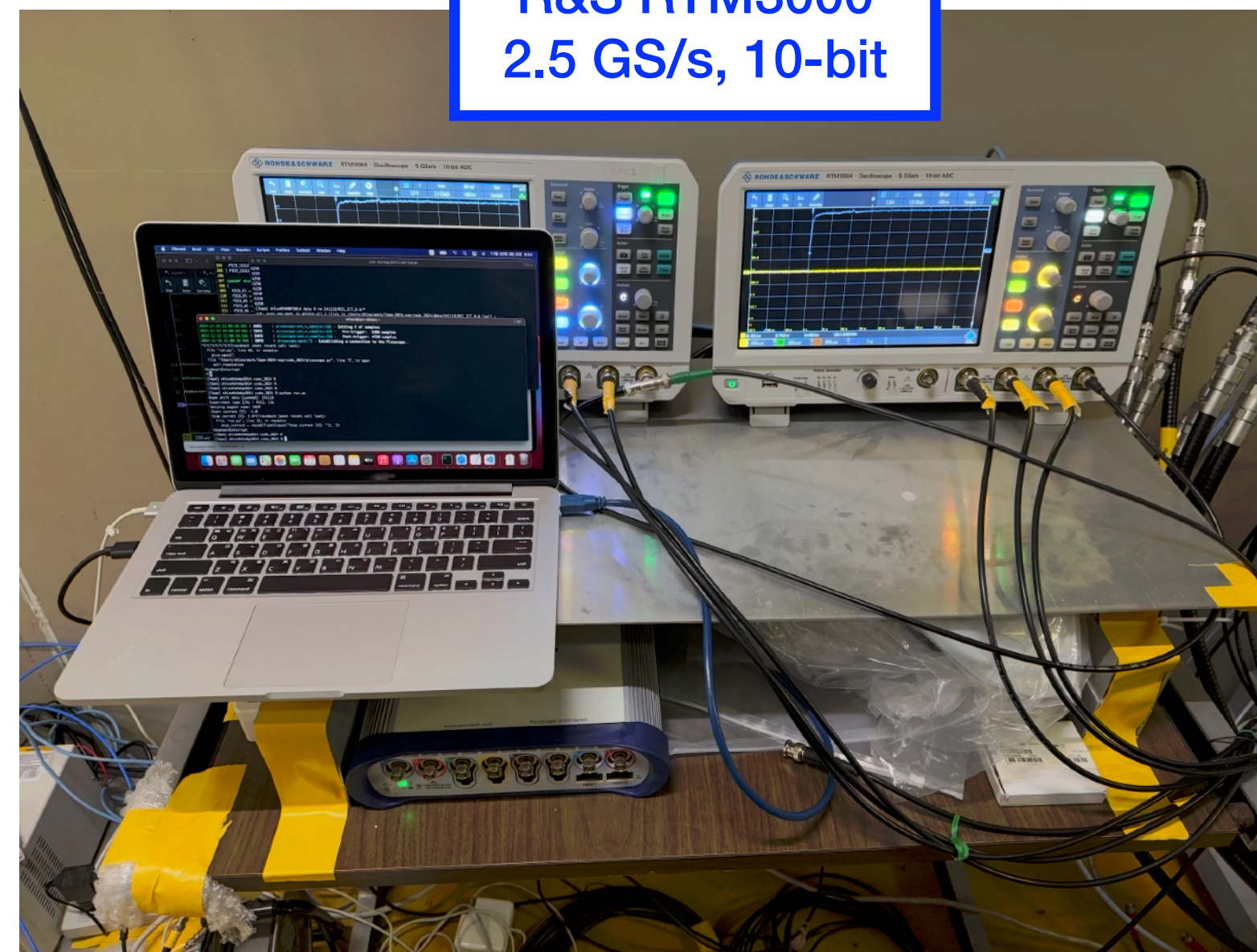
Unlocked LO



Amplifiers for LOs



R&S RTM3000
2.5 GS/s, 10-bit

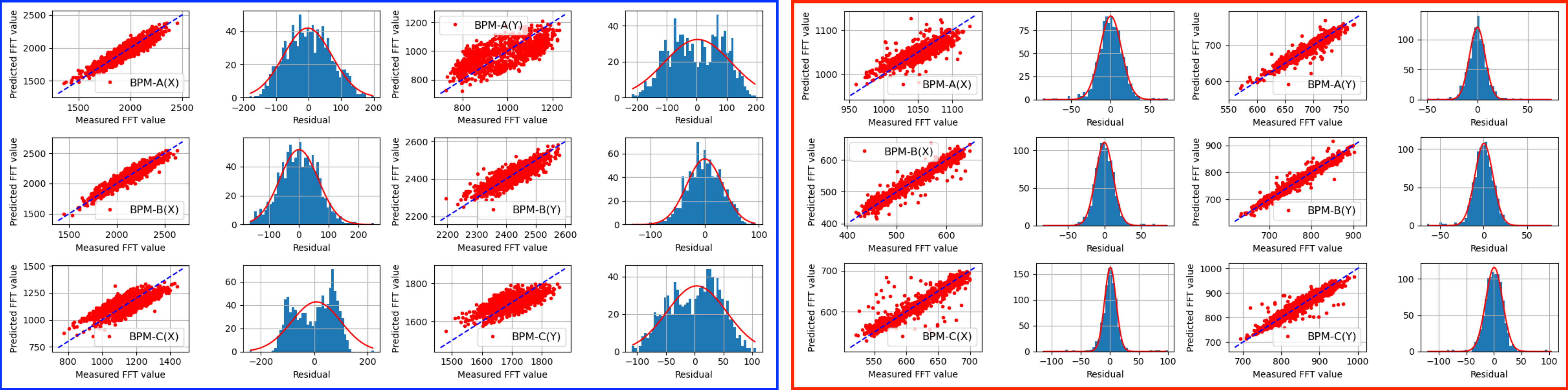


Picoscope 6824E
625 MS/s, 10-bit



Resolution Measurements in 2025

IF=45 MHz with 3 different LO's | Different IF's with a common LO



[nm]	2019	2023	2024 (w/ osc.'s)	2024 (w/ pico.)	2025 (IF=45 MHz)	2025 (Common LO)
BPM-A (X)	3750	2062	2033	3068	1552	768
BPM-B (X)	2680	1984	824	1904	2165	1188
BPM-C (X)	985	1831	470	1167	2277	404
BPM-A (Y)	698	3450	1457	1392	1843	319
BPM-B (Y)	2260	1948	1234	1198	756	951
BPM-C (Y)	324	712	162	309	838	469

arxiv:2512.13022 [physics.acc-ph]

BPM Comparisons



	Button BPM	Stripline BPM	Cavity BPM
Coupling to the beam	Capacitive	Electromagnetic	Resonant
Signal shape	Bipolar differentiated pulse	Doublet pulse pair	Decaying sinusoid (ring-down)
Transfer impedance	$\sim 0.5 - 2 \Omega$	$\sim 1 - 5 \Omega$	$R_s \sim \text{k}\Omega/\text{mm}$
Frequency response	Broadband	Broadband	Narrowband
Typical resolution	10 - 50 μm (single bunch) $\sim 1 \mu\text{m}$ (multi-turn average)	1 - 10 μm ($< 1 \mu\text{m}$ with optimization)	100 nm - 1 μm (typical) Best: 8.7 nm (KEK ATF2)
Beam coupling impedance	Low	Medium	High
Electrode size	Small	Medium	Large
Optimal bunch length	Short bunches	Long bunches	Short bunches
Beam type	Electron beam	Proton and Ion beams	Electron beam
DC/debunched beam	No	No	No
Primary use cases	Storage rings, damping rings, collider rings	Proton linacs, Ion accelerators, hadron synchrotrons	FEL undulator sections, high-precision linac sections, IP

Summary

- BPM is the “eyes” of accelerators
- Fundamentals and the most popular BPMs (Button/Stripline/Cavity) are visited



BPMs, Maketh, Beam

20th Century Studios