

HW1

March 10, 2026

Problem 1

Consider a simple harmonic oscillator (SHO) consisting of a particle of mass m attached to an ideal spring with spring constant k . The particle oscillates along the x -axis with amplitude A and initial phase ϕ_0 . Define

$$\omega_0 \equiv \sqrt{\frac{k}{m}}.$$

Assume there is no friction and no gravitational force.

- 1) Write down the equation of motion for this SHO system.
- 2) Find the general solution $x(t)$ of the equation of motion in the form

$$x(t) = A \cos(\omega_0 t + \phi_0).$$

- 3) Compute the velocity $\dot{x}(t)$.
- 4) Evaluate $x(0)$ and $\dot{x}(0)$, and express them in terms of A , ω_0 , and ϕ_0 . Denote these initial values by

$$x(0) = x_0, \quad \dot{x}(0) = \dot{x}_0.$$

- 5) Using the trigonometric identities

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta,$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta,$$

rewrite $x(t)$ and $\dot{x}(t)$ in terms of A , $\cos(\omega_0 t)$, $\sin(\omega_0 t)$, $\cos \phi_0$, and $\sin \phi_0$.

- 6) Using the results from part 4, rewrite $x(t)$ and $\dot{x}(t)$ entirely in terms of x_0 , \dot{x}_0 , ω_0 , $\cos(\omega_0 t)$, and $\sin(\omega_0 t)$.

Problem 2

Consider a particle of mass m moving in one dimension under the restoring force

$$F(x) = -m\omega_0^2 x,$$

where ω_0 is a positive constant. Assume there is no damping and no external driving force.

1) Using Newton's second law, derive the differential equation of motion.

2) Show that the function

$$x(t) = C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t)$$

satisfies the equation of motion.

3) Find the velocity $\dot{x}(t)$.

4) Suppose the initial conditions are

$$x(0) = x_0, \quad \dot{x}(0) = v_0.$$

Determine the constants C_1 and C_2 in terms of x_0 , v_0 , and ω_0 .

5) Hence show that the motion can be written as

$$x(t) = x_0 \cos(\omega_0 t) + \frac{v_0}{\omega_0} \sin(\omega_0 t).$$

6) Rewrite the solution in the amplitude-phase form

$$x(t) = A \cos(\omega_0 t + \phi_0),$$

and express A and ϕ_0 in terms of x_0 and v_0 .

7) Show that the total energy

$$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega_0^2 x^2$$

is constant in time.