HW3

March 24, 2026

Problem 1

1) Starting from

$$L = -mc^2\sqrt{1 - \frac{v^2}{c^2}} + q\mathbf{A}\cdot\mathbf{v} - q\phi,$$

derive the canonical momentum.

2) Perform the Legendre transform explicitly and show that

$$H = \gamma mc^2 + q\phi.$$

3) Show that expressing H in canonical variables gives

$$H(\mathbf{r}, \mathbf{p}, t) = \sqrt{m^2c^4 + c^2(\mathbf{p} - q\mathbf{A})^2} + q\phi.$$

4) Starting from the leading dipole vector potential

$$A_s = -B_0 x,$$

derive the leading transverse dipole Hamiltonian and show how the condition

$$\frac{1}{\rho} = \frac{qB_0}{p_0}$$

removes the linear term in x.

5) Starting from the leading quadrupole vector potential

$$A_s = \frac{Q}{2}(y^2 - x^2),$$

derive the quadratic quadrupole Hamiltonian and the equations of motion.

6) Show that the sextupole Hamiltonian

$$H_S = \frac{1}{2}p_x^2 + \frac{1}{2}p_y^2 + \frac{k_2}{6}x^3 - \frac{k_2}{2}xy^2$$

leads to the standard sextupole kicks.

7) Explain why skew quadrupoles generate linear transverse coupling.

8) For a 2D linear map

$$R = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

show that symplecticity implies

$$ad - bc = 1.$$

9) Write a short paragraph explaining why beam dynamics is fundamentally a phase-space theory.