HW2

March 17, 2026

Problem 1

Consider a charged particle with charge q moving along the z-axis through an RF accelerating gap of length d. Assume that the longitudinal electric field inside the gap is spatially uniform and varies sinusoidally in time according to

$$E_z(t) = E_0 \cos(\omega t + \phi),$$

where E_0 is the peak field amplitude, ω is the angular RF frequency, and ϕ is the phase of the RF field when the particle enters the gap.

Assume that the particle velocity remains constant while crossing the gap:

$$v = \text{constant}.$$

Let the particle enter the gap at $z = 0$ and exit at $z = d$.

1) Write the expression for the energy gain ΔW of the particle in terms of the longitudinal electric field E_z.

2) Using the relation

$$z = vt,$$

rewrite the electric field as a function of position z.

3) Substitute this expression into the integral for ΔW, and evaluate the integral explicitly over the interval $0 \le z \le d$.

4) Use the trigonometric identity

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

to simplify your result.

5) Define the peak gap voltage as

$$V_0 = E_0 d.$$

Show that the energy gain can be written in the form

$$\Delta W = qV_0 T \cos\left(\phi + \frac{\omega d}{2v}\right),$$

and identify the quantity T.

6) Show that the transit-time factor is

$$T = \frac{\sin(\omega d/2v)}{\omega d/2v}.$$

7) Using

$$\beta = \frac{v}{c}, \qquad \lambda = \frac{2\pi c}{\omega},$$

rewrite the transit-time factor in the form

$$T = \frac{\sin(\pi d/\beta\lambda)}{\pi d/\beta\lambda}.$$

8) For a pillbox cavity with

$$d = \frac{\lambda}{2}, \qquad v \approx c,$$

evaluate the transit-time factor.

9) Explain physically why the transit-time factor is always less than or equal to 1, and why it becomes smaller when the gap length is too large.