Cavity Beam Position Monitors

2025 Accelerator and Beam Line Field Training
Jul 8 2025, Korea University Sejong Campus

Soohyung Lee, Ph. D (Korea University)

Course Overview

- Jul 8, 2025 (9:30 - 12:15)
  - Cavity beam position monitor basics
  - Measurements for position resolutions of cavity BPM
  - Python installation for data analyses
  - (hands-on) cBPM Position Resolution Measurement in 2023

- Jul 9, 2025 (9:30 - 12:15)
  - (hands-on) cBPM Position Resolution Measurement in 2024
  - (hands-on) cBPM Position Resolution Measurement in 2025

Cavity Beam Position Monitor

Beam Position Monitors (BPM)

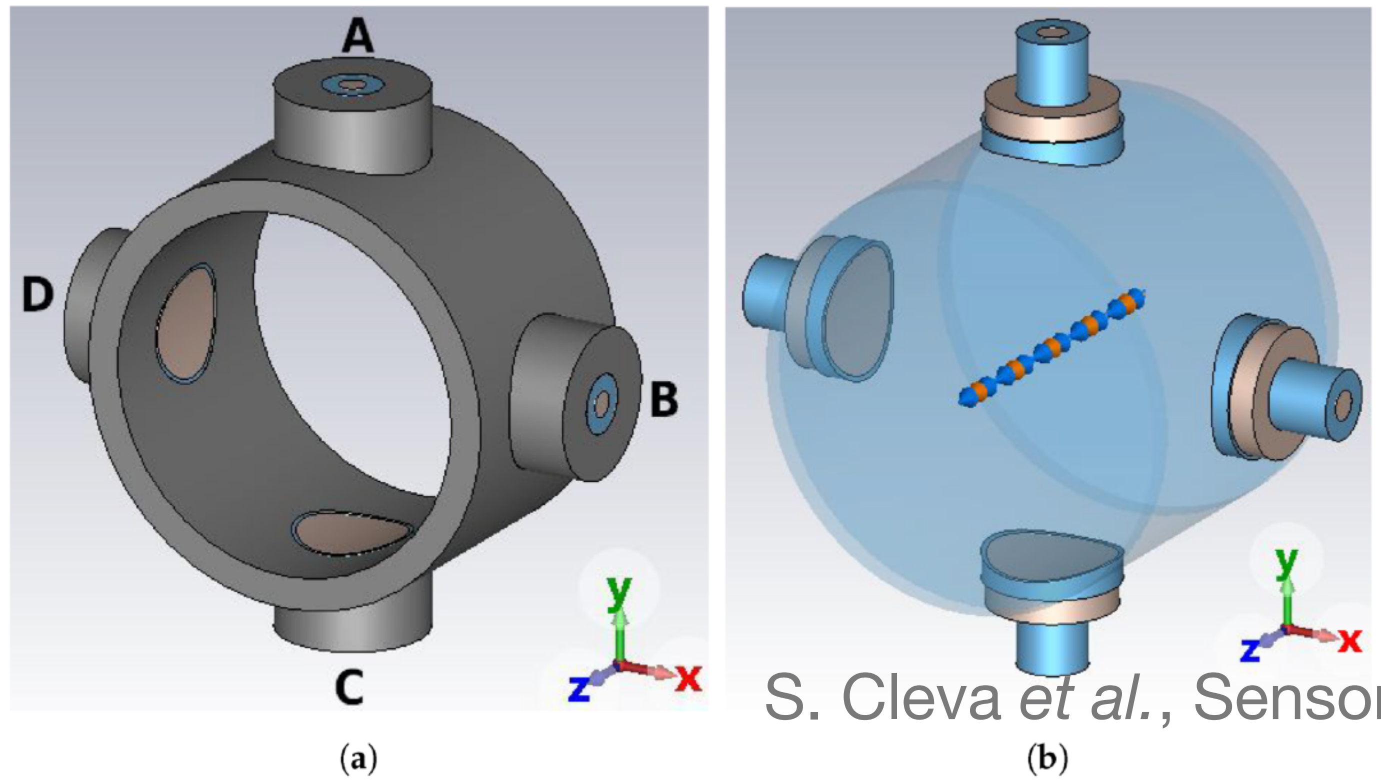
- Beam position monitors measure the beam position inside the beam pipe

- Measurements of beam positions are crucial in the accelerators
  - Provides feedbacks for the beam orbit corrections
  - Diagnoses of components in the accelerator

- Since beam sizes are usually very small, precise position measurements are required
  - e.g. Interaction point (IP) in the International Linear Collider (ILC)
    - $\sigma_y = 5.9\,\mathrm{nm} \rightarrow$ requires a few nano-meter position resolution

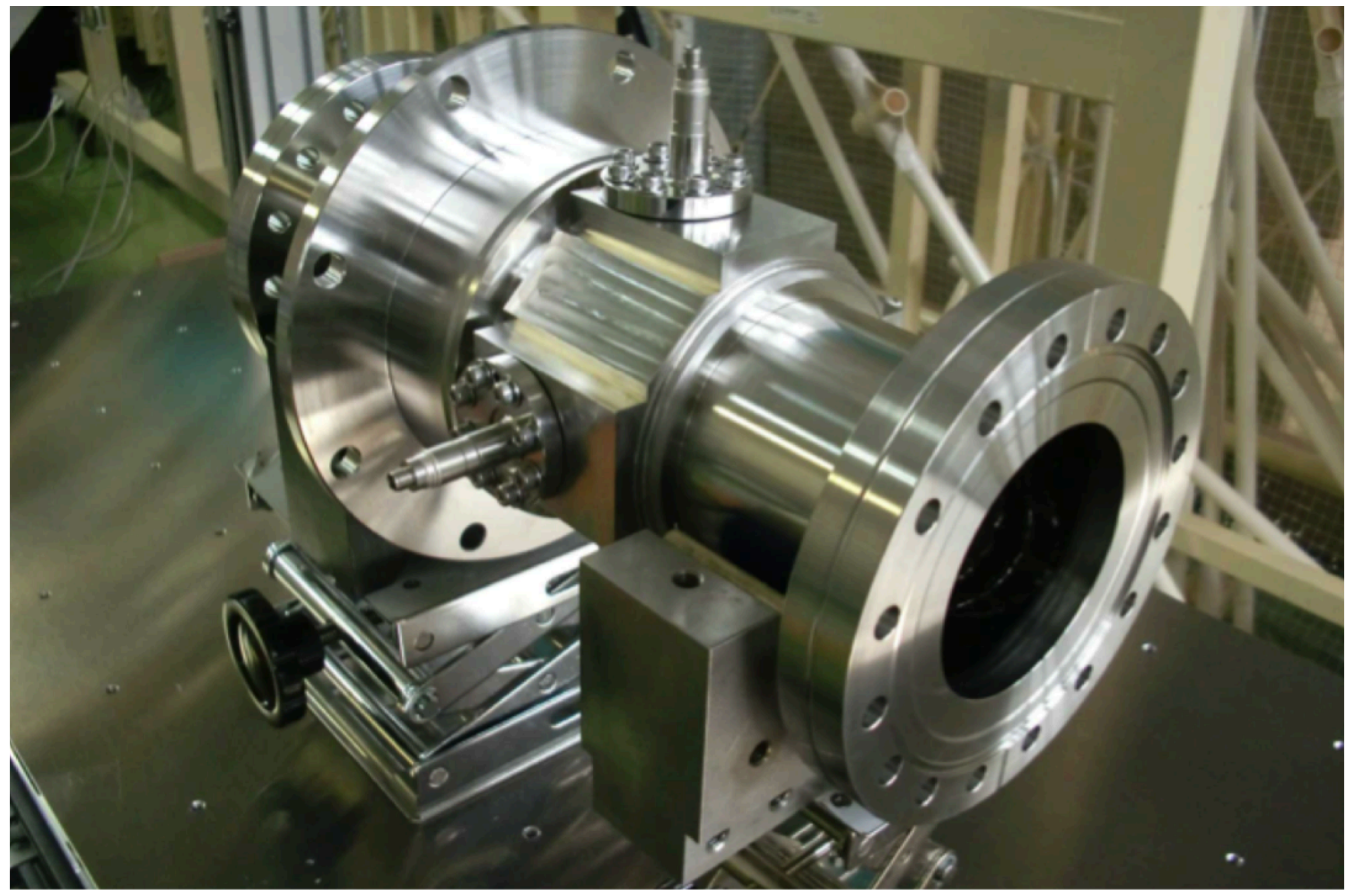
Types of BPMs

- Button BPM
  - Calculates beam position by comparing signals from four small electrodes
  - Simple, cheap, fast, but position resolution is limited ~1 μm
- Stripline BPM
  - Uses a long stripline electrodes along the beam direction
  - Sensitive to the signal direction, phase can be measured too
  - Limited resolution by $\mathcal{O}(10)\,\mu\text{m}$
- Cavity BPM
  - Utilizes the electromagnetic modes induced by the beam passing through
  - Very good resolutions (nano-meter scale)
  - Complicated, expensive



S. Cleva *et al.*, Sensors **2024**, 2726 (2024)

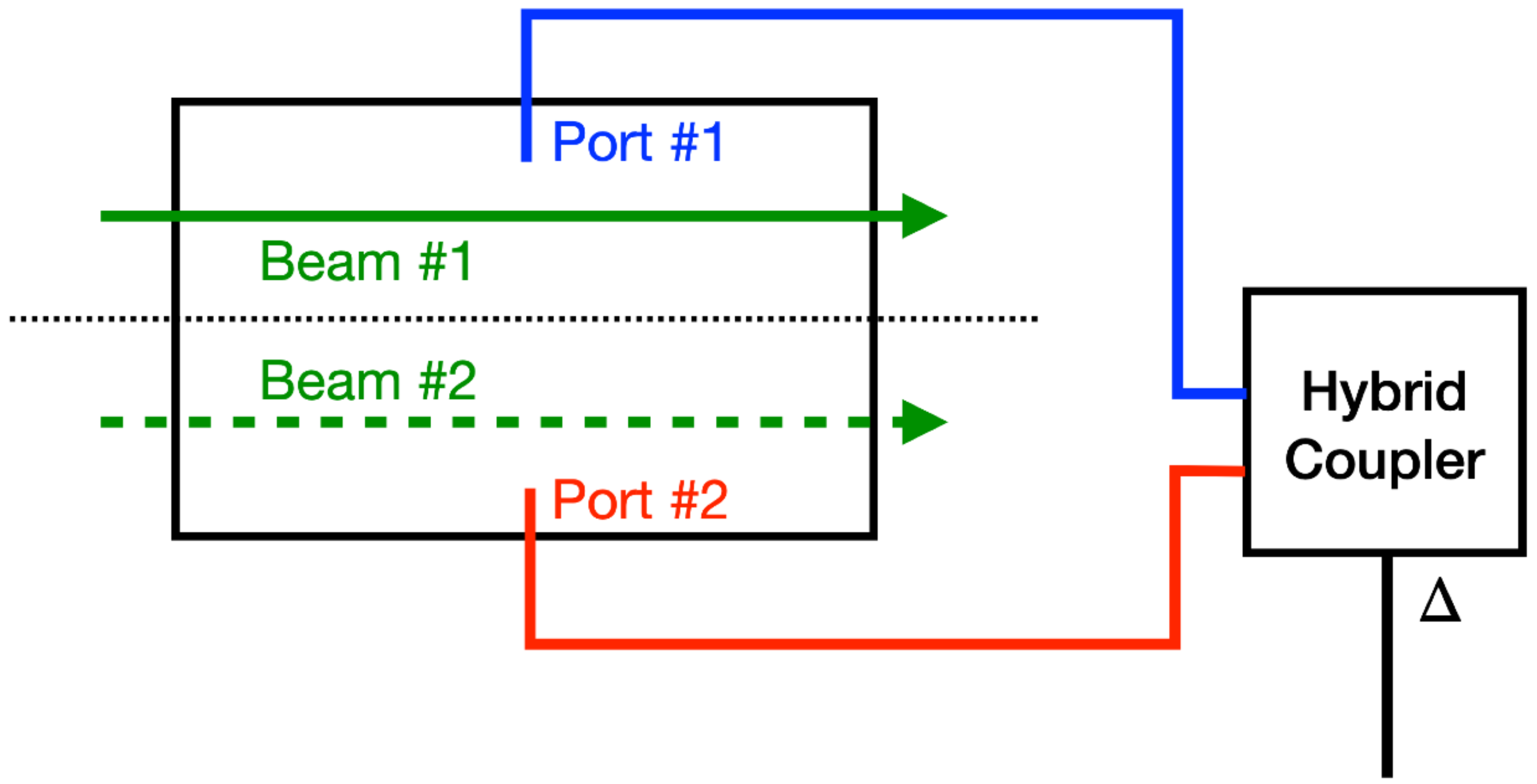
T. Kamps *et al.*, JACoW eConf C0508213 (2005)

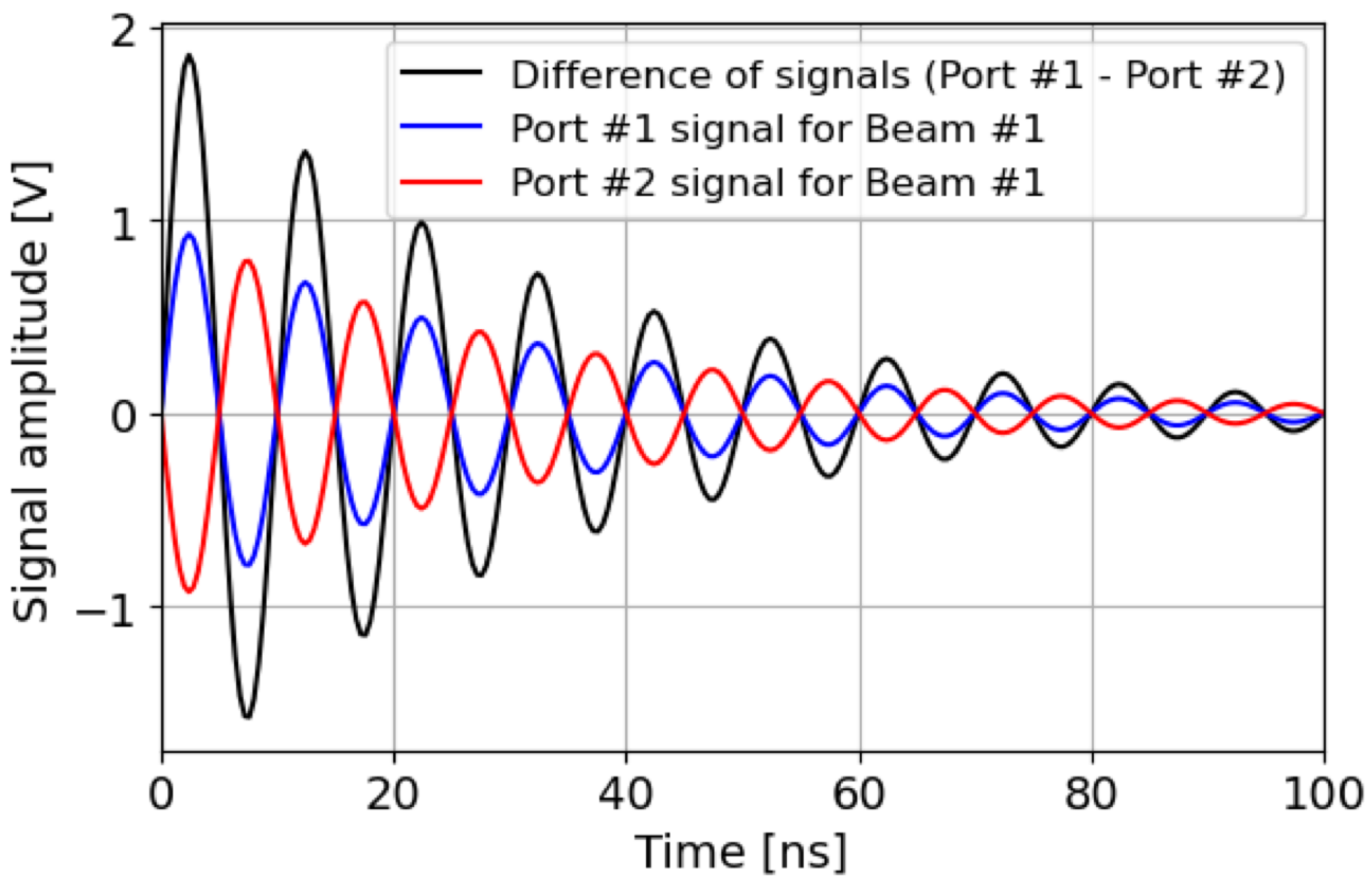


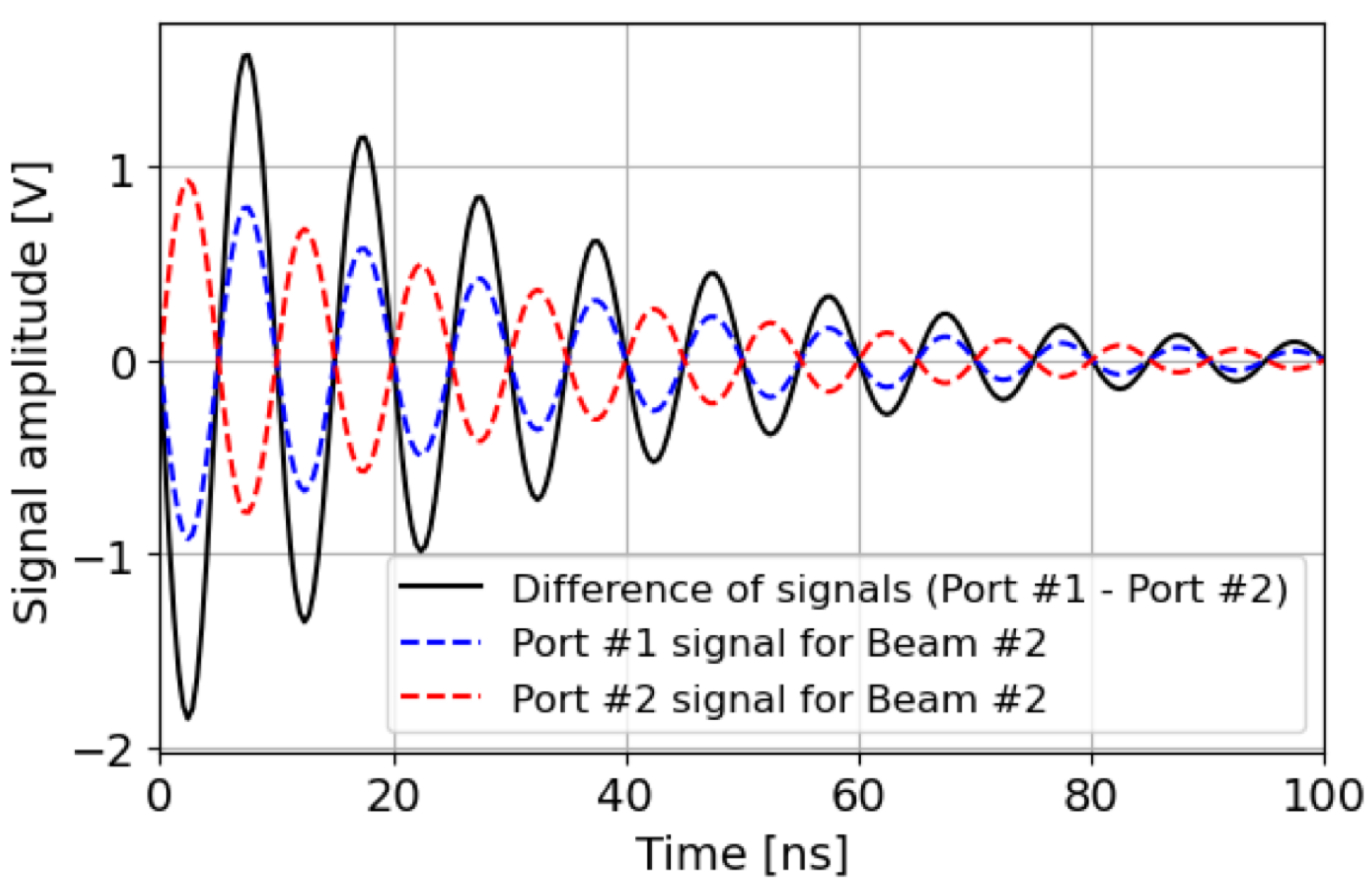
ILC Technical Design Report (2013)

Cavity BPM Basics

- Dipole mode (TM_{110}) is used since the dipole mode signal is proportional to the offset of the beam from the cavity center
  - Same offset, opposite direction → same amplitude, opposite phase
  - If beam passes through the center of the cavity, no signal appears
  - Single cavity BPM can't determine the absolute position, but an offset from the center
  - To determine the absolute position, another cavity BPM (reference cavity) is necessary





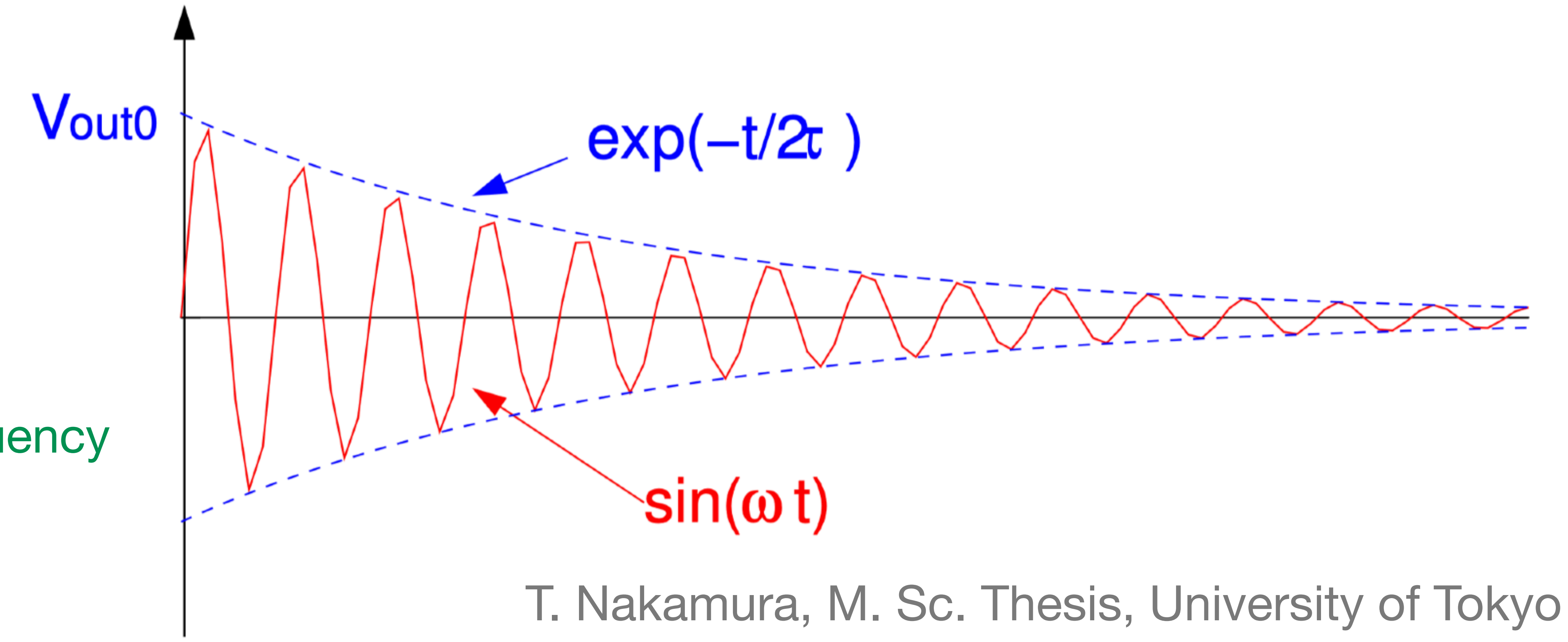


Output Signals

- Pickup signal from a cavity BPM

$$V_{\text{out}} = V_0 e^{-\frac{t}{2\tau}} \sin(\omega t + \phi)$$

ω: Cavity TM_{110} resonant frequency
ϕ: Phase of BPM signal



T. Nakamura, M. Sc. Thesis, University of Tokyo (2008)

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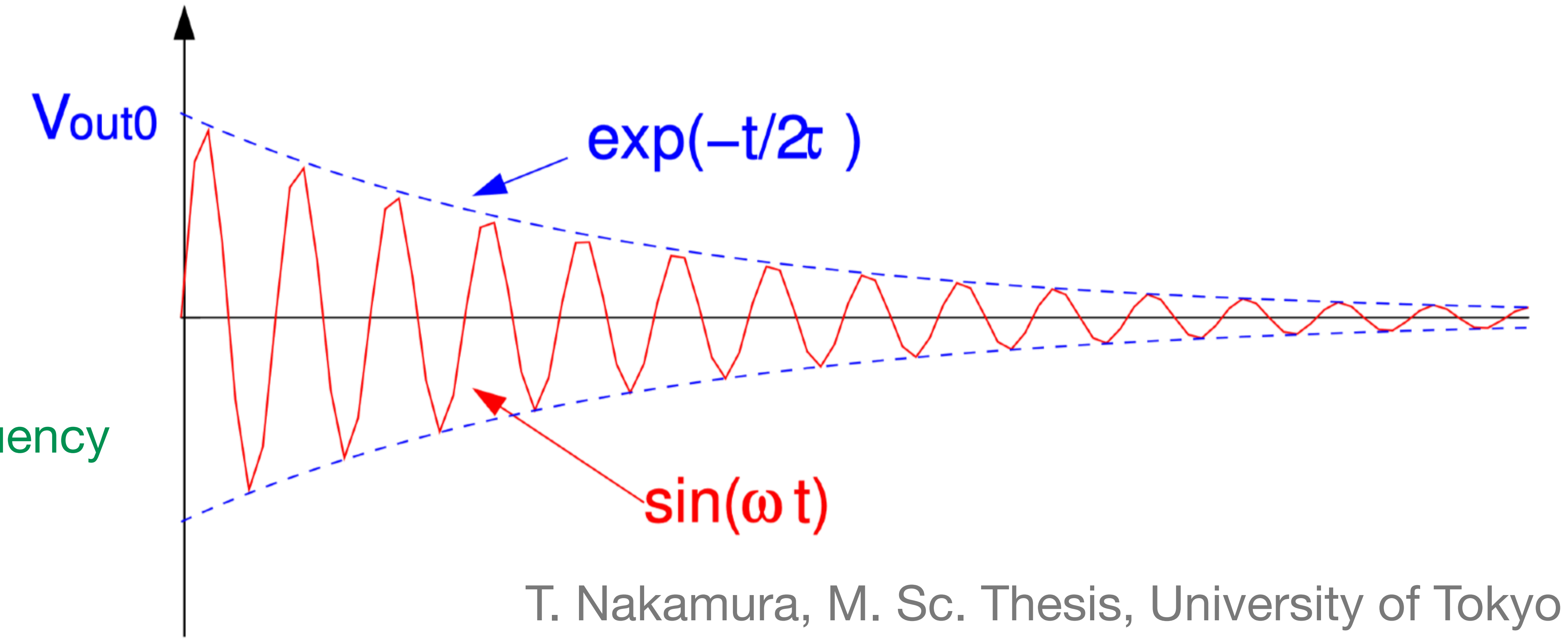
$$\boxed{V_0 = \frac{\omega q}{2}\sqrt{\frac{Z}{Q_{\text{ext}}}\left(\frac{R}{Q}\right)} \exp\left(-\frac{\omega^2 \sigma_z^2}{2c^2}\right)}$$

q: Charge of the beam
Z: Load impedance (usually 50 Ω)



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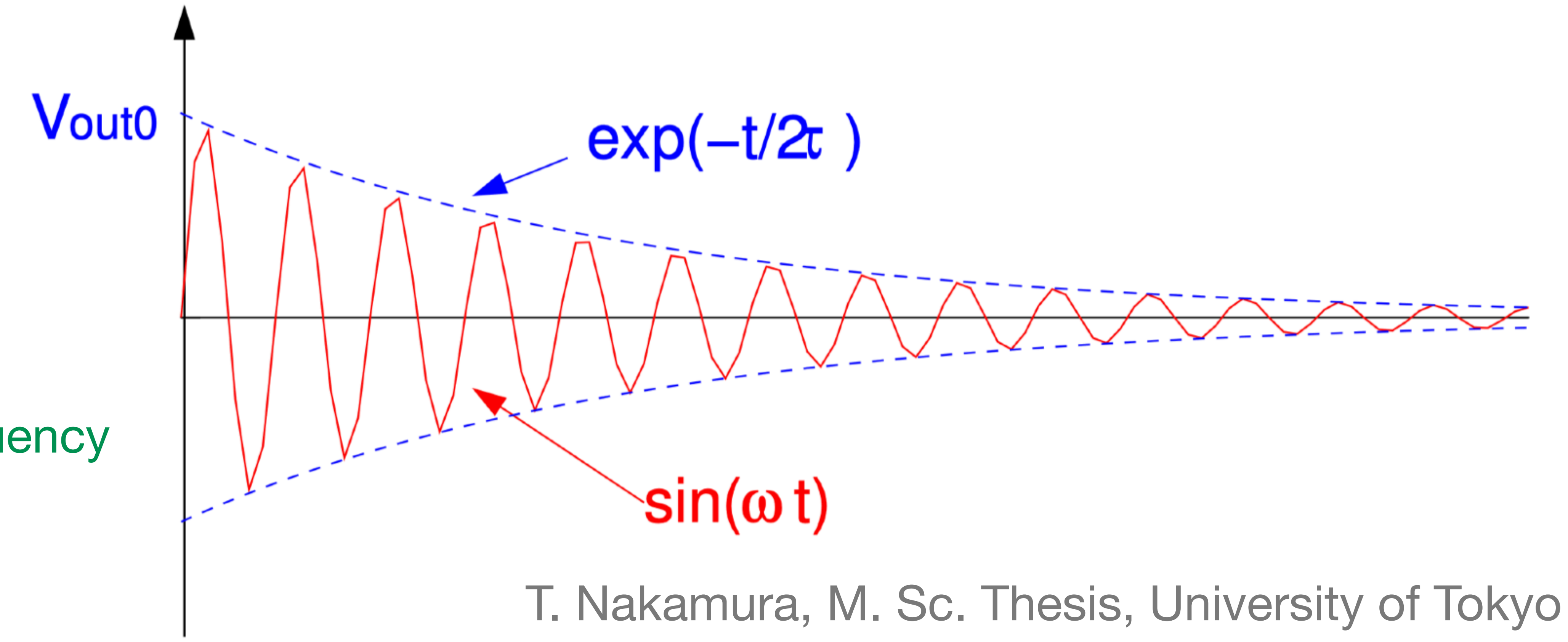

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Z: Load impedance (usually 50 Ω)

$$\frac{R}{Q} \simeq 50.3 \times \left(\frac{\omega}{c}\right)^3 LT^2x^2$$

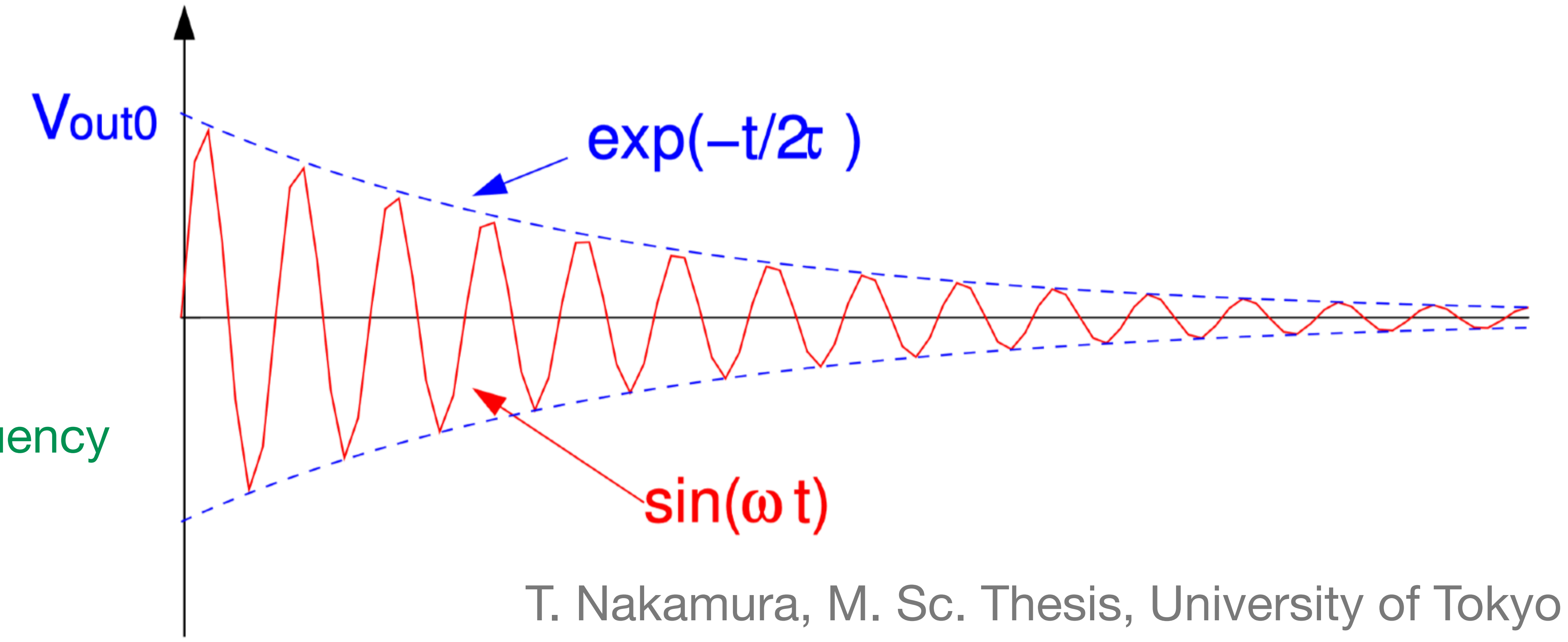
L: Cavity length
c: Speed of light
x: Beam offset from the cavity center
T: Transit time factor (phase transition during the beam passing time)

Output Signals

- Pickup signal from a cavity BPM

$$V_{\text{out}} = V_0 e^{-\frac{t}{2\tau}} \sin(\omega t + \phi)$$

ω: Cavity TM_{110} resonant frequency
ϕ: Phase of BPM signal



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From the Gaussian beam shape
σ_z: Beam bunch length

q: Charge of the beam
Z: Load impedance (usually 50 Ω)

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Output Signals

- Pickup signal from a cavity BPM

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ω: Cavity TM_{110} resonant frequency
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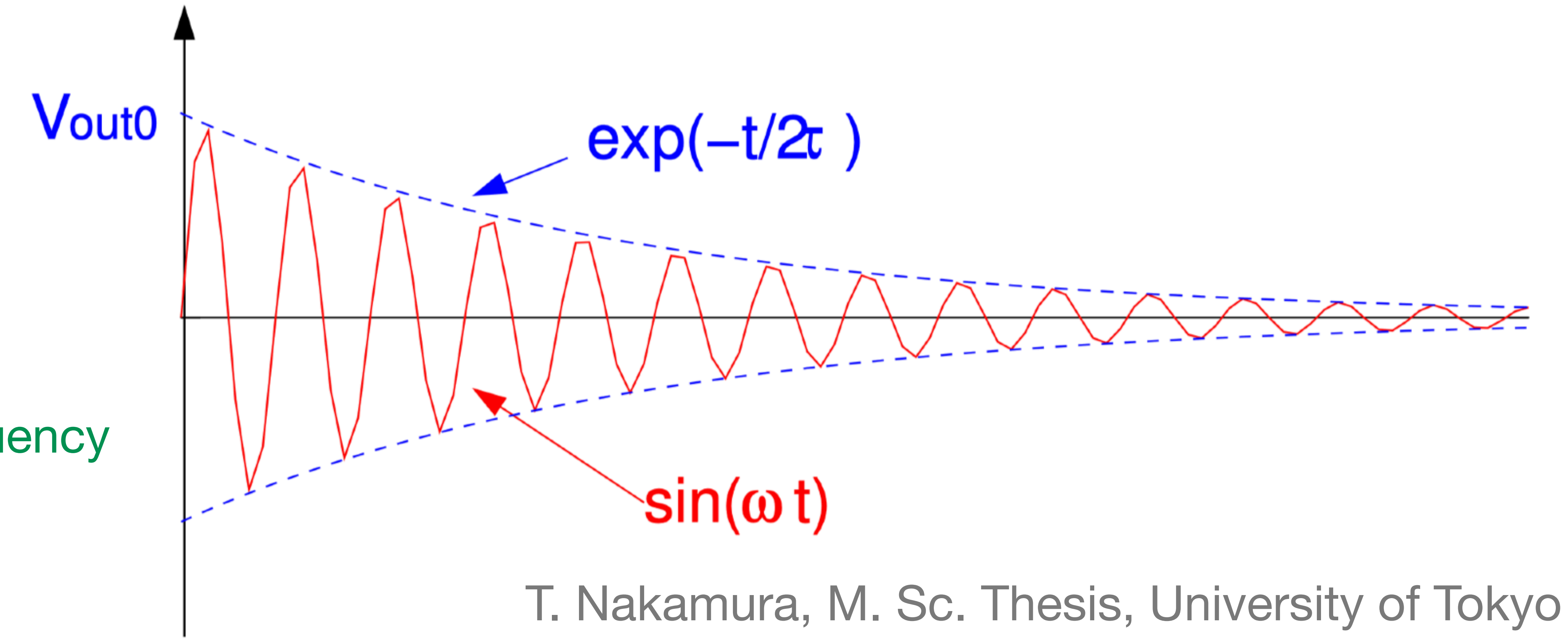

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From the Gaussian beam shape
σ_z: Beam bunch length

q: Charge of the beam
Z: Load impedance (usually 50 Ω)

$$Q_{\text{ext}} = Q_0/\beta$$

Q_0: Unloaded quality factor of the cavity TM_{110} mode
β: Coupling coefficient

$$\frac{R}{Q} \simeq 50.3 \times \left(\frac{\omega}{c}\right)^3 L T^2 x^2$$

L: Cavity length
c: Speed of light
x: Beam offset from the cavity center
T: Transit time factor (phase transition during the beam passing time)

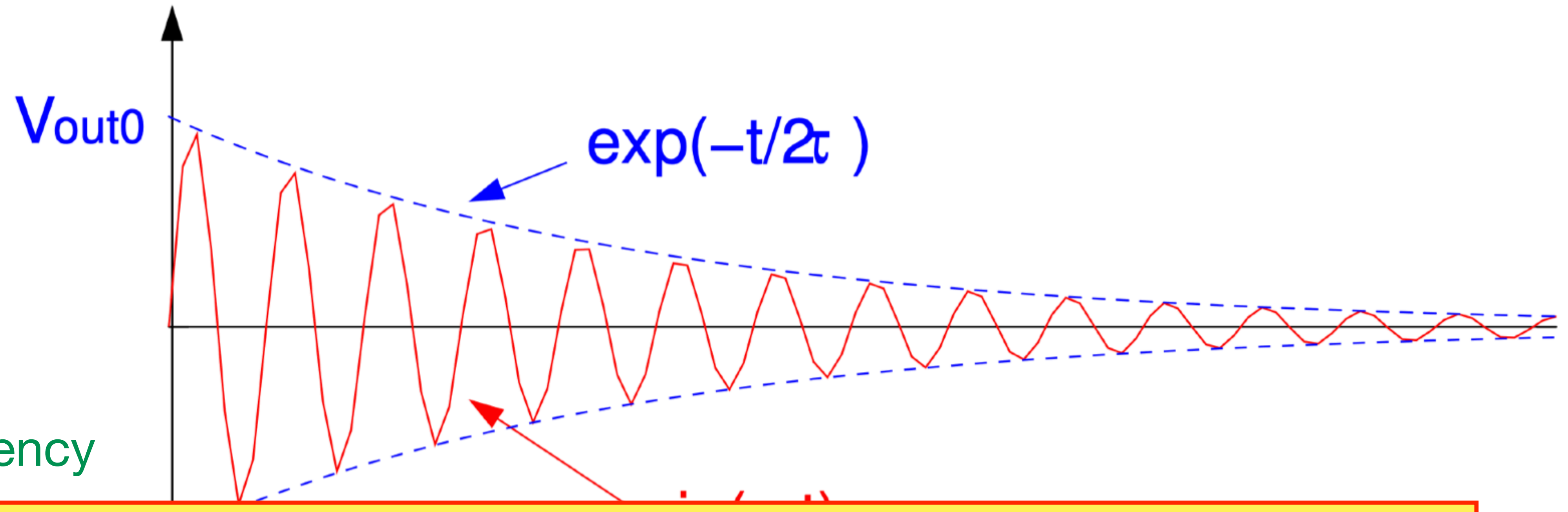
or,

$$Q_0 \equiv \frac{\omega U}{P_{\text{wall}}} \text{ and } Q_{\text{ext}} \equiv \frac{\omega U}{P_{\text{out}}}$$

U: Stored energy in a cavity eigenmode
P_{wall}: Dissipated power at the cavity wall
P_{out}: Output power to the circuit

Output Signals

- Pickup signal from a cavity BPM



...okyo (2008)

$$V_{\rm out} = V_0 e^{-\frac{t}{2\tau}} \sin(\omega t + \phi)$$

ω: Cavity TM_{110} resonant frequency
ϕ: Phase of B...

$$V_0 = \frac{\omega q}{2}\sqrt{\frac{Z}{Q_{\rm ext}}\left(\frac{R}{Q}\right)}\exp\left(-\frac{\omega^2\sigma_z^2}{2c^2}\right)$$

From t...
σ_z: Be...

The output signal from the cavity BPM is proportional to q and x !

q: Charge of the beam
Z: Load impedance (usually 50 Ω)

$$Q_{\rm ext} = Q_0/\beta$$

Q_0: Unloaded quality factor of the cavity TM_{110} mode
β: Coupling coefficient

$$\frac{R}{Q} \simeq 50.3 \times \left(\frac{\omega}{c}\right)^3 LT^2x^2$$

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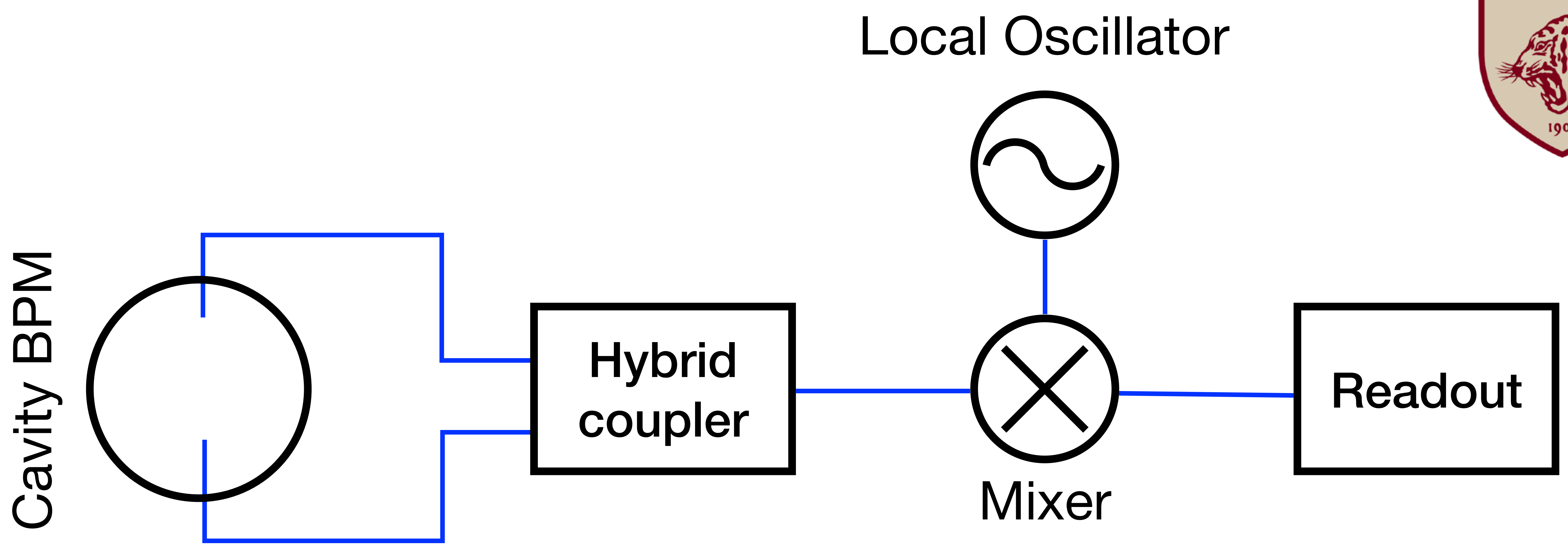
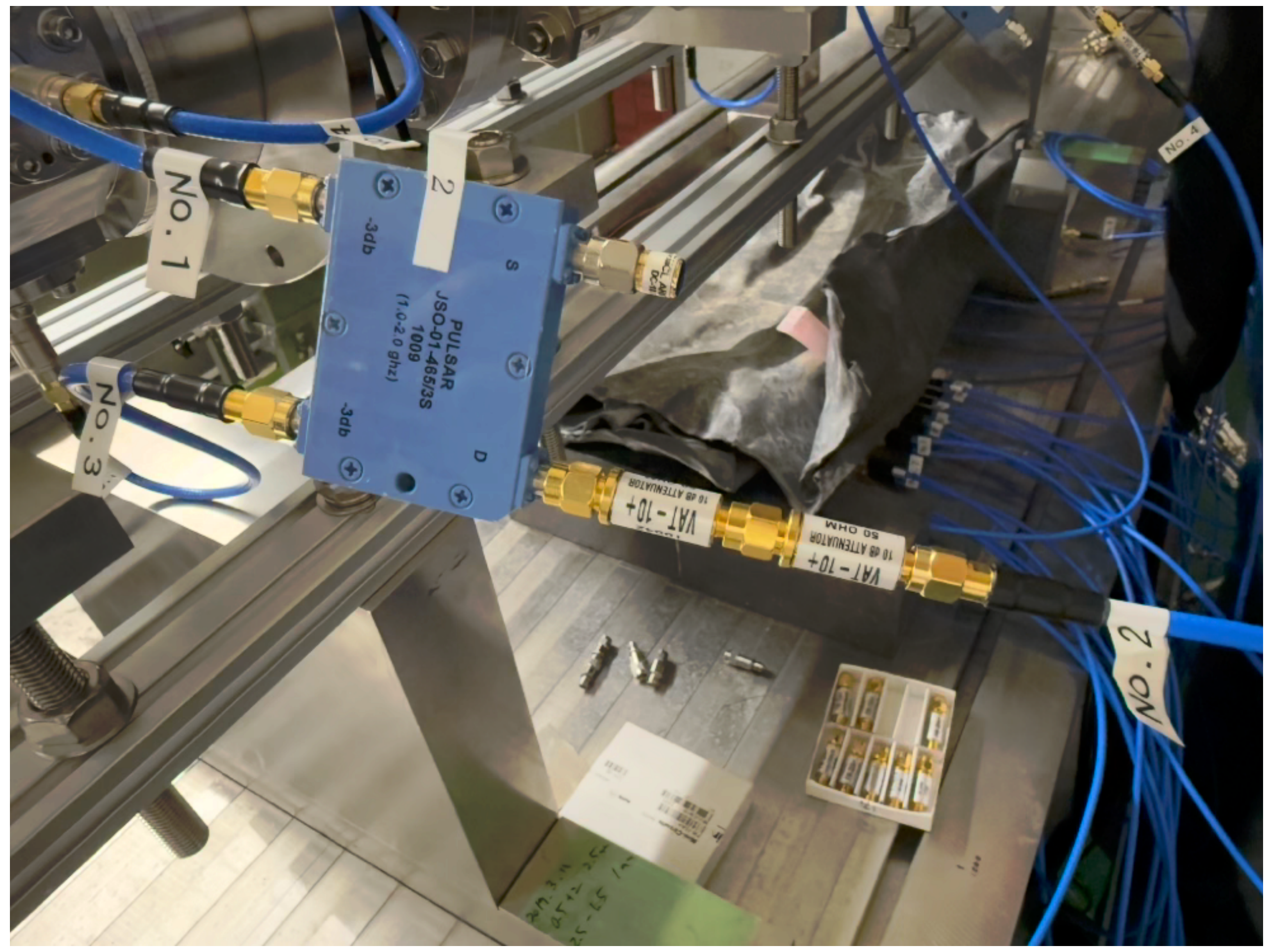
Signal Processing

- BPM signals are combined by a hybrid coupler, downconverted by a mixer, and finally acquired by a readout



- Hybrid coupler
  - Takes summation (Σ-port) or subtraction (Δ-port) of two input signals
  - For BPM, Δ-port is used since the opposite pickup antennas have opposite phase
  - Combining signals of two ports increases signal power, therefore, better sensitivity



Signal Processing

Local Oscillator

Readout

- BPM signals are combined by a
  hybrid coupl
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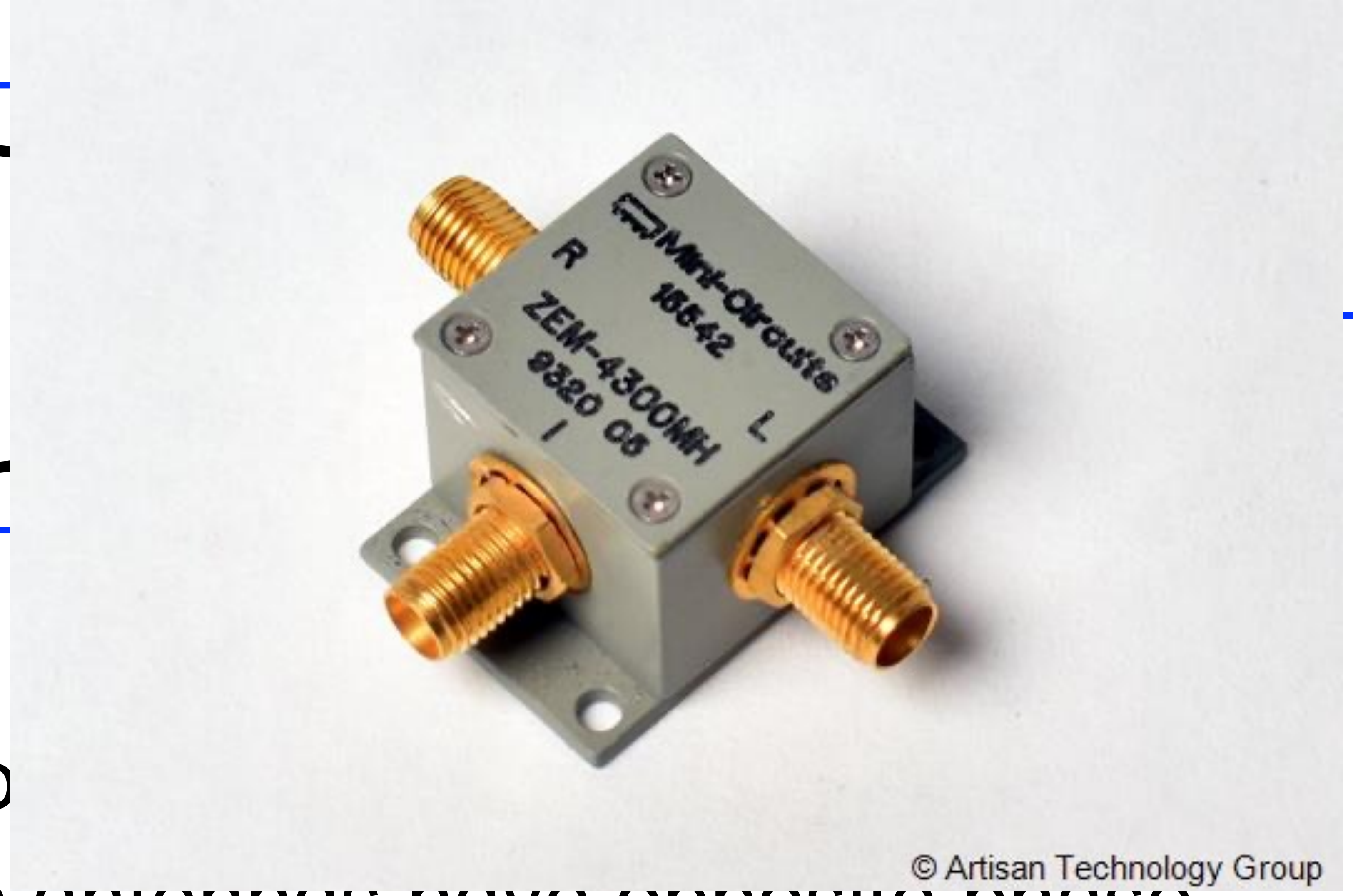

- Hybrid coup
  - Takes summ rt) o
  - For BPM, Δ- kup antennas have opposite phase
  - Combining signals of two ports increases signal power, therefore, better sensitivity
- Mixer and local oscillator
  - BPM signal and local oscillator signal are mixed by mixer
    - $S_{\mathrm{BPM}} \times S_{\mathrm{LO}} = A\left[\cos((\omega_{\mathrm{BPM}} - \omega_{\mathrm{LO}})t + (\phi_{\mathrm{BPM}} - \phi_{\mathrm{LO}}) + \cos((\omega_{\mathrm{BPM}} + \omega_{\mathrm{LO}})t + (\phi_{\mathrm{BPM}} + \phi_{\mathrm{LO}})\right]$
  - Downconversion: translate BPM signal frequency (RF) to desired frequency (IF)

Signal Processing

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Local Oscillator

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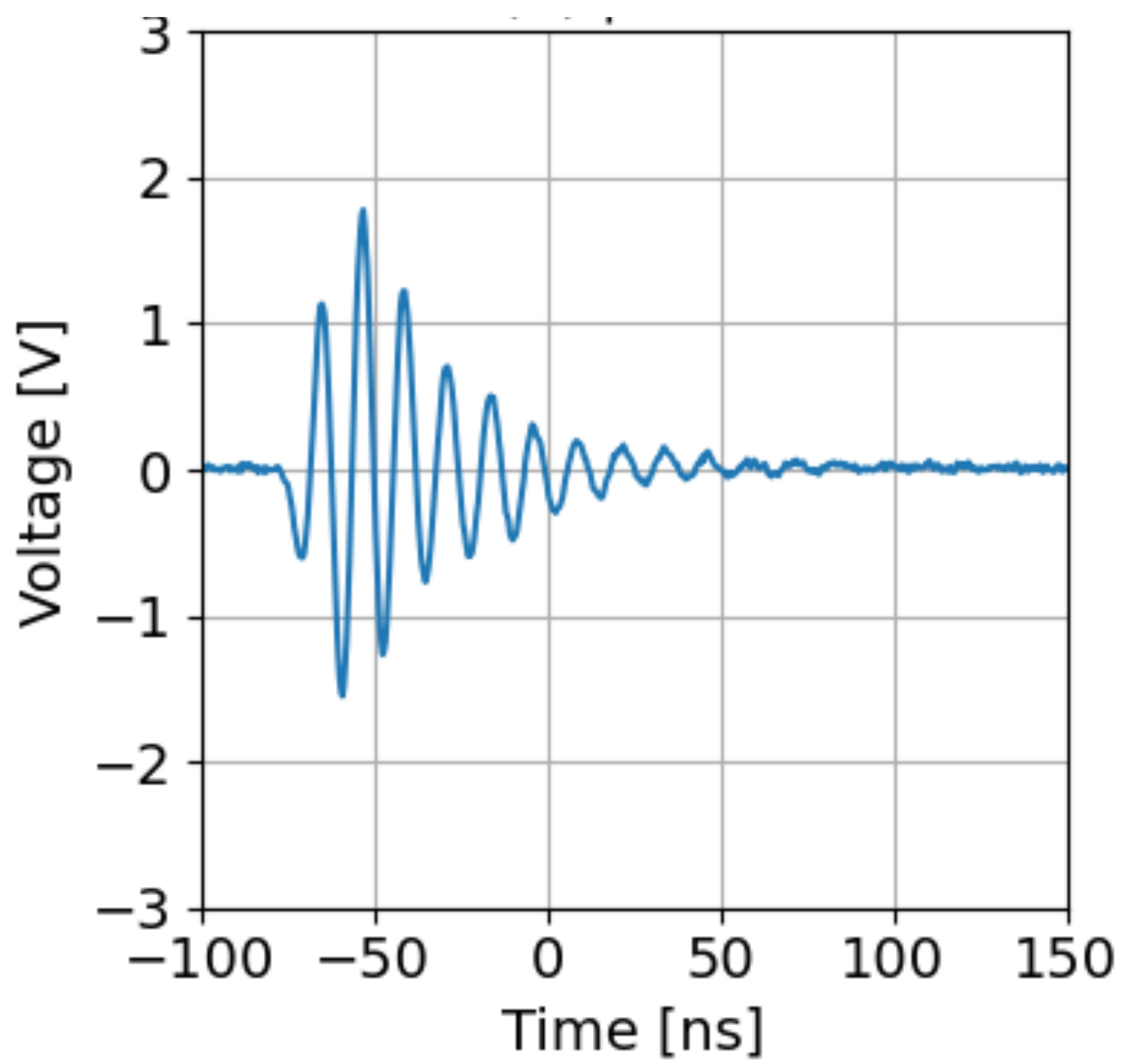
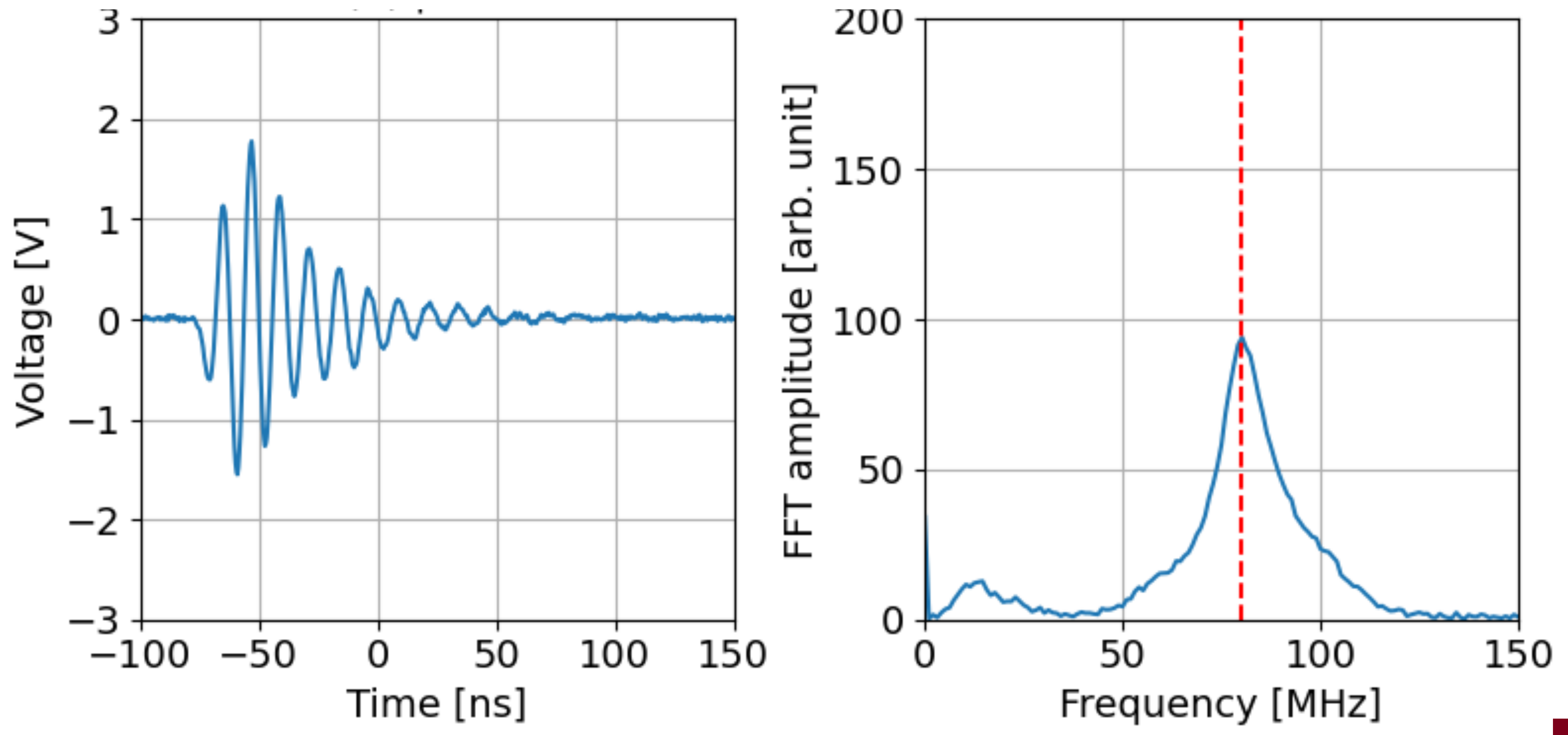
  - Downconversion: translate BPM signal frequency (RF) to desired frequency (IF)
- Readout (e.g. oscilloscope)
  - Retrieves downconverted signal and stores into digital data

Signal Processing

- Received data is a time-domain data
  - i.e. signal voltage vs time



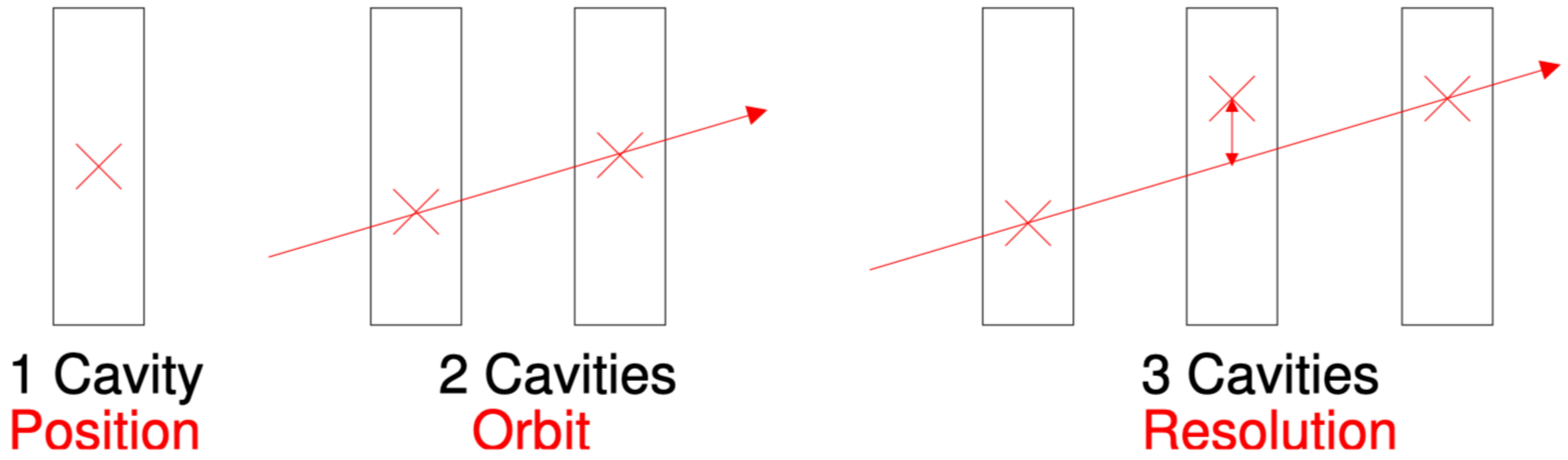
- In the analysis stage, it is more convenient to deal with data in frequency-domain
  - Fast Fourier transform



Position Resolution of Cavity BPM

Position Resolution of BPM

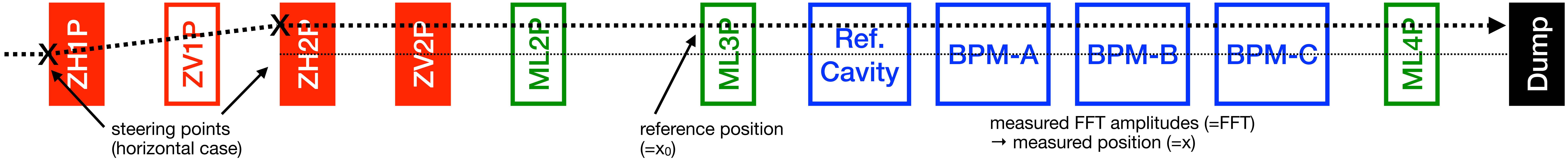
- The position resolution is a critical factor of a BPM
  - Precision of the position measurement should be comparable to beam size
    - e.g. σ_y=5.9 nm at ILC IP
- Cavity BPM is the only easy option to achieve <1 μm position resolution
  - Button BPM: $\mathcal{O}(1) - \mathcal{O}(10)\,\mu\mathrm{m}$
  - Stripline BPM: $\mathcal{O}(10)\,\mu\mathrm{m}$
- Position resolution measurements → "3 BPM method"
  - Calibration run: Obtaining a translation factor from arbitrary FFT value to real world quantity
  - Resolution run: Data at a beam position to measure the resolution



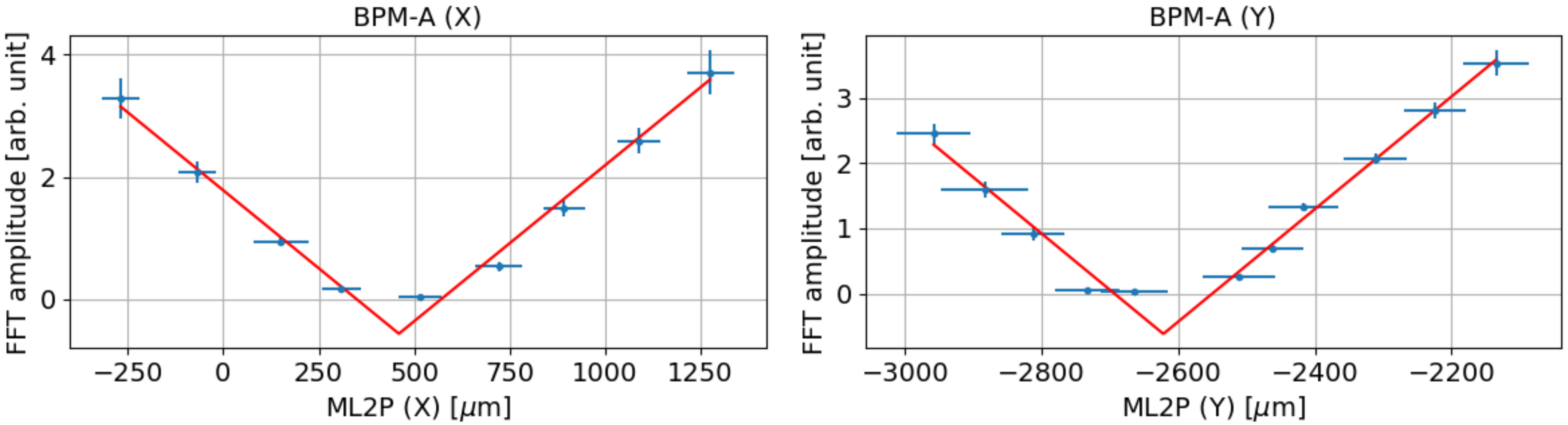
T. Nakamura, M. Sc. Thesis, University of Tokyo (2008)

Resolution Measurements of Cavity BPMs

- Calibrations
  - Changing the beam orbit by steering magnets
  - Referring stripline BPMs for the calibration



  - Mapping FFT to x_0: obtaining $\Delta\mathrm{FFT}/\Delta x_0$
    - V-shape appears since the cavity BPM does not know the absolute position of the beam



Resolution Measurements of Cavity BPMs

- Resolution measurements
  - Finding a correlation between all channels and the one of interest (k-th channel):

$$\mathbf{d}_k = \mathbf{D}_{\not{k}} \cdot \mathbf{v}$$

$\mathbf{d}_k$: measured positions of the k-th channel

$\mathbf{D}_{\not{k}}$: measured positions not of the k-th channel but of the others

$\mathbf{v}$: correlation coefficients

    - Once we get $\mathbf{v}$, predictions of measurements can be made by:

$$\mathbf{d}_k^{\text{pred.}} = \mathbf{D}_{\not{k}} \cdot \mathbf{v}$$

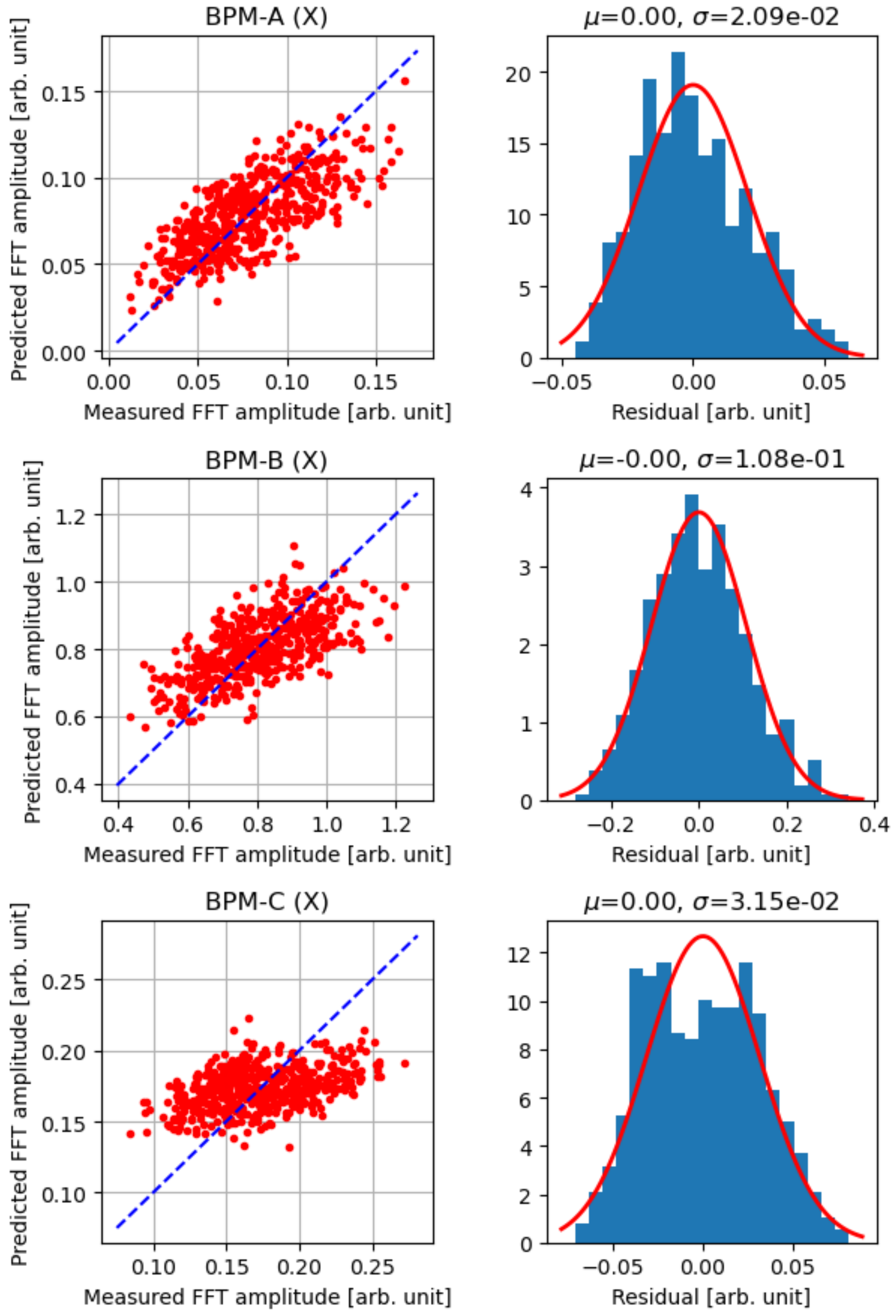
    - Singular value decomposition (SVD)

$$\mathbf{D}_{\not{k}} = U\Sigma V^{\top} \rightarrow \mathbf{v} = (V\Sigma^{-1}U^{\top}) \cdot \mathbf{d}_k$$

  - $\mathbf{d}_k$ is still in FFT amplitude, so it needs to be translated by the calibration factors

$$\mathbf{d}^{\mu\text{m}} = \mathbf{d}^{\text{FFT}} \cdot \frac{\Delta x_0}{\Delta \text{FFT}}$$

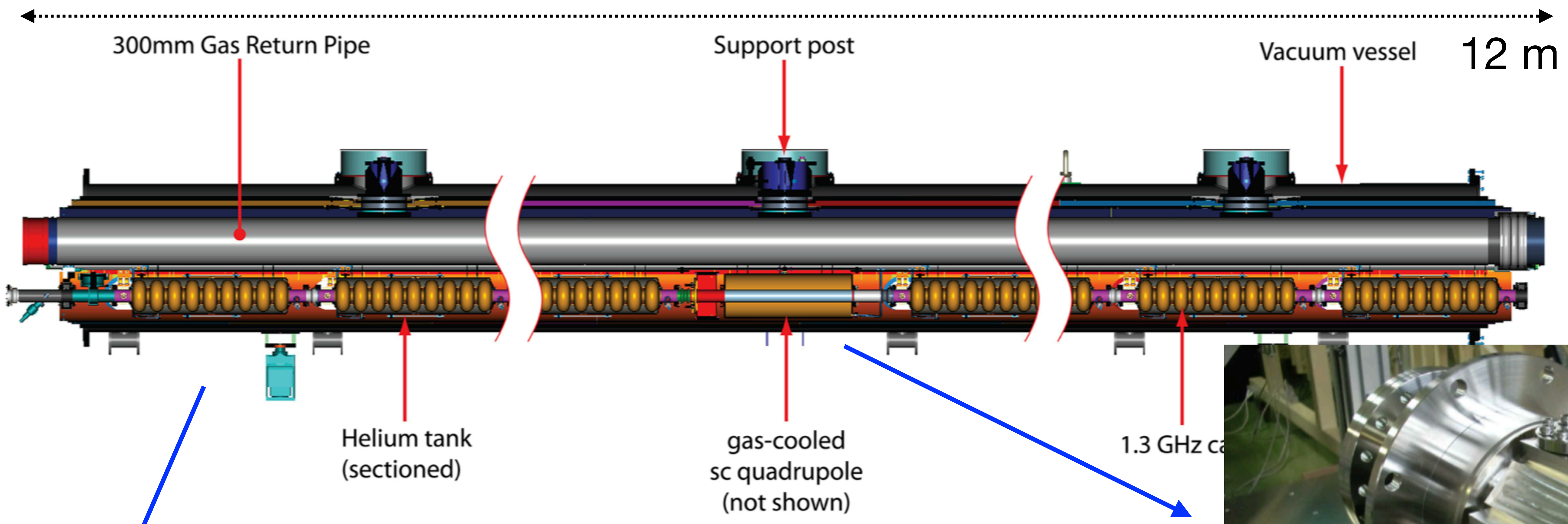
  - From the residual ($\mathbf{R} \equiv \mathbf{d}_k^{\text{pred.}} - \mathbf{d}_k^{\text{meas.}}$), the resolutions (σ) are obtained

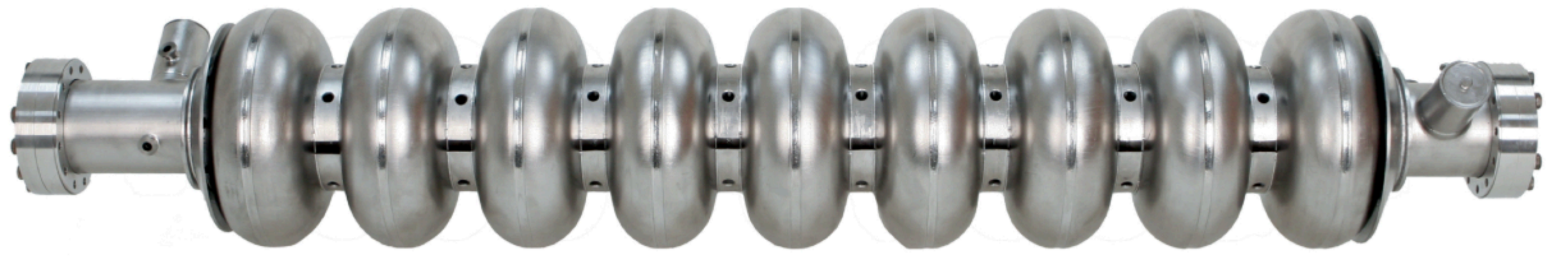


Example:
L-band Cavity BPM for ILC

L-band Cavity BPM for ILC

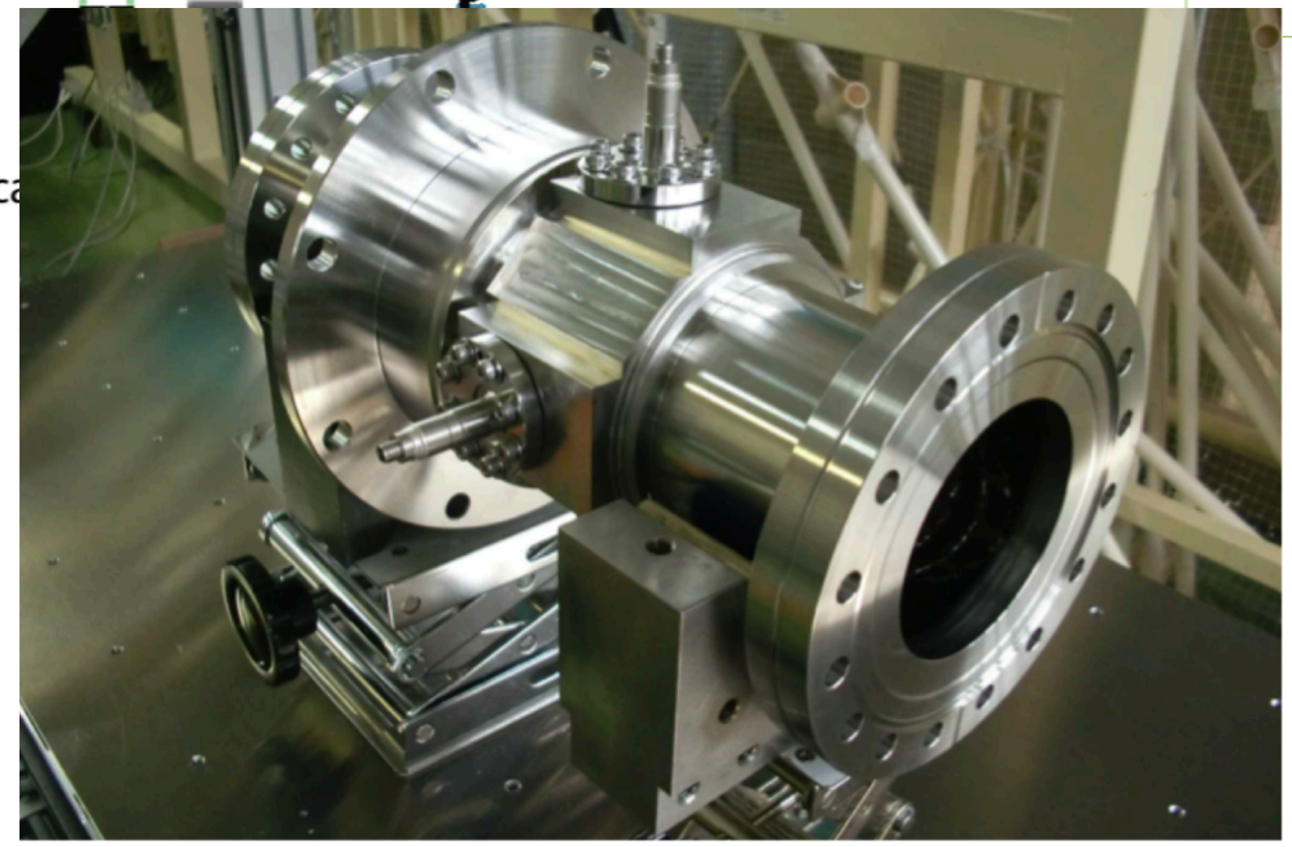
- Two types of cryomodule in the main LINAC of ILC
  - Type A (~1,100 in total): 1.3 GHz 9-cell cavities * 9
  - Type B (~600 in total, at every 3 modules): 1.3 GHz 9-cell cavities * 8 + a SC quadrupole package (quadrupole/corrector/BPM)
- BPM resolution requirement: $5\ \mu\text{m}$ (TDR), $\mathcal{O}(100)\ \text{nm}$ (goal)

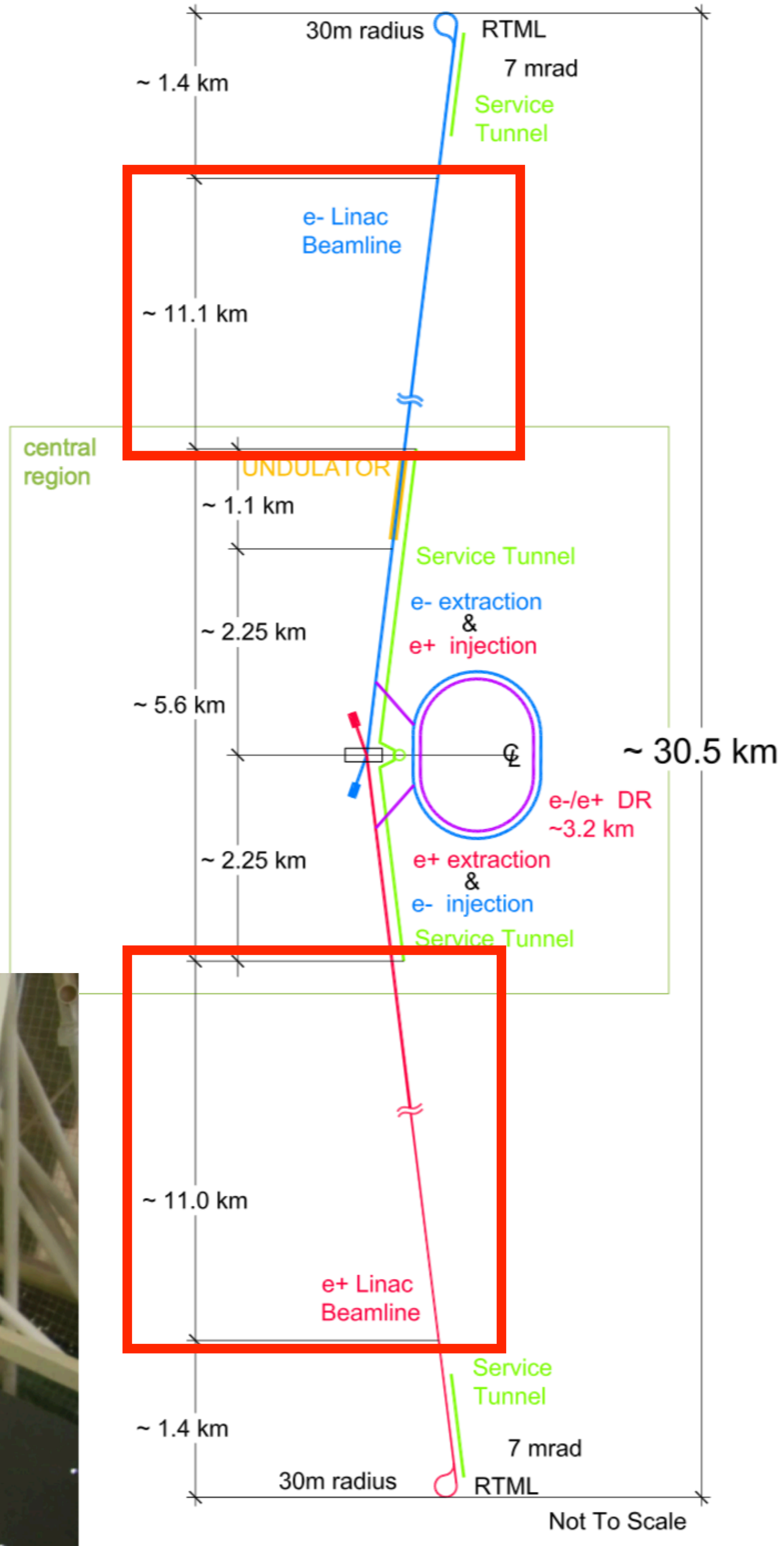




1.3 GHz 9-cell cavity (Nb)

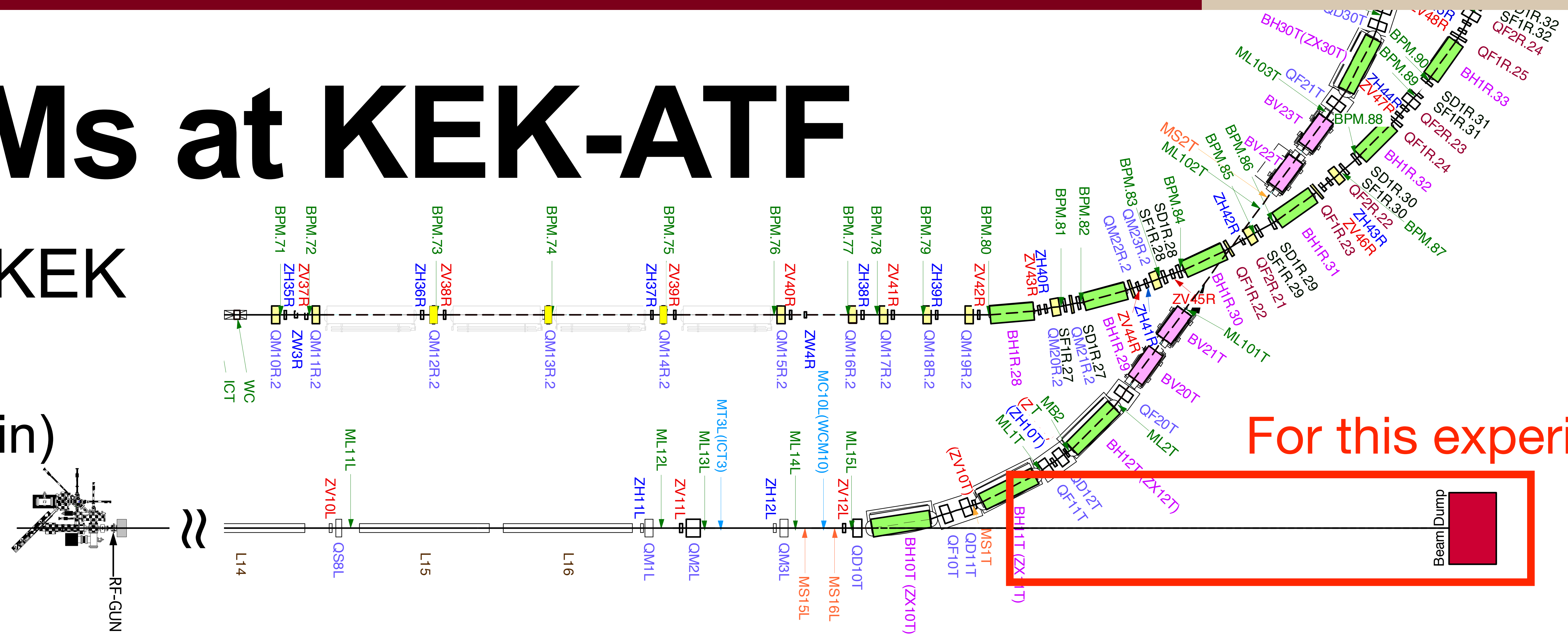
L-band re-entrant cavity BPM



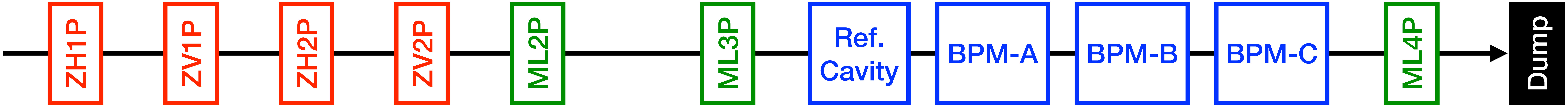


L-band Cavity BPMs at KEK-ATF

- Accelerator Test Facility (ATF) at KEK
  - 1.3 GeV electron LINAC of 110 m long
  - ~10^{10} electrons/bunch (1~20 bunches/train)
  - Repetition: 3.125 Hz



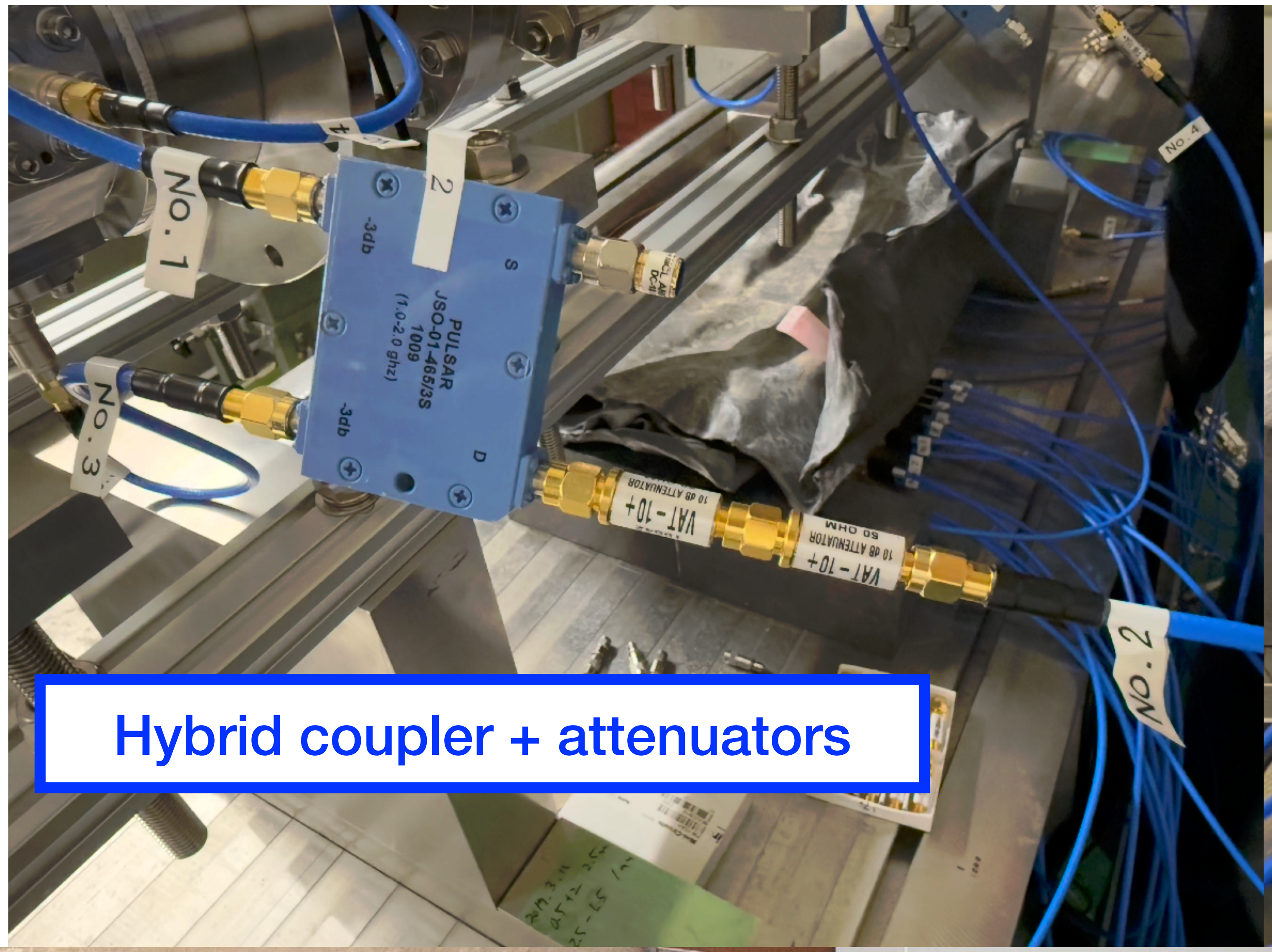
- At the end of the LINAC, 4 steering magnets, 4 stripline BPMs, and 4 cavity BPMs are located

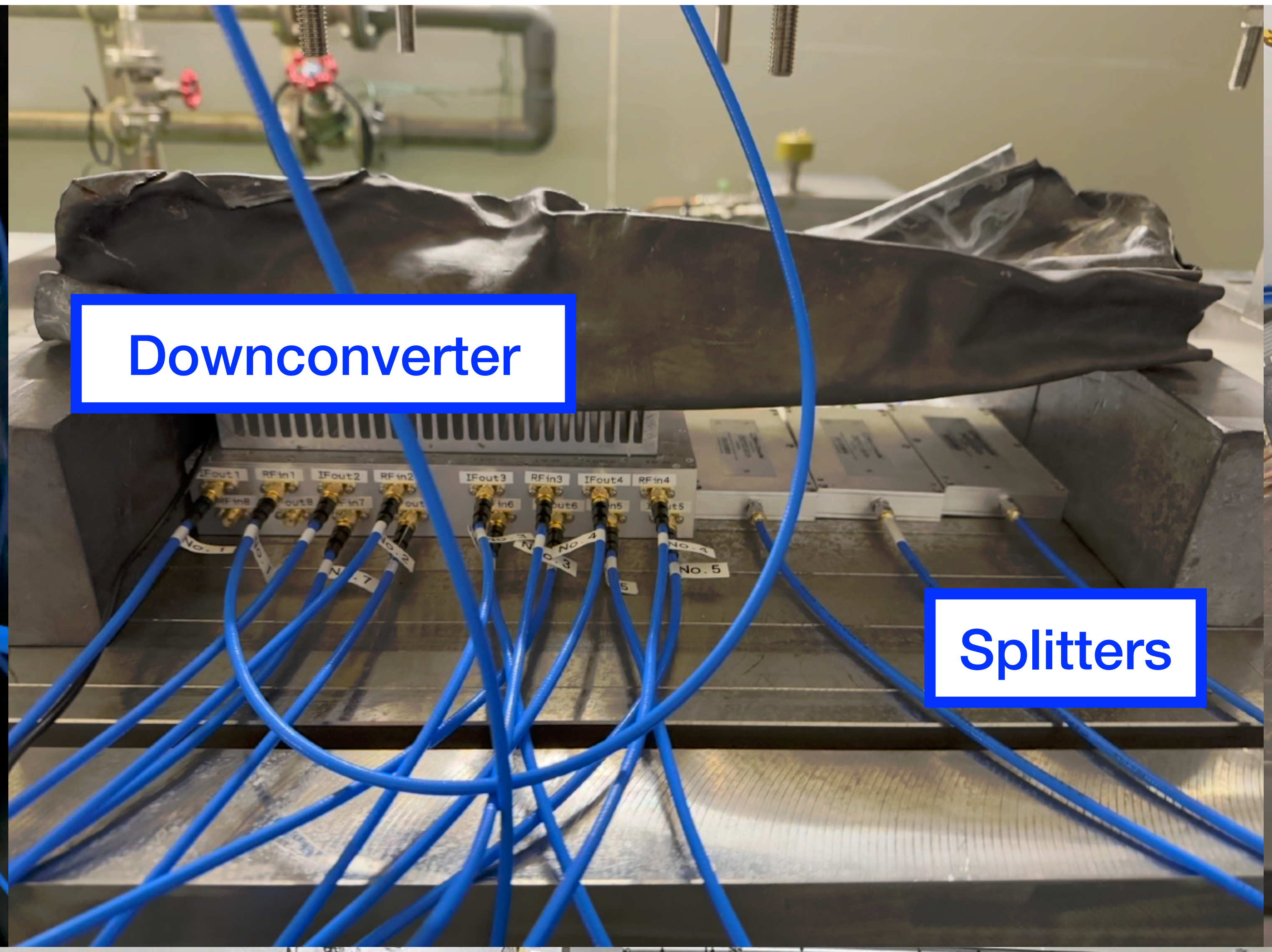


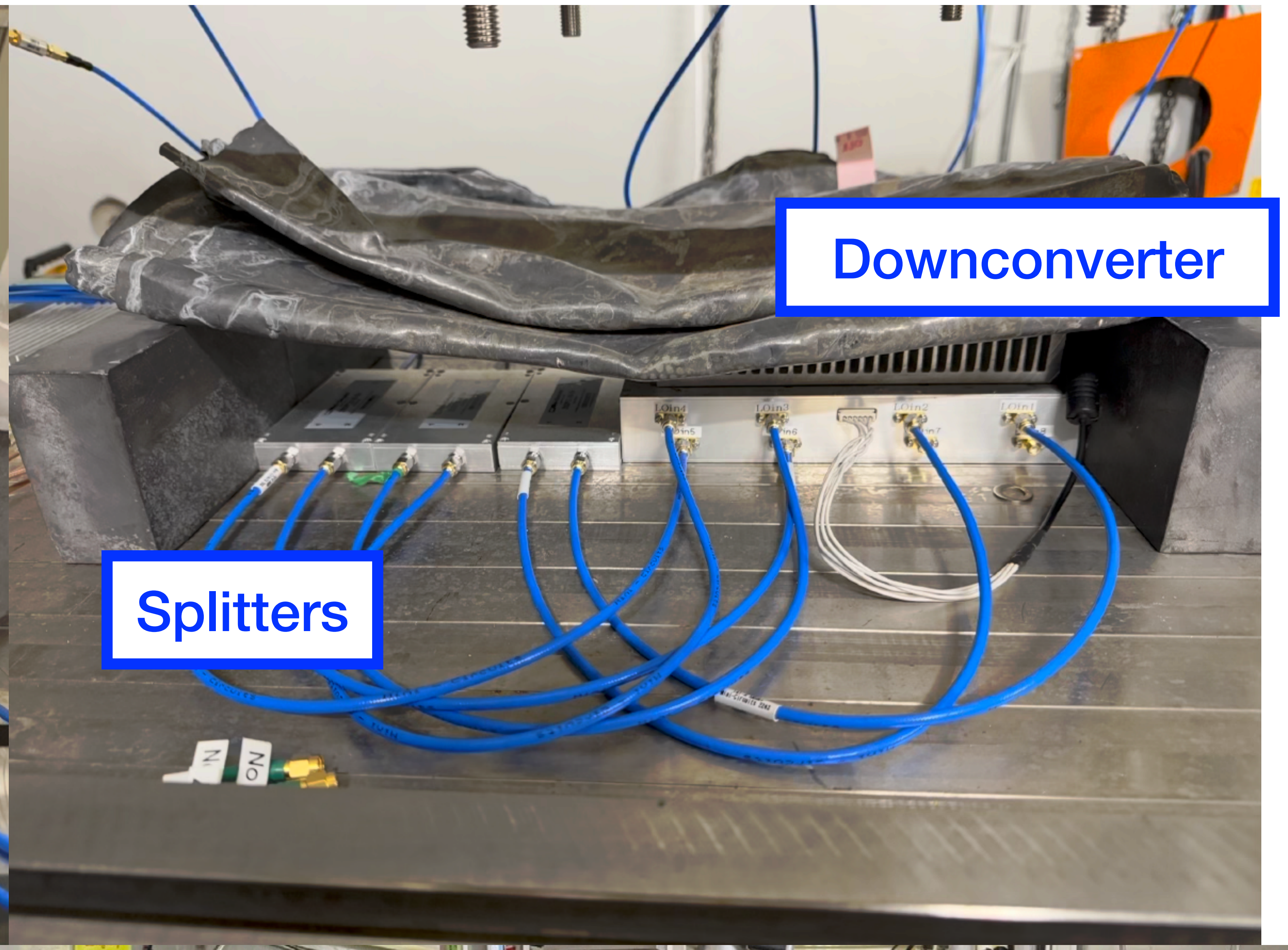
L-band Cavity BPM Readouts

- Signals are merged by hybrid couplers and pass through a downconverter

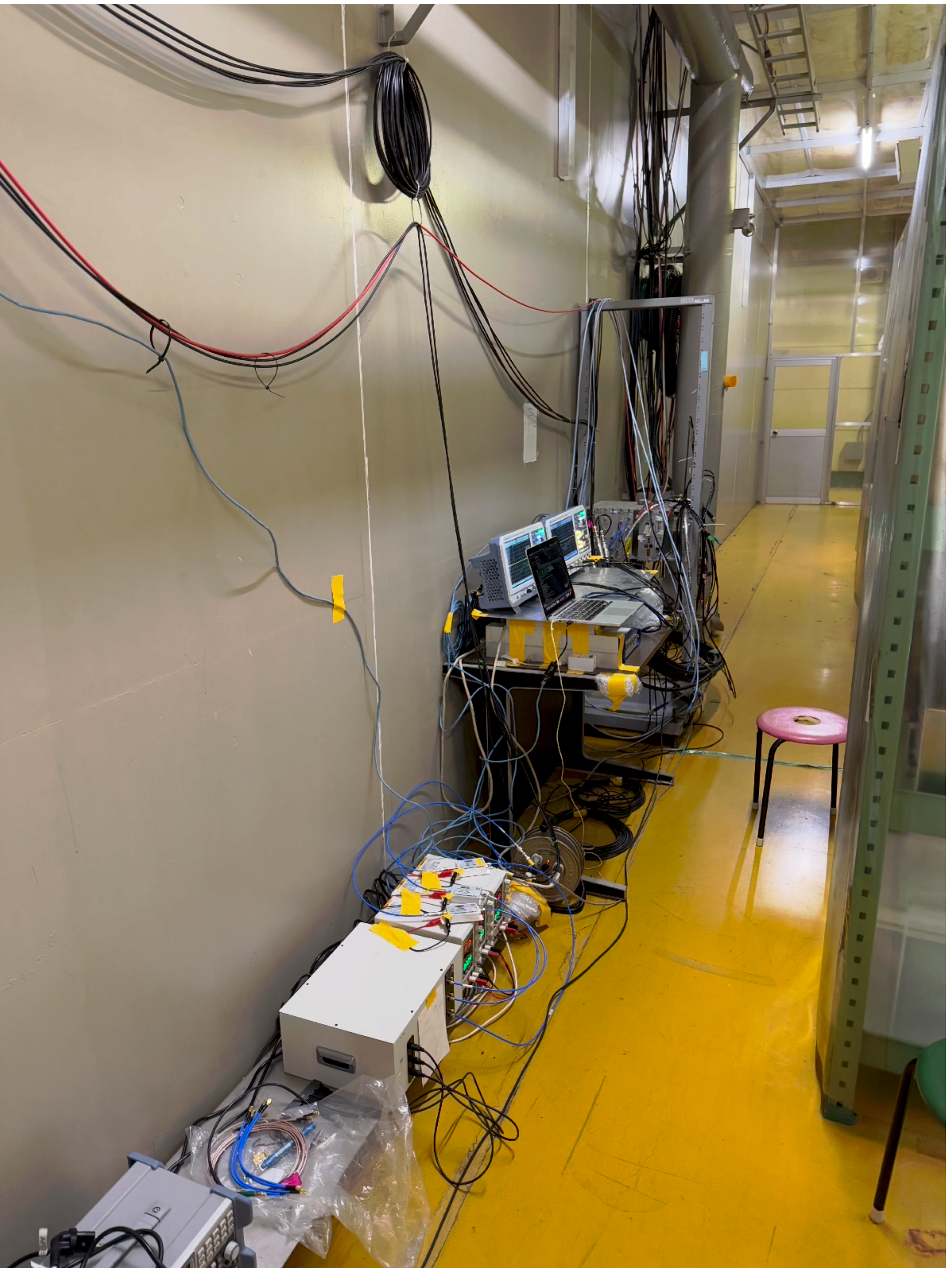
L-band Cavity BPM Readouts (inside the tunnel)

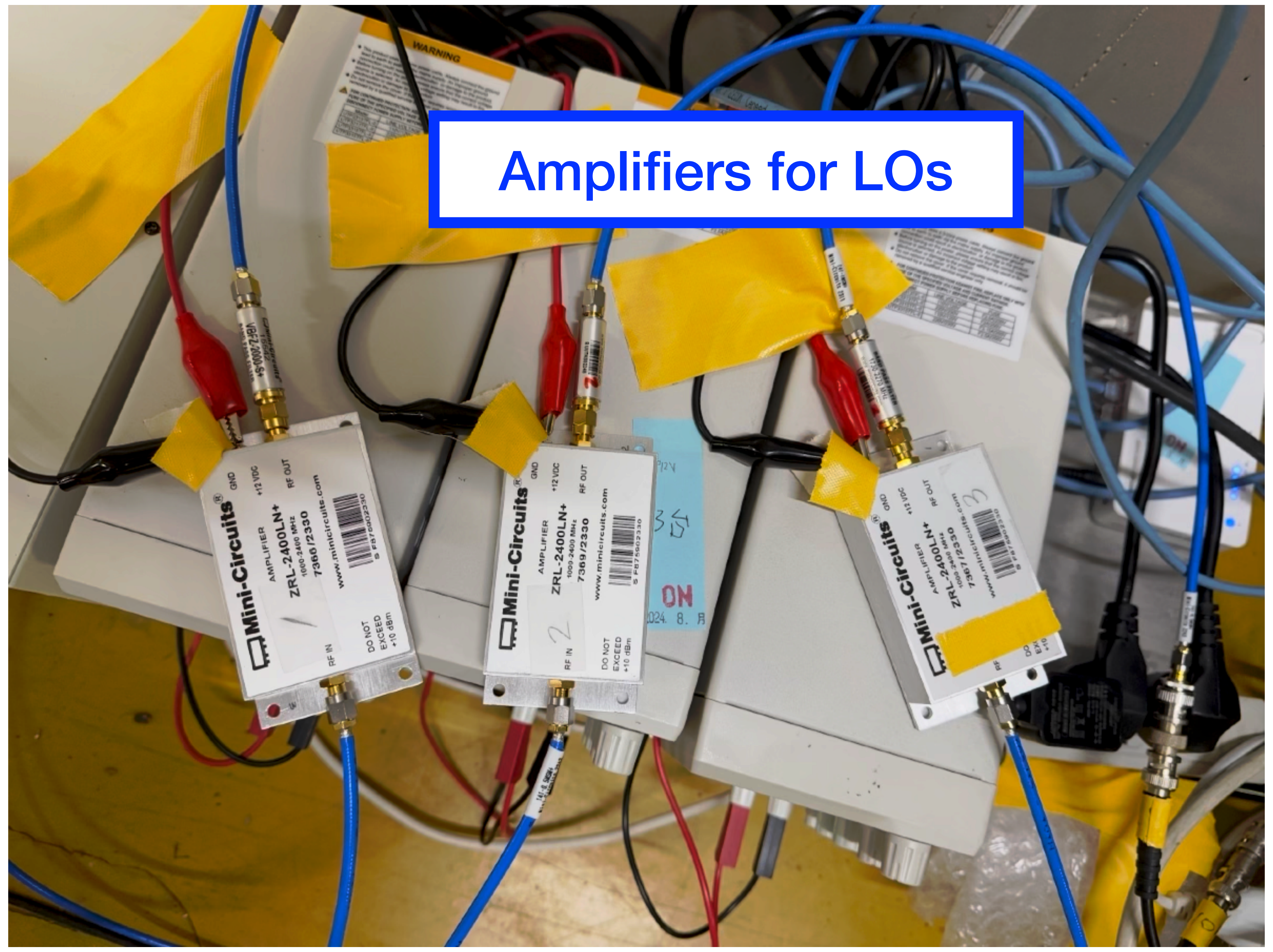






L-band Cavity BPM Readouts (outside the tunnel)

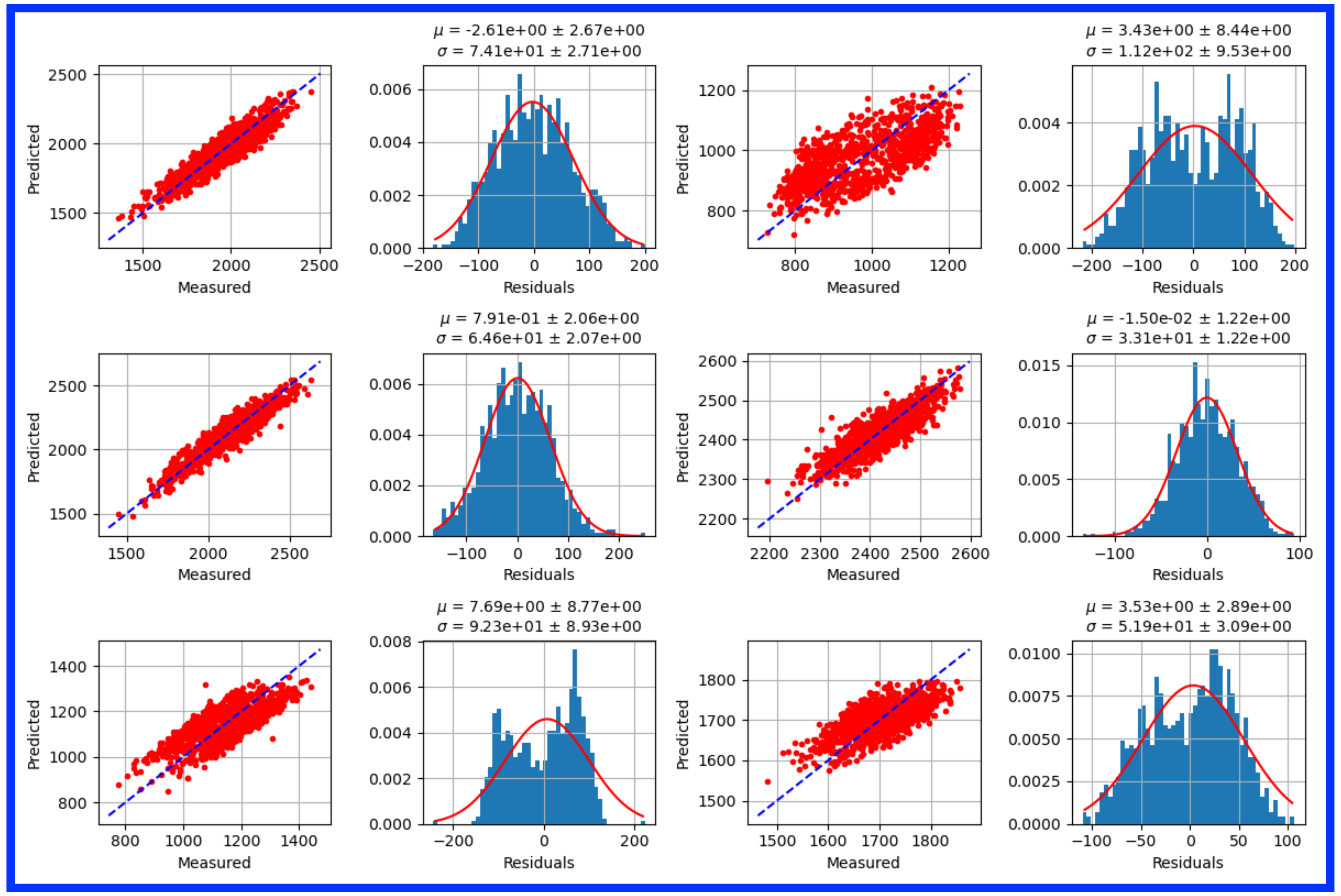


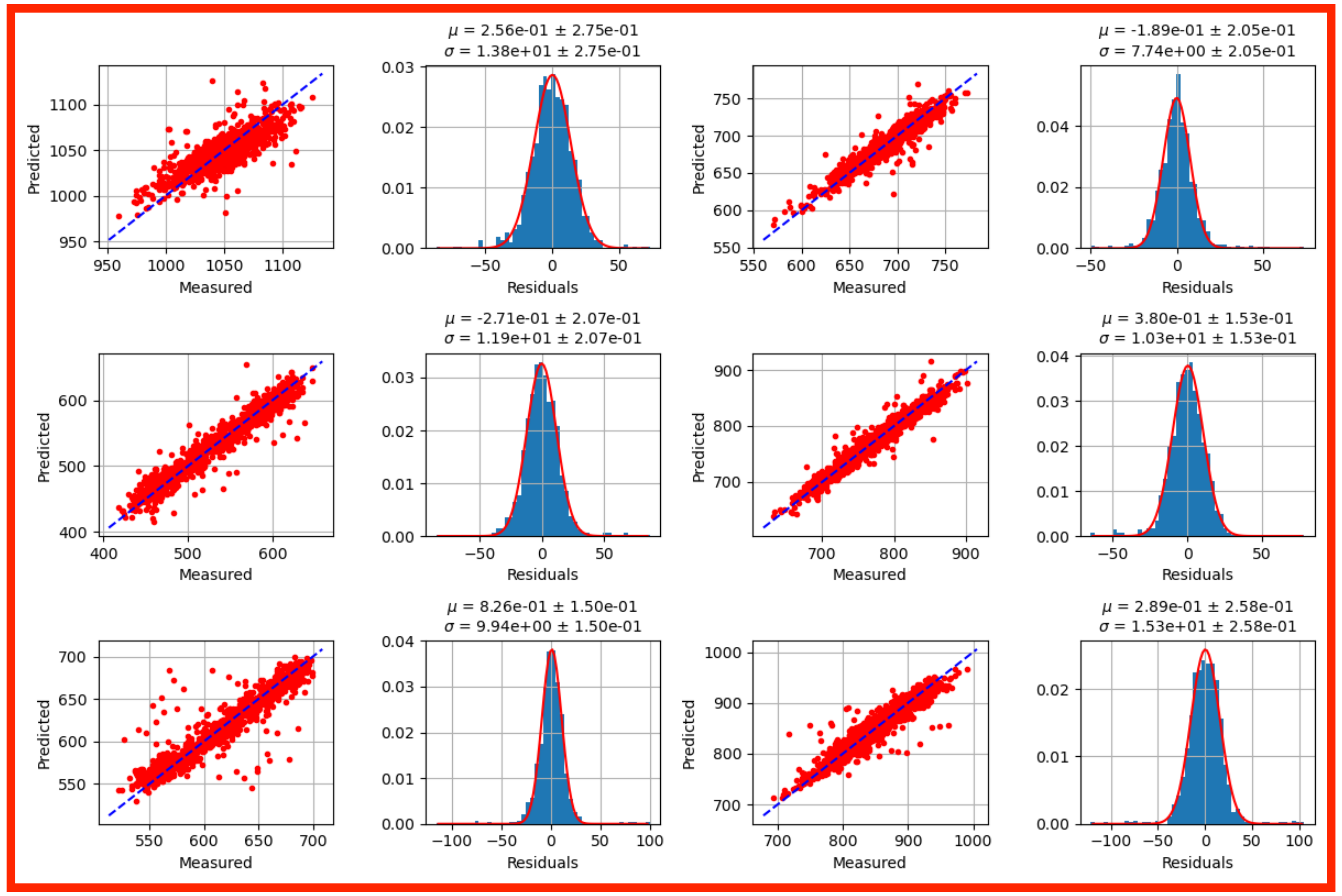


R&S RTM3000
2.5 GS/s, 10-bit

Resolution Measurements in 2025

- IF=45 MHz with 3 different LO's | Different IF's with a common LO





[nm]	2019	2023	2024 (w/ osc.'s)	2024 (w/ pico.)	2025 (IF=45 MHz)	2025 (Common LO)
BPM-A (X)	3750	2062	2033	3068	1372	731
BPM-B (X)	2680	1984	824	1904	1933	1123
BPM-C (X)	985	1831	470	1167	1927	383
BPM-A (Y)	698	3450	1457	1392	1716	317
BPM-B (Y)	2260	1948	1234	1198	695	902
BPM-C (Y)	324	712	162	309	767	485