KOREA UNIVERSITY
SEJONG CAMPUS

KU-The Future

Accelerating cavities
(현장실습)

July 7, 2025

2025 가속기 및 빔라인 미래인재 양성 교육단 여름학교

가속기과학과 고병록

Charged particle acceleration

•Lorentz force $\vec{F} = q\vec{E} + q\left(\vec{v} \times \vec{B}\right) = \vec{F}_E + \vec{F}_B$

•work-kinetic energy theorem, $W = \int_0^d \vec{F} \cdot d\vec{s} = \Delta K = (\gamma - 1)mc^2$ if $v_i = K_i = 0$

∘ $W_B = \int_0^d \vec{F}_B \cdot d\vec{s} = q\int_0^d d\vec{s} \cdot \left(\vec{v} \times \vec{B}\right) = q\int_0^d \vec{B} \cdot \left(d\vec{s} \times \vec{v}\right) = 0 \ \left(\because d\vec{s} \parallel \vec{v}\right) \Leftrightarrow \gamma = 1$ (or $v_f = 0$) $\rightarrow \Delta v = 0$

∘ $W_E = \int_0^d \vec{F}_E \cdot d\vec{s} = q\int_0^d \vec{E} \cdot d\vec{s} = (\gamma - 1)mc^2$ if $K_i = 0$ (or $v_i = 0$) $\rightarrow \gamma = \dfrac{1}{\sqrt{1 - \dfrac{v_f^2}{c^2}}} = \dfrac{q}{mc^2}\int_0^d \vec{E} \cdot d\vec{s} + 1$

$\rightarrow v$ increases if E increases

•particle energy $\varepsilon = \gamma mc^2 = \dfrac{mc^2}{\sqrt{1-\beta^2}} = \dfrac{mc^2}{\sqrt{1-\left(\dfrac{v}{c}\right)^2}}$, c : speed of light in vacuum, $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$, and $\beta = \dfrac{v}{c}$

$= mc^2 + K \rightarrow K = (\gamma - 1)mc^2$

E(t) vs. E (Radio Frequency (RF) E vs. Static E)

- for circular accelerators

$$\circ W_E = q\oint \vec{E}\cdot d\vec{s} = q\int_A (\vec{\nabla}\times\vec{E})\cdot d\vec{A}$$

$$* \text{ for static } E \text{ field } \vec{E} = -\vec{\nabla}\phi,\ W_E = -q\int_A (\vec{\nabla}\times\vec{\nabla}\phi)\cdot d\vec{A} = 0$$

$$* \text{ for RF } E \text{ field } \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t},\ W_E = -q\int_A \frac{\partial\vec{B}}{\partial t}\cdot d\vec{A} = (\gamma-1)mc^2$$

- multipass acceleration
- electrical breakdown

- RF E field preferable for high energy accelerators

Electric fields in cavities

9-cell Superconducting RF (SRF) cavity, Nb (niobium)
→ **accelerating cavity** with accelerating electric field whose frequency is 1.3 GHz

No net charge, no battery (generator), no electrodes → no potential

radiation energy

$$\varepsilon = h\nu\left(\frac{1}{e^{h\nu/k_B T_{cavity}}-1}+\frac{1}{2}\right)$$

→ Electromagnetic wave according to J. C. Maxwell

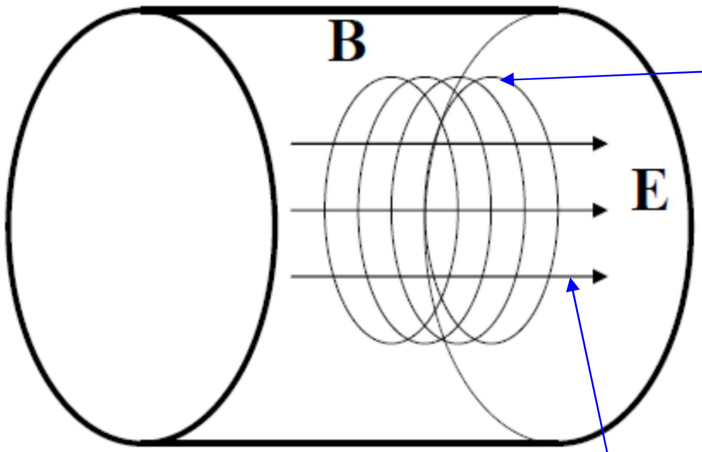
$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{B}\end{Bmatrix} = 0,$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \rightarrow \text{constant}$$

RF electric fields in a cavity even at $T_{cavity} = 0\ \mathrm{K}$

EM profiles in cylindrical cavities

- cylindrical RF cavities → exact solutions by the Maxwell's eqs. and the relevant boundary conditions



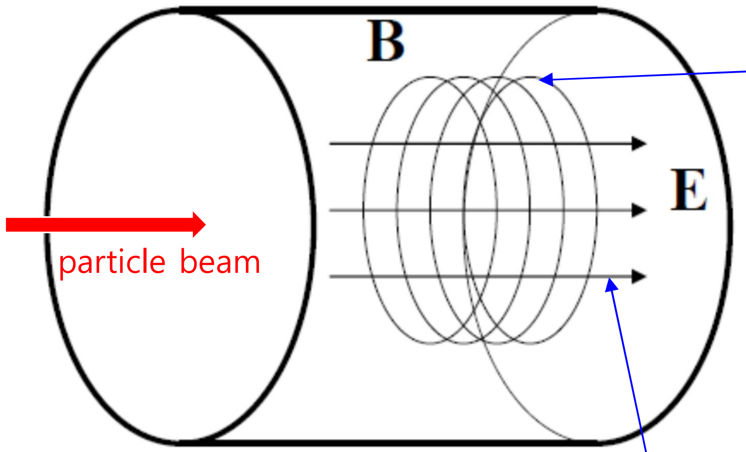
Transverse **M**agnetic field only

standing wave
TM_{010} mode

Longitudinal Electric field only

EM profiles in cylindrical cavities

- cylindrical RF cavities → exact solutions by the Maxwell's eqs. and the relevant boundary conditions



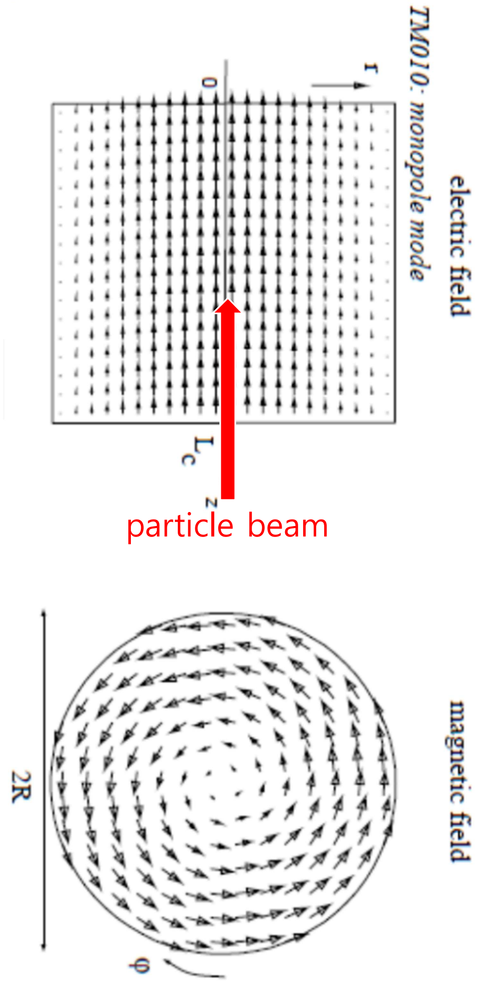
Transverse **M**agnetic field only

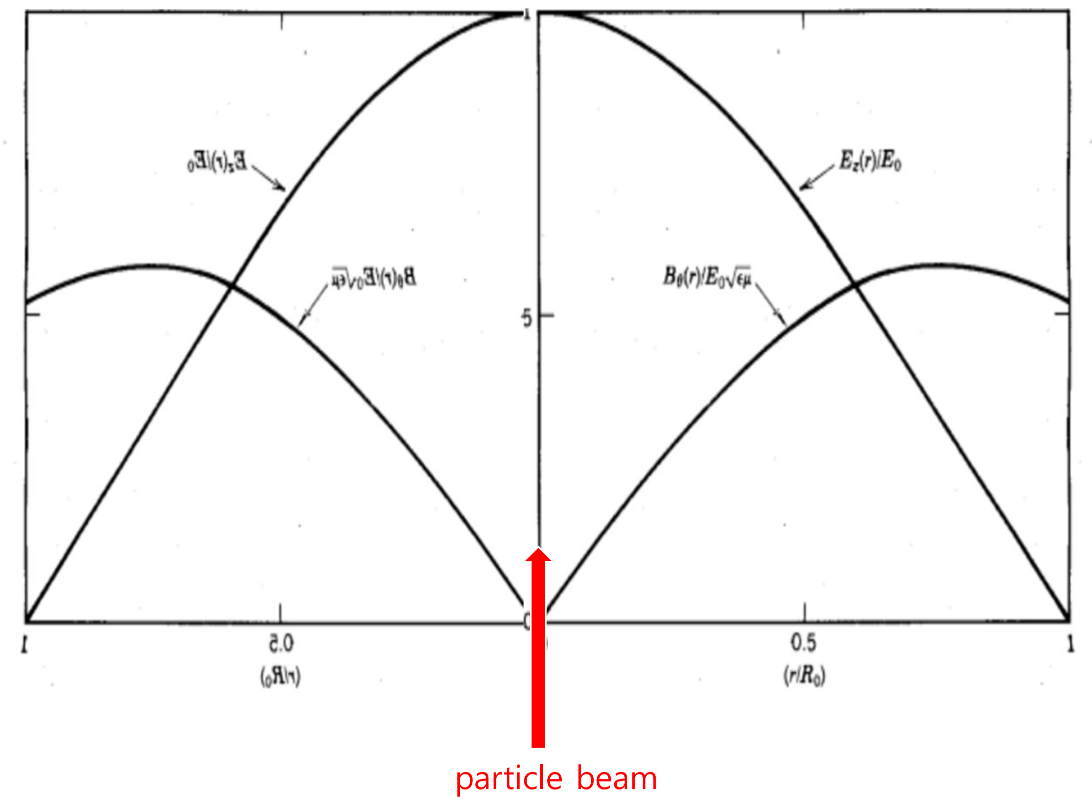
standing wave
TM_{010} mode → acceleration mode

Longitudinal Electric field only

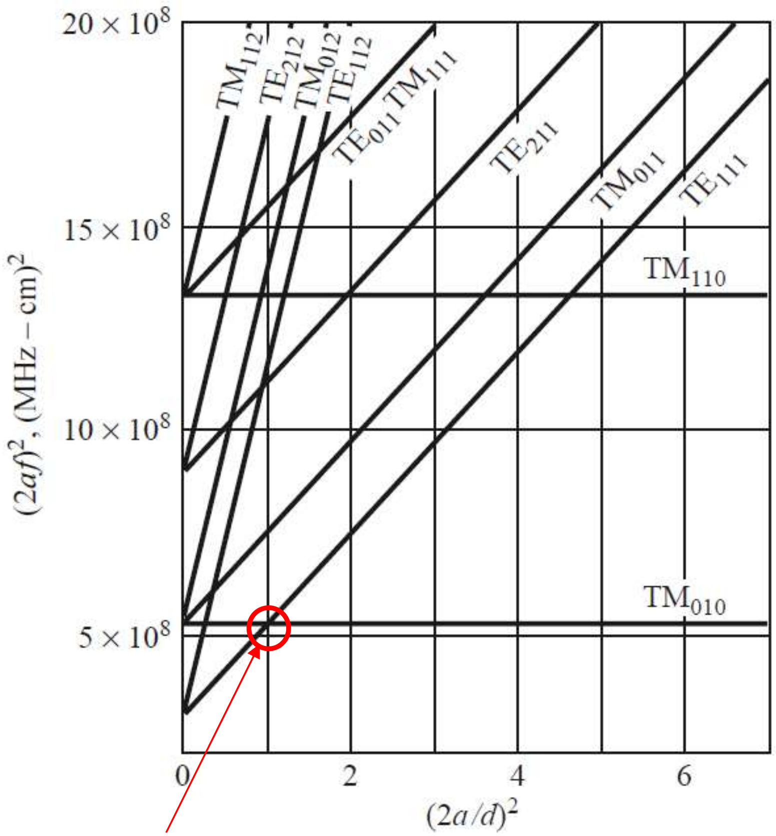
accelerating field

EM profiles in cylindrical cavities

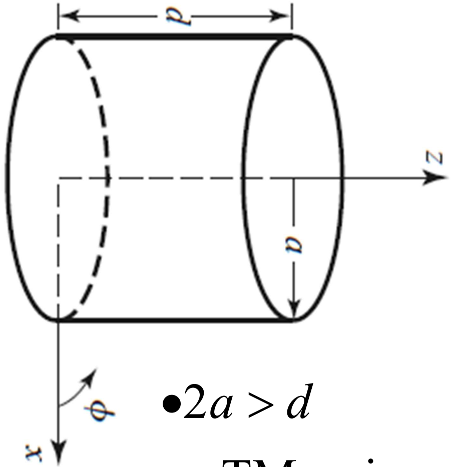




Cavity modes

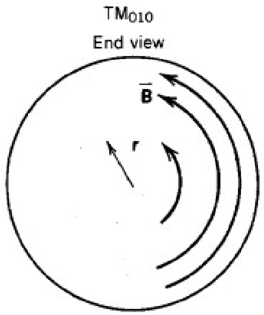


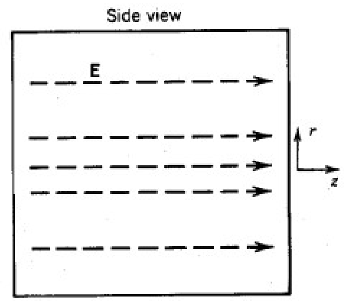
mode crossing
$TM_{010} = TE_{111}$



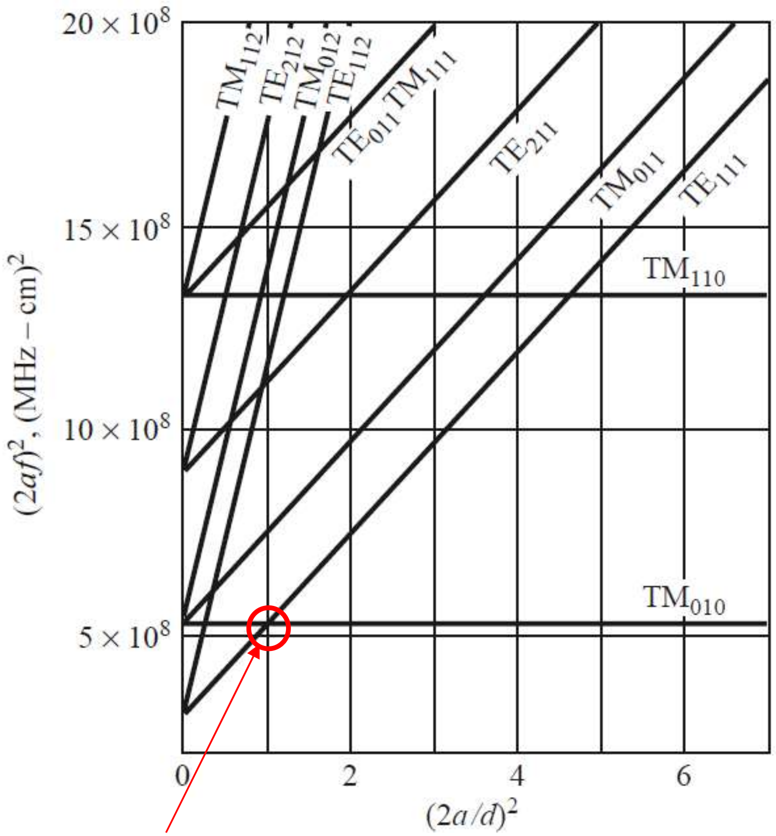
- $2a > d$

$\rightarrow TM_{010}$ is the fundamental cavity mode

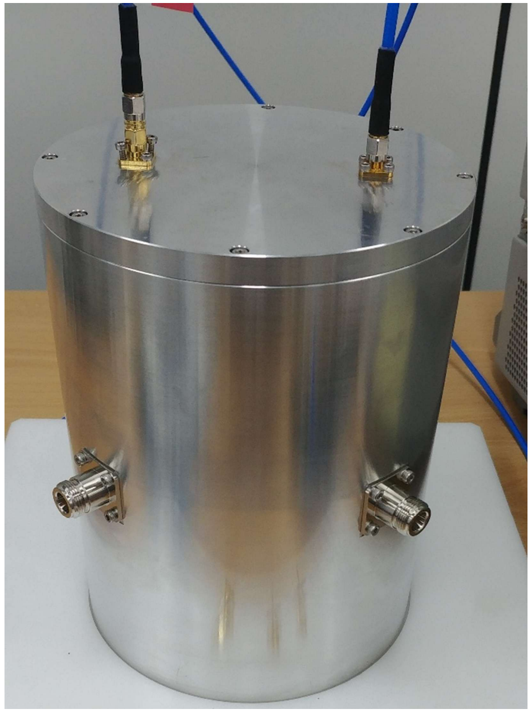


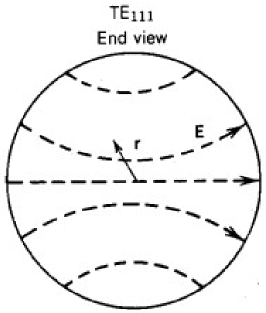


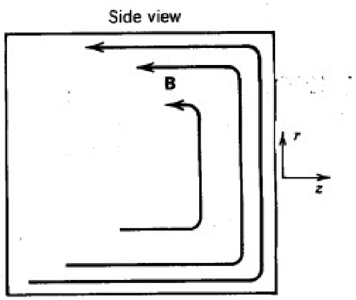
Cavity modes



mode crossing
$TM_{010}=TE_{111}$







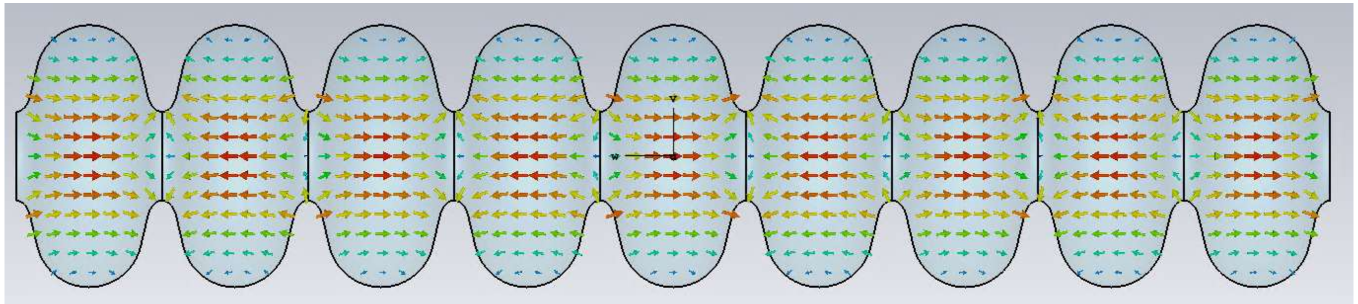
• $2a < d \rightarrow TE_{111}$ is the fundamental cavity mode

EM profiles in the TESLA (TeV Superconducting Linear Accelerator) cavity

← 9 X

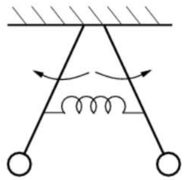
←

- 9-cell RF cavity inherited from a cylinder RF cavity



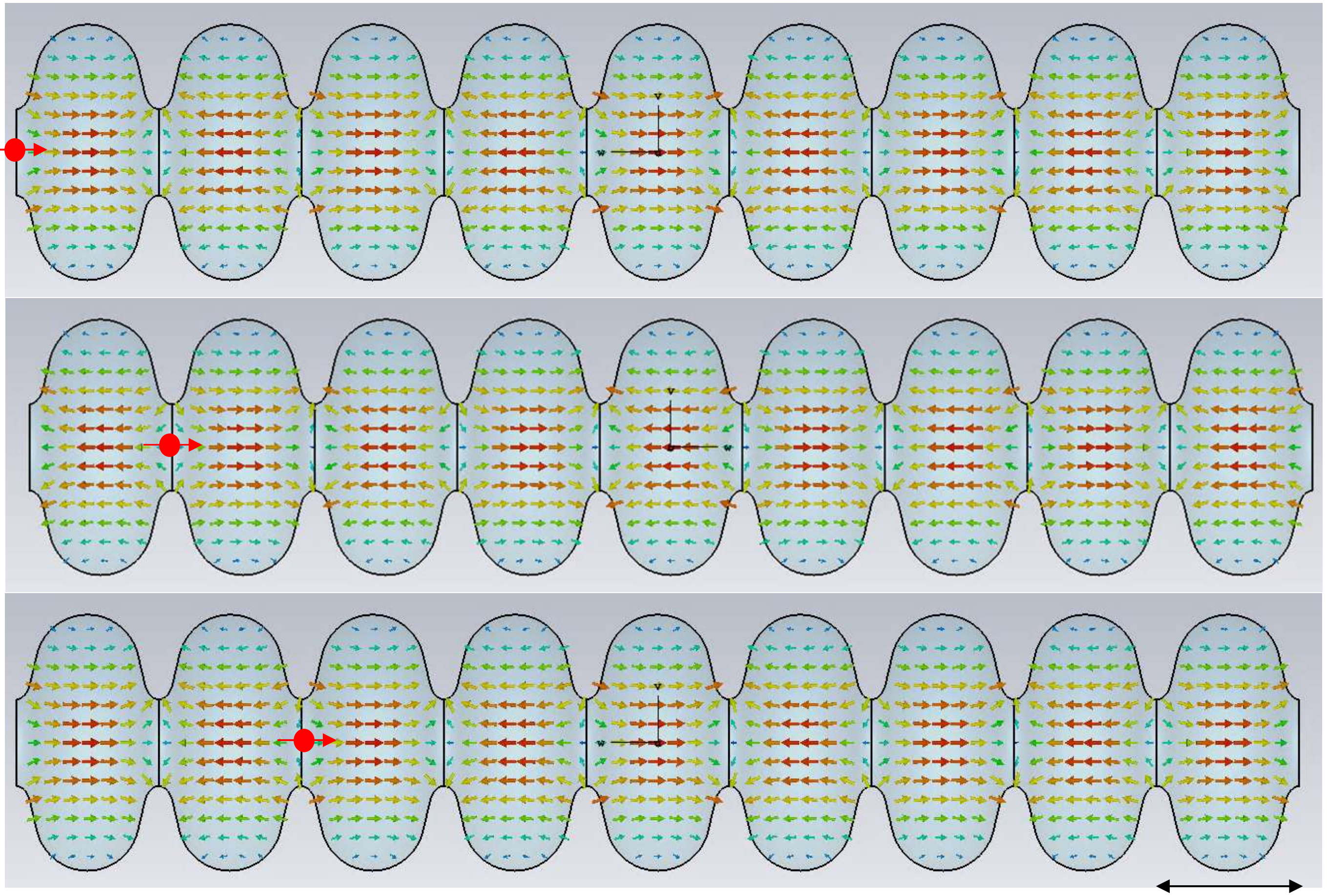
coupled harmonic oscillator
9 masses attached with springs

highest normal mode



- TM_{010}(-like) mode → acceleration mode

EM profiles in the TESLA (TeV Superconducting Linear Accelerator) cavity



$$t_0 = 0$$

$$t_{L_{cell}=\frac{\lambda}{2}} = \frac{1}{2}\frac{1}{f_{TM_{010}}} = \frac{1}{2}\frac{1}{1.3\ \text{GHz}} \simeq 0.3846\ \text{ns}$$

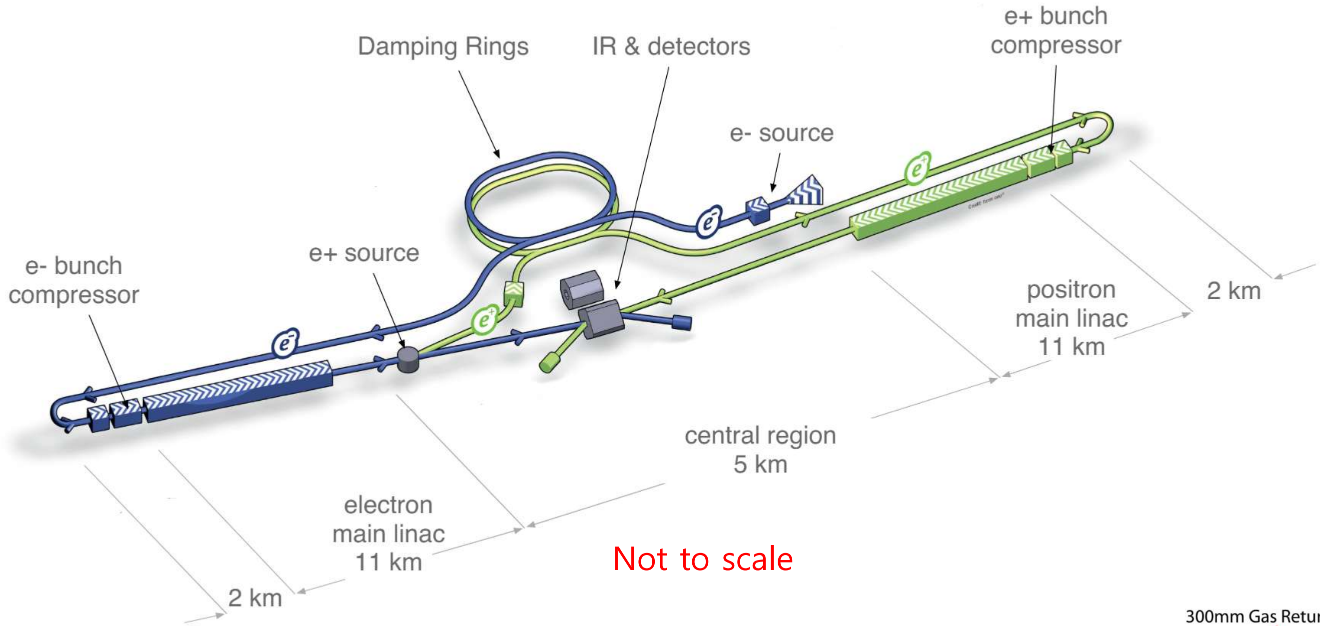
$$t_{2L_{cell}=\lambda} = \frac{1}{f_{TM_{010}}} = \frac{1}{1.3\ \text{GHz}} \simeq 0.7692\ \text{ns}$$

L_{cell}

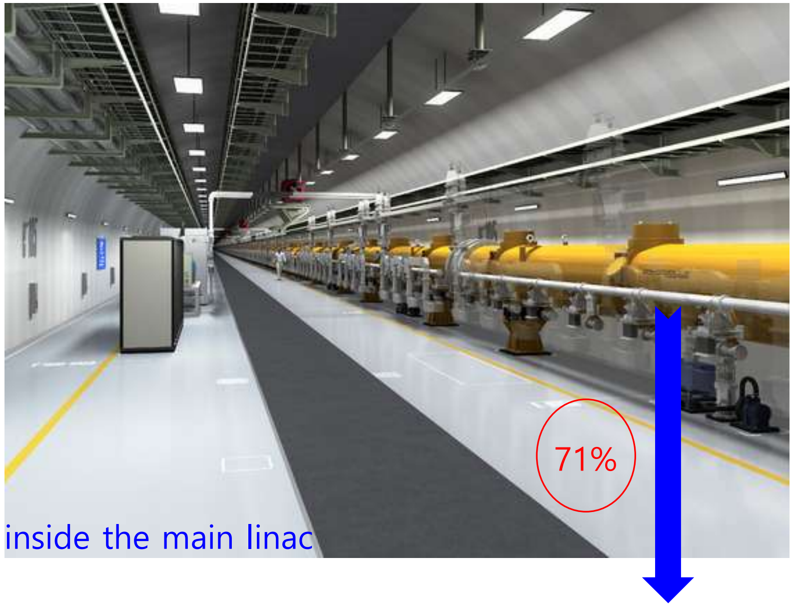
$$\lambda = 2L_{cell} = \frac{c}{1.3\ \text{GHz}} \rightarrow L_{cell} = 115.304\ \text{mm}$$

- works for v~c (or β~1) particles

Accelerating cavities in the International Linear Collider (ILC)



Not to scale

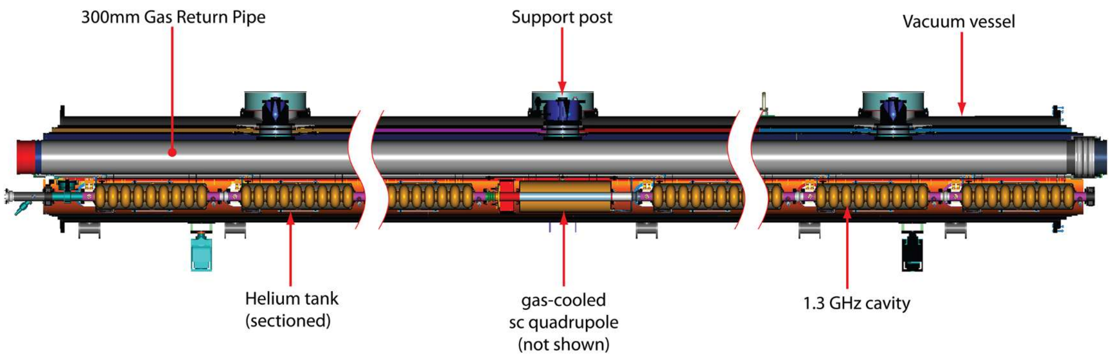


inside the main linac

9-cell Superconducting Radio Frequency (SRF) cavity, Nb (niobium)

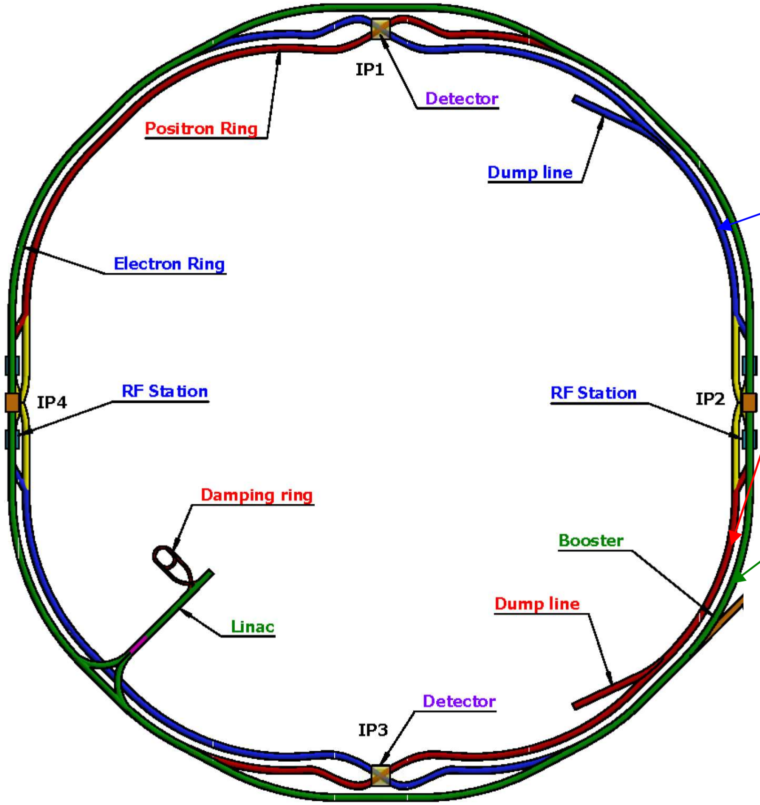
→ **accelerating cavity** with electric field whose frequency is 1.3 GHz

~8000 9-cell cavities for the ILC



lateral cross section of the ILC cryomodule

Accelerating cavities in the Circular Electron Positron Collider (CEPC)

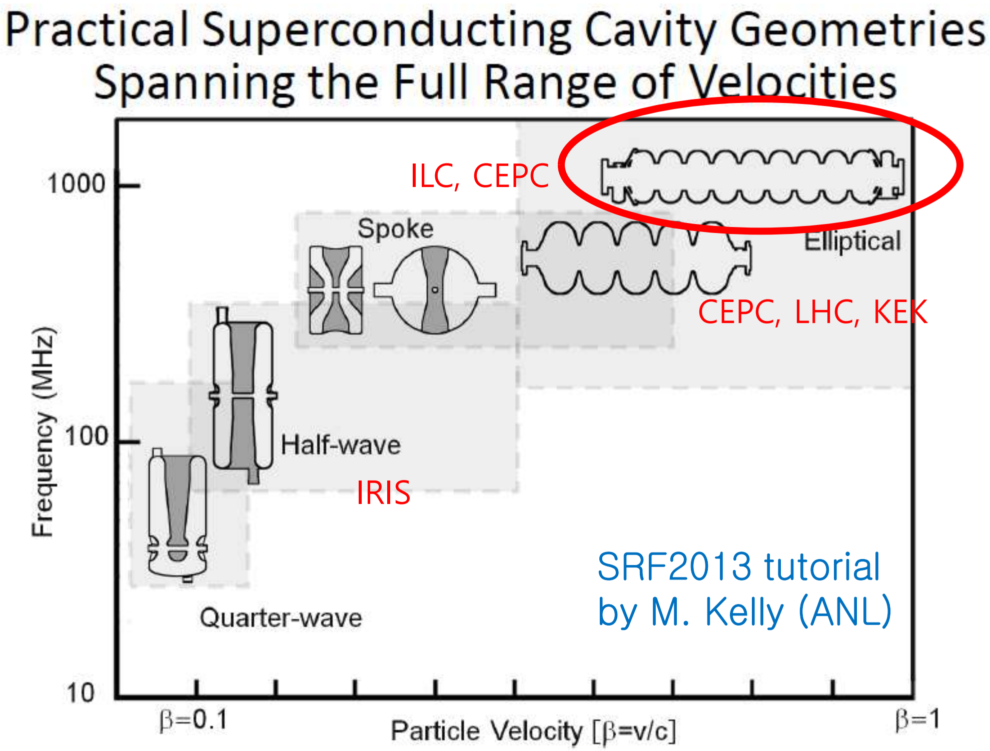


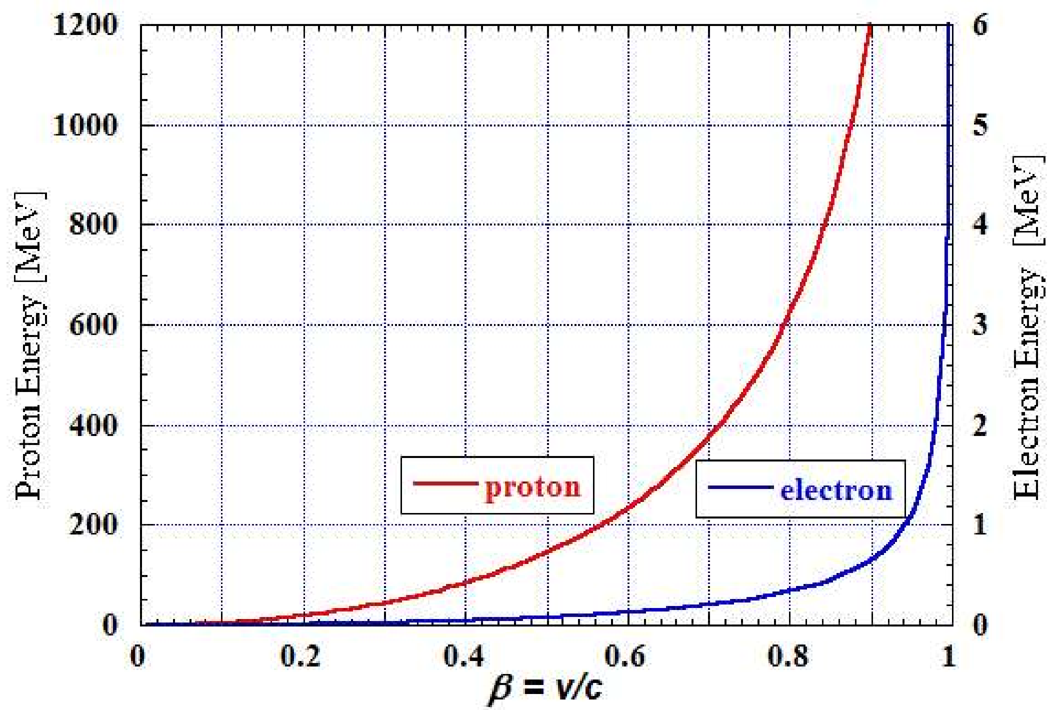
circumference of 100 km

	Baseline			Power Upgrade			Energy Upgrade			
	Higgs	W	Z	Higgs	W	Z	$t\bar{t}$			
Collider SR power / beam [MW]	30			50			30		50	
Beam energy [GeV]	120	80	45.5	120	80	45.5	180			
Luminosity / IP [10^{34} $cm^{-2}s^{-1}$]	5	16	115	8.3	26.7	192	0.5		0.8	
Collider 650 MHz cavities	**2-cell**		**1-cell**	**2-cell**		**1-cell**	**Add 5-cell**	**Existing 2-cell**	**Add 5-cell**	**Existing 2-cell**
RF voltage [GV]	2.2	0.7	0.12	2.2	0.7	0.1	10 (6.1 + 3.9)		10 (6.1 + 3.9)	
Beam current / ring [mA]	16.7	84	801	27.8	140	1345	3.4		5.6	
Cavity number	192	96×2	30×2	336	168×2	50×2	192	336	192	336
Cryomodule number	32	32	60	56	56	100	48	56	48	56
Klystron number	96	96	60	168	168	100	48	168	96	168
Klystron power [kW]	800	800	1200	800	800	1200	800	800	800	800
Collider 4.5 K equiv. heat load [kW]	44.4	28.1	15.2	41.9	20	20.1	128.3		128.3	
Booster 1.3 GHz cavities	**9-cell**						**Add 9-cell**	**Existing 9-cell**	**Add 9-cell**	**Existing 9-cell**
Extraction RF voltage [GV]	2.17	0.87	0.46	2.17	0.87	0.46	9.7 (7.53 + 2.17)		9.7 (7.53 + 2.17)	
Beam current [mA]	1	3.1	16	1.4	5.3	30	0.12		0.19	
Cavity number	96	96	32	96	96	32	256	96	256	96
Cryomodule number	12	12	4	12	12	4	32	12	32	12
SSA number	96	96	32	96	96	32	256	96	256	96
SSA power [kW]	25	25	25	30	30	40	10	10	10	10
Booster 4.5 K equiv. heat load [kW]	7.8	3.1	3.5	8.1	3.2	3.7	11.4		11.4	
Total RF length [m]	704	704	384	1088	1088	608	2368		2368	
Total 4.5 K equiv. heat load [kW]	52.2	31.2	18.7	50.0	23.2	23.8	139.7		139.7	

→ accelerating cavity

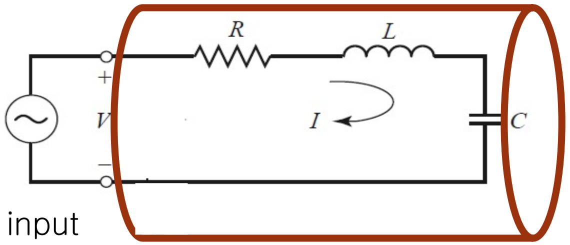
RF accelerating cavities





• for ILC, electron with energy 5 GeV to the TESLA-like multi-cell cavity → safe to put $\beta \sim 1$

Equivalent circuit of RF cavities



input power

•damped simple harmonic oscillator why?

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=\frac{d^2q}{dt^2}+2\alpha\frac{dq}{dt}+\omega_0^2 q=0,\ \text{where}\ \omega_0=\sqrt{\frac{1}{LC}}\ \text{and}\ \alpha=\frac{R}{2L}$$

$$q(t)=q_1e^{-\alpha t}\cos(\omega' t+q_2),\ \text{where}\ \omega'^2=\omega_0^2-\alpha^2$$

$$\simeq q_1e^{-\alpha t}\cos(\omega_0 t+q_2)\ \because \omega'^2=\omega_0^2-\alpha^2\simeq\omega_0^2$$

•stored electric energy $W_E(t)=\frac{q^2}{2C}=\frac{q_1^2e^{-2\alpha t}}{2C}\cos^2(\omega_0 t+q_2)\rightarrow W(t)=\frac{q_1^2e^{-2\alpha t}}{2C}\rightarrow$ *total* EM energy stored in the circuit/cavity,

then after a period $T=\frac{2\pi}{\omega_0},\ W(t+T)=\frac{q_1^2e^{-2\alpha(t+T)}}{2C}$

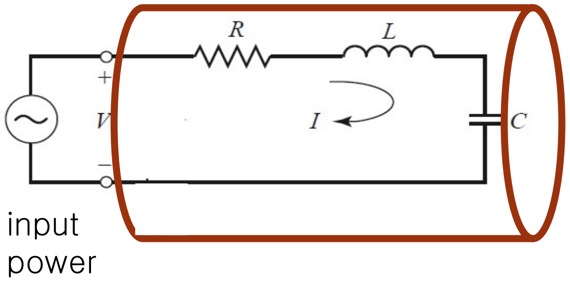
•energy loss after T or energy loss per cycle, $W(t)-W(t+T)$

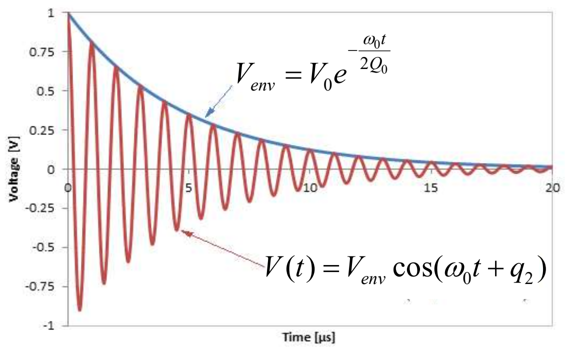
$$\bullet\ \frac{\text{energy loss per cycle}}{\text{stored energy}}=\frac{W(t)-W(t+T)}{W(t)}=1-\frac{W(t+T)}{W(t)}=1-\frac{\frac{q_1^2e^{-2\alpha(t+T)}}{2C}}{\frac{q_1^2e^{-2\alpha t}}{2C}}=1-e^{-2\alpha T}\simeq1-(1-2\alpha T)=2\alpha T=2\alpha\frac{2\pi}{\omega_0}$$

$$\frac{\text{stored energy}}{\text{energy loss per cycle}}=\frac{\omega_0}{2\alpha 2\pi}\Leftrightarrow\frac{\omega_0}{2\alpha}=\frac{2\pi}{T}\frac{\text{stored energy}}{\text{energy loss per cycle}/T}=\omega_0\frac{W}{P_{loss\ \text{per cycle}}}\equiv Q_0\rightarrow\alpha=\frac{\omega_0}{2Q_0}$$

$$\bullet Q_0=\omega_0\frac{W}{P_{loss\ \text{per cycle}}}=\omega_0\frac{\frac{1}{4}I^2L}{\frac{1}{4}I^2R}=\omega_0\frac{L}{R}=\omega_0\frac{1}{2\alpha}$$

Equivalent circuit of RF cavities -- Q_0 in the time domain





•damped simple harmonic oscillator why?

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = \frac{d^2q}{dt^2} + 2\alpha\frac{dq}{dt} + \omega_0^2 q = 0, \text{ where } \omega_0 = \sqrt{\frac{1}{LC}} \text{ and } \alpha = \frac{R}{2L} = \frac{\omega_0}{2Q_0}$$

$$q(t) = q_1 e^{-\alpha t}\cos(\omega_0 t + q_2) = CV(t) \to V(t) = \frac{q_1}{C}e^{-\alpha t}\cos(\omega_0 t + q_2) = V_0 e^{-\frac{\omega_0}{2Q_0}t}\cos(\omega_0 t + q_2)$$

•envelope $V_0 e^{-\frac{\omega_0}{2Q_0}t} \to V_0 e^{-\frac{\omega_0}{2Q_0}t_1}$ at t_1 and $V_0 e^{-\frac{\omega_0}{2Q_0}t_2}$ at t_2

$$\bullet \frac{V_0 e^{-\frac{\omega_0}{2Q_0}t_2}}{V_0 e^{-\frac{\omega_0}{2Q_0}t_1}} = \frac{A_2}{A_1} \Leftrightarrow e^{-\frac{\omega_0}{2Q_0}(t_2 - t_1)} = \frac{A_2}{A_1}$$

$$\Leftrightarrow -\frac{\omega_0}{2Q_0}(t_2 - t_1) = \ln\left(\frac{A_2}{A_1}\right) \Leftrightarrow Q_0 = \frac{\omega_0(t_2 - t_1)}{2\ln\left(\frac{A_1}{A_2}\right)}$$

•if $t_1 = 0$ and $t_2 = t$, and $\frac{A_2}{A_1} = \frac{1}{2}$, $Q_0 = \frac{\omega_0 t}{2\ln 2}$

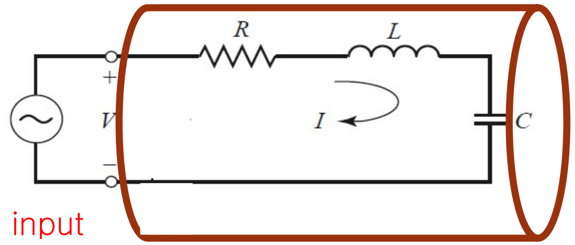
•$t = NT$ and $\omega_0 = \frac{2\pi}{T} \to \omega_0 t = 2N\pi$

$$Q_0 = \frac{2N\pi}{2\ln 2} = \frac{\pi N}{\ln(2)} \simeq 4.53N$$

$N = 3$ for $\frac{A_2}{A_1} = \frac{1}{2} \to Q_0 \sim 13.6$ for the time data $V(t)$

$$\bullet\omega_0^2 = \omega'^2\left(1 + \frac{1}{4Q_0^2}\right) \simeq 1.00135\omega'^2 \simeq \omega'^2 \text{ with such a low } Q_0 \to \omega'^2 = \omega_0^2 - \alpha^2 \simeq \omega_0^2$$

Equivalent circuit of RF cavity and input power -- Cavity excitation



•forced damped simple harmonic oscillator

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=\frac{d^2q}{dt^2}+2\alpha\frac{dq}{dt}+\omega_0^2 q=V_{in}\sin\omega_{in}t,\ \text{where }\omega_0=\sqrt{\frac{1}{LC}}\text{ and }\alpha=\frac{R}{2L}=\frac{\omega_0}{2Q_0}$$

$$q_c(t)=q_1e^{-\alpha t}\cos(\omega_0 t+q_2)\xrightarrow{t\to\infty}0,\ \text{complementary solution}$$

•particular solution

$$q_p(t)=\frac{V_{in}}{\sqrt{\left(\omega_0^2-\omega_{in}^2\right)^2+4\alpha^2\omega_{in}^2}}\sin\left(\omega_{in}t-\phi\right)\rightarrow\text{steady-state solution}$$

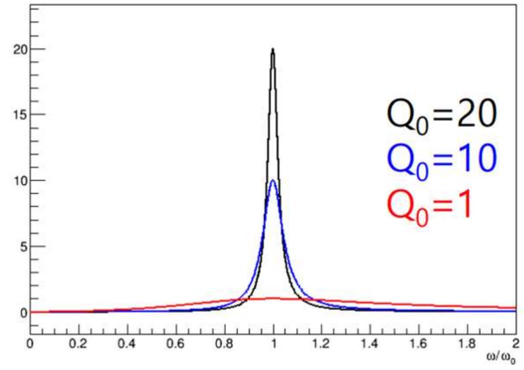
maximum $q_p(t)$ with $\omega_{in}^2=\omega_0^2-2\alpha^2$

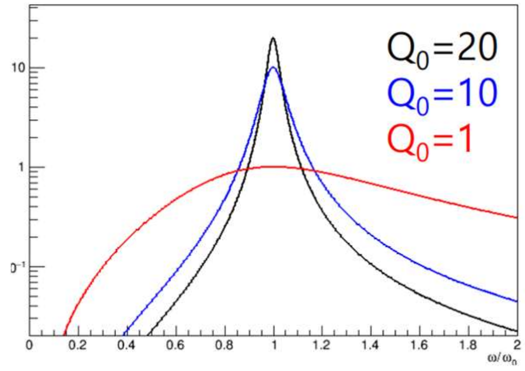
•input power , $\mathcal{P}_{in}=V_{in}I_{in}$

$$\frac{dq_p(t)}{dt}=I_p(t)=I_{in}\propto\frac{\omega_{in}}{\sqrt{\left(\omega_0^2-\omega_{in}^2\right)^2+4\alpha^2\omega_{in}^2}}\rightarrow\text{maximum } I_p(t)\text{ with }\omega_{in}=\omega_0$$

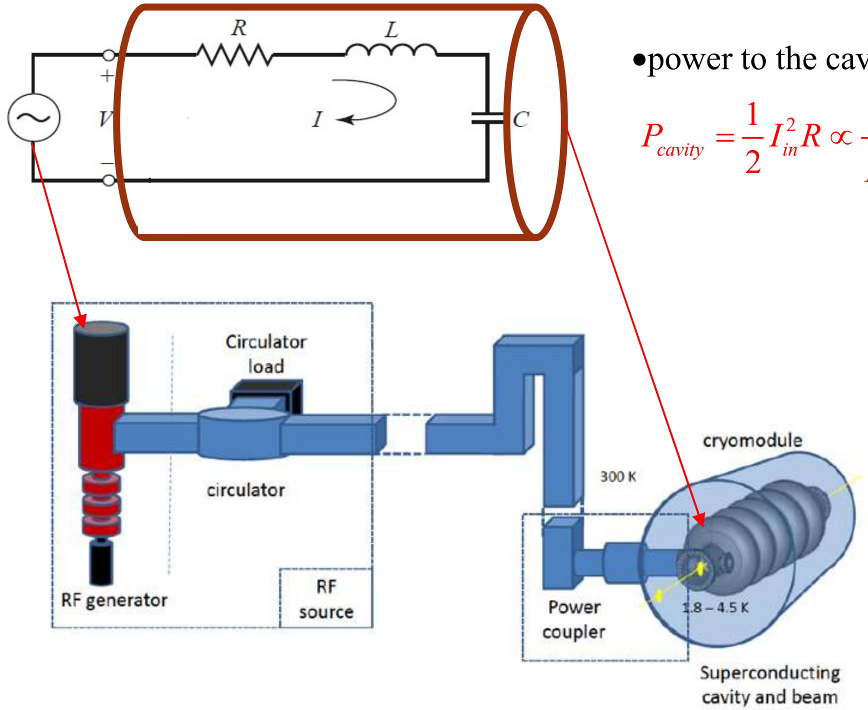
$$I_p(t)=I_{in}\propto\frac{\omega_{in}}{\sqrt{\left(\omega_0^2-\omega_{in}^2\right)^2+\frac{R^2}{L^2}\omega_{in}^2}}\propto\frac{\omega_{in}}{\sqrt{R^2\omega_{in}^2+L^2\left(\omega_0^2-\omega_{in}^2\right)^2}}$$

•power to the load (cavity), $P_{load}=\frac{1}{2}I_{in}^2R\propto\frac{\omega_{in}^2}{R^2\omega_{in}^2+L^2\left(\omega_0^2-\omega_{in}^2\right)^2}$



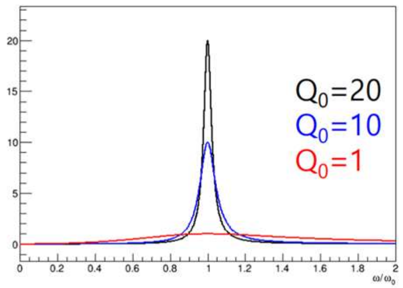


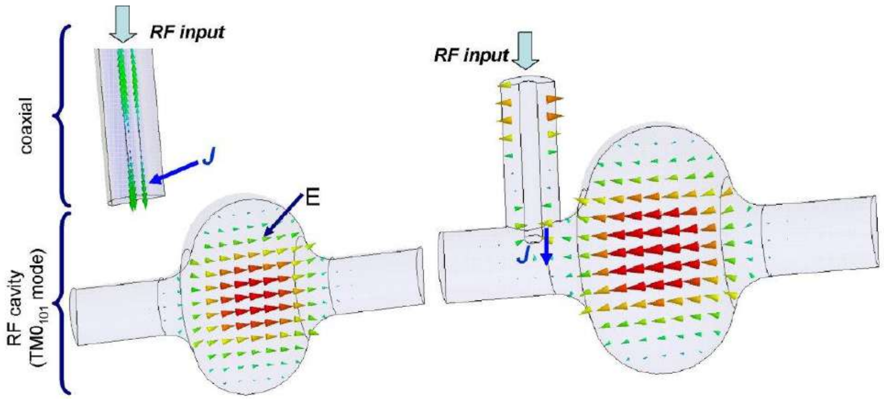
Equivalent circuit of RF cavity, input power, coupler



•power to the cavity,

$$P_{cavity} = \frac{1}{2} I_{in}^2 R \propto \frac{\omega_{in}^2}{R^2\omega_{in}^2 + L^2\left(\omega_0^2 - \omega_{in}^2\right)^2}$$



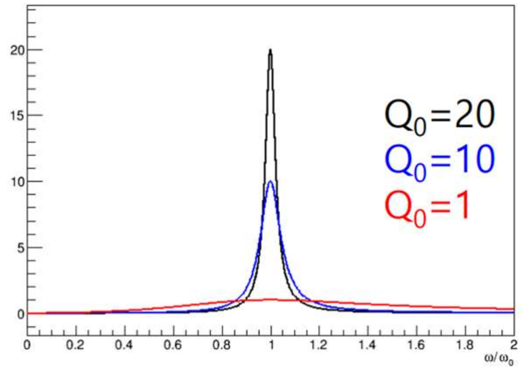


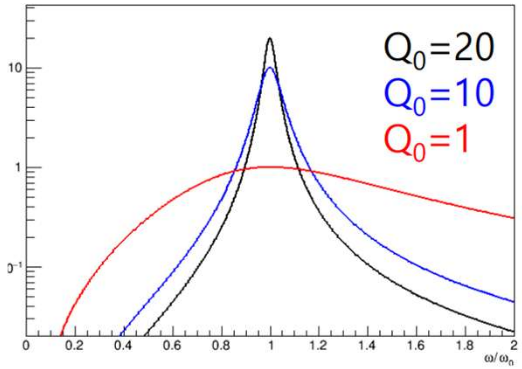
•cavity mode frequencies $\omega_0 = \omega_{TM_{010}} = 2\pi f_{TM_{010}}$,

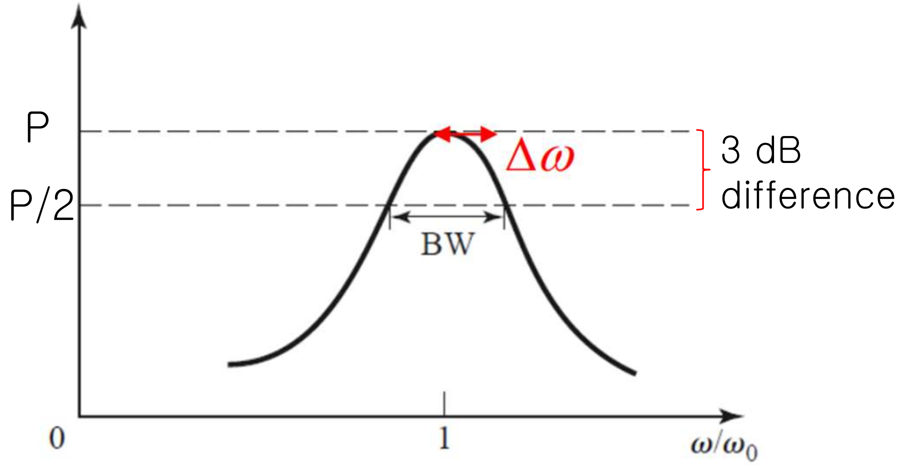
-RF input power frequency $\omega_{in} = \omega_{TM_{010}} \rightarrow$ power to the TM_{010} mode $\rightarrow \mathrm{TM}_{010}$ mode excited $\rightarrow$ typical resonant behavior

•also can probe the cavity parameters

Q_0 in the frequency domain







$$\bullet Q_0 = \frac{\omega_0}{\mathrm{BW}} = \frac{\omega_0}{2\Delta\omega}$$

$\bullet \omega_0 = 2\pi \times 1.3\ \mathrm{GHz}$ and $Q_0 \sim 10^{10} = \dfrac{\omega_0}{\mathrm{BW}} = \dfrac{\omega_0}{2\Delta\omega}$

$\mathrm{BW} = \dfrac{1.3\ \mathrm{GHz}}{10^{10}} < 1\ \mathrm{Hz} \rightarrow$ can't be done in the frequency domain

$\rightarrow Q_0$ in the time domain with an oscilloscope

dB and dBm

•$\text{dB} = 10\log_{10}\left(\frac{P_2}{P_1}\right)$

$0 \text{ dB} \Leftrightarrow P_2 = P_1$

$3 \text{ dB} \Leftrightarrow P_2 \simeq 2P_1,$ $\quad -3 \text{ dB} \Leftrightarrow P_2 \simeq P_1 / 2$

$10 \text{ dB} \Leftrightarrow P_2 \simeq 10P_1,$ $\quad -10 \text{ dB} \Leftrightarrow P_2 \simeq P_1 / 10$

$20 \text{ dB} \Leftrightarrow P_2 \simeq 100P_1,$ $\quad -20 \text{ dB} \Leftrightarrow P_2 \simeq P_1 / 100$

$30 \text{ dB} \Leftrightarrow P_2 \simeq 1000P_1$ $\quad -30 \text{ dB} \Leftrightarrow P_2 \simeq P_1 / 1000$

•$30 \text{ dB} - 20 \text{ dB} = 10 \text{ dB}$

•$\text{dBm} = 10\log_{10}\left(\frac{P}{10^{-3}}\right)$

$0 \text{ dBm} \Leftrightarrow 1 \text{ mW},$

$10 \text{ dBm} \Leftrightarrow 10 \text{ mW},$ $\quad -10 \text{ dBm} \Leftrightarrow 100\ \mu\text{W}$

$20 \text{ dBm} \Leftrightarrow 100 \text{ mW},$ $\quad -20 \text{ dBm} \Leftrightarrow 10\ \mu\text{W}$

$30 \text{ dBm} \Leftrightarrow 1 \text{ W},$ $\quad -30 \text{ dBm} \Leftrightarrow 1\ \mu\text{W}$

•$30 \text{ dBm} - 20 \text{ dBm} = 10 \text{ dB}$

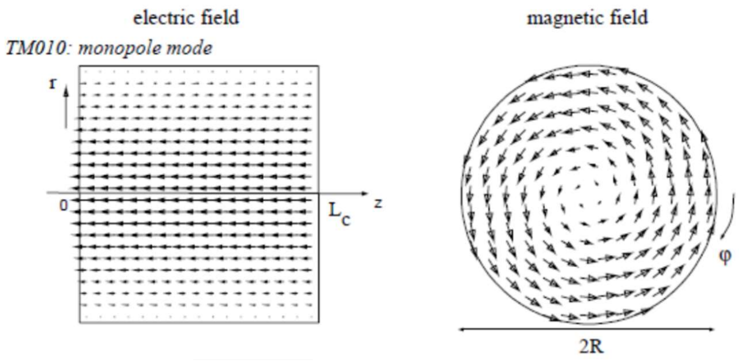
Cavity parameters

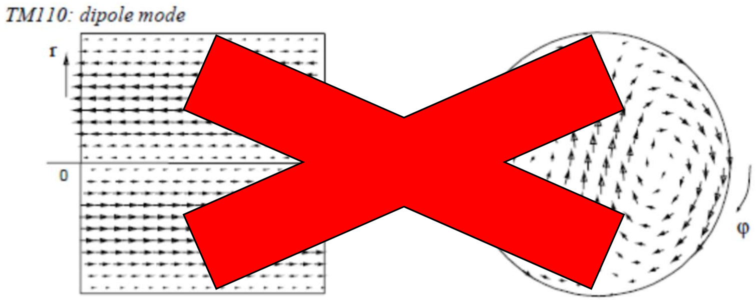
- quality factor of the cavity mode, TM_{010}

stored energy in the cavity mode $U = \frac{1}{2}\varepsilon_0 \int_V \vec{E}_{010}^2 dV = \frac{1}{2}\mu_0 \int_V \vec{H}_{010}^2 dV$

$P_{cavity} \equiv P_0 = \frac{1}{2} R_S \int_S \vec{H}_{010}^2 dS$, R_S : surface resistance of the metal

$$Q_0 = \frac{\omega_{010} U}{P_0} = \frac{\omega_{010}\mu_0 \int_V \vec{H}_{010}^2 dV}{R_S \int_S \vec{H}_{010}^2 dS}$$





Cavity parameters

• shunt impedance

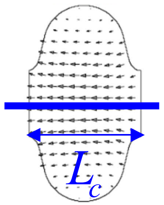
$$R_{sh} = \frac{V_{acc}^2}{P_0}$$: how much acceleration a particle can get for a given power dissipation in a cavity

• R over Q

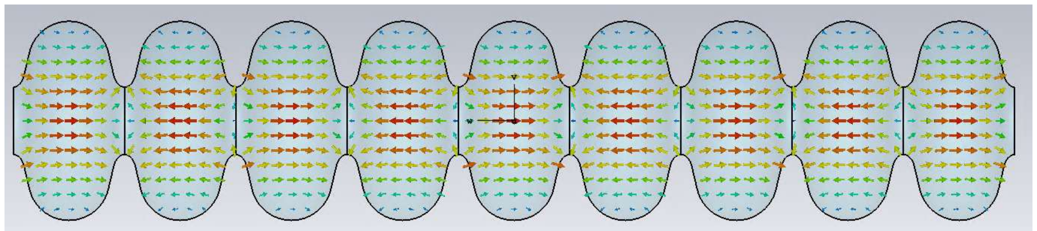
$$\frac{R_{sh}}{Q_0} = \frac{\frac{V_{acc}^2}{P_0}}{\frac{\omega_{010} U}{P_0}} = \frac{V_{acc}^2}{\omega_{010} U} = \frac{2 E_{acc}^2 L_c^2}{\varepsilon_0 \omega_{010} E_{acc}^2 L_c \delta A} = \frac{2 L_c}{\varepsilon_0 \omega_{010} \delta A}$$ for δA $\rightarrow$ independent of the R_S

$\rightarrow \sim$ independent of Q_0, $\because Q_0 \propto \frac{1}{R_S}$

δA



$$\frac{R_{sh}}{Q_0} \sim 114\ \Omega$$ from simulation

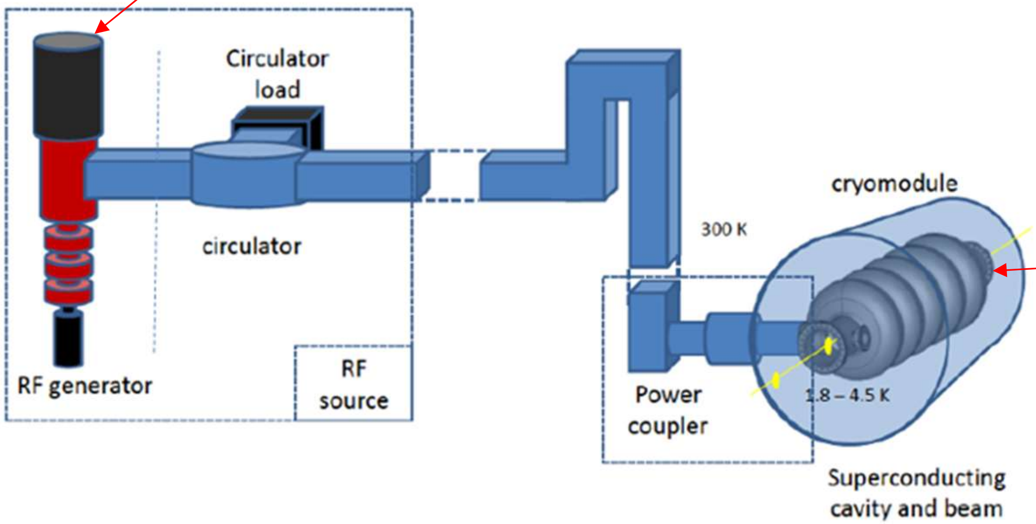


$$\frac{R_{sh}}{Q_0} \sim 1039\ \Omega$$

Accelerating gradient, E_{acc}

- $$P_0 = \frac{V_{acc}^2}{R_{sh}} \rightarrow E_{acc} = \frac{V_{acc}}{L_c} = \frac{\sqrt{R_{sh} P_0}}{L_c} = \frac{1}{L_c}\sqrt{\frac{R_{sh}}{Q_0}}\sqrt{P_0 Q_0}$$

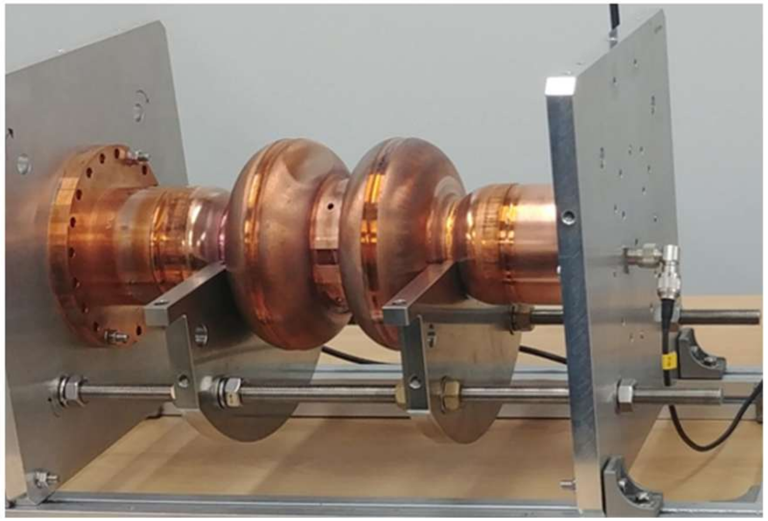
power to the cavity P_0



cavity Q_0

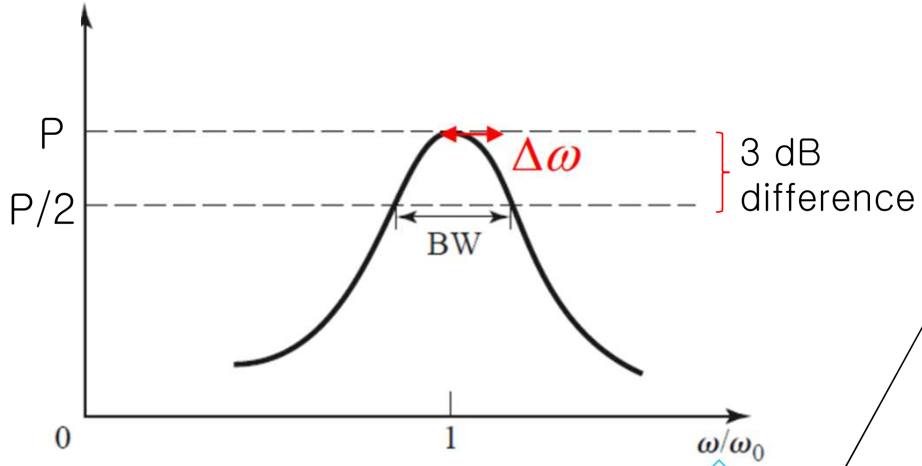
What you are doing today

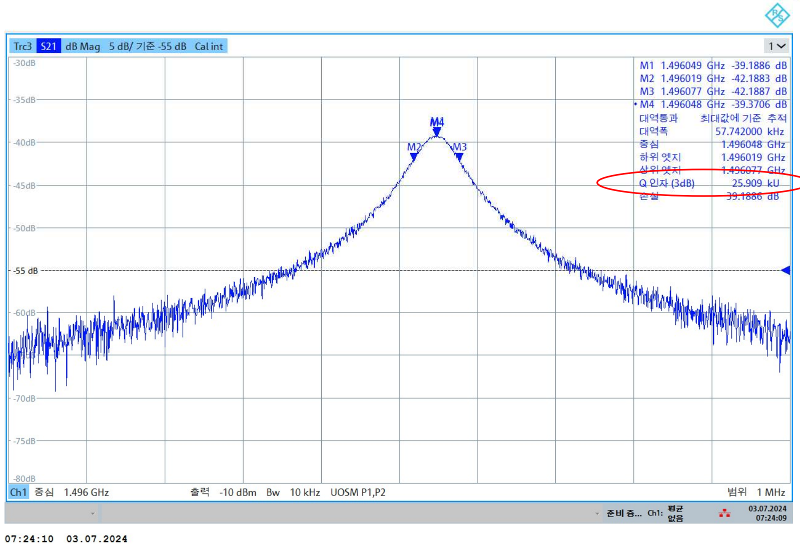
- TM_{010} mode Q_0 measurement at the frequency and time domains, and compare the two Q_0s



- Four Q_0 values should be measured

TM_{010} mode Q_0 measurement at the frequency domain

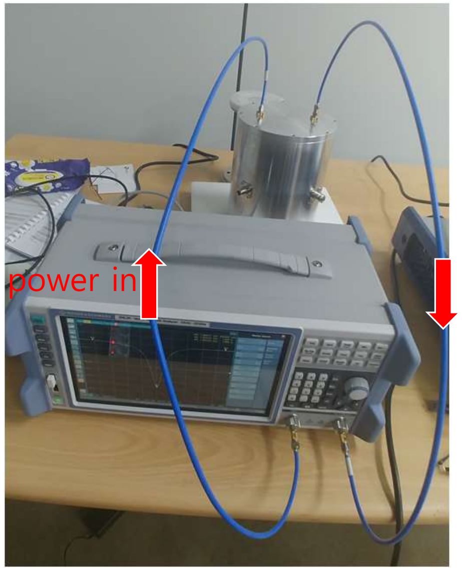




- $Q_0 = \dfrac{\omega_0}{\mathrm{BW}} = \dfrac{\omega_0}{2\Delta\omega}$: unloaded Q
- $Q_L = \dfrac{\omega_0}{\mathrm{BW}'} = \dfrac{\omega_0}{2\Delta\omega'}$: loaded Q

this is Q_L from

the two-port measurement (transmission measurement)

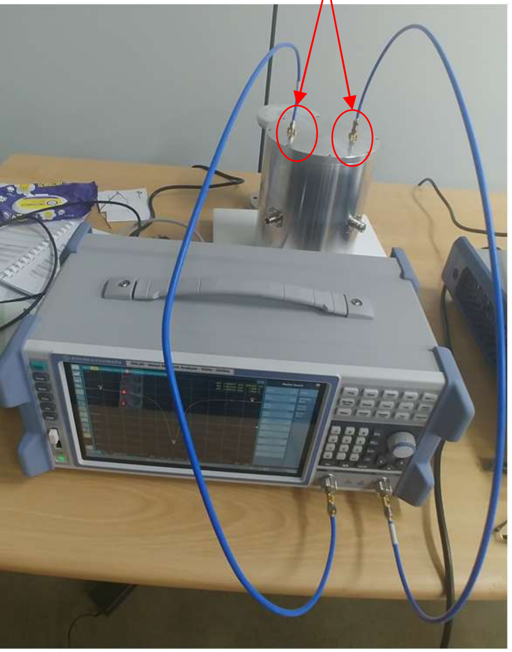


power in

power out

power in and out by a network analyzer

$\mathrm{TM_{010}}$ mode $\mathrm{Q_0}$ measurement at the frequency domain



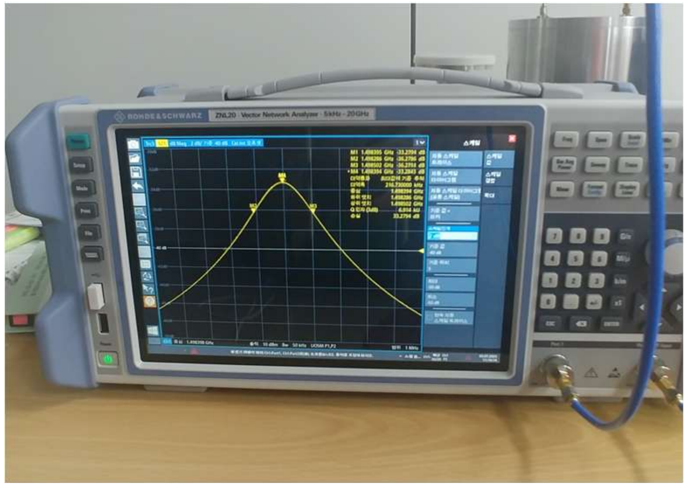
- total power loss $P_{total} = P_0 + P_{ext1} + P_{ext2}$ and $\beta_{ext} \equiv \dfrac{Q_0}{Q_{ext}}$

$$P_{total} = \frac{\omega_0 U}{Q_L} \text{ and } P_0 + P_{ext1} + P_{ext2} = \frac{\omega_0 U}{Q_0} + \frac{\omega_0 U}{Q_{ext1}} + \frac{\omega_0 U}{Q_{ext2}}$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext1}} + \frac{1}{Q_{ext2}}$$

$$= \frac{1}{Q_0} + \frac{\beta_{ext1}}{Q_0} + \frac{\beta_{ext2}}{Q_0} = \frac{1}{Q_0}\left(1 + \beta_{ext1} + \beta_{ext2}\right)$$

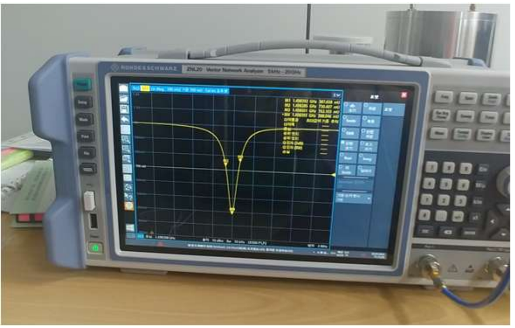
$$\rightarrow Q_0 = Q_L\left(1 + \beta_{ext1} + \beta_{ext2}\right)$$



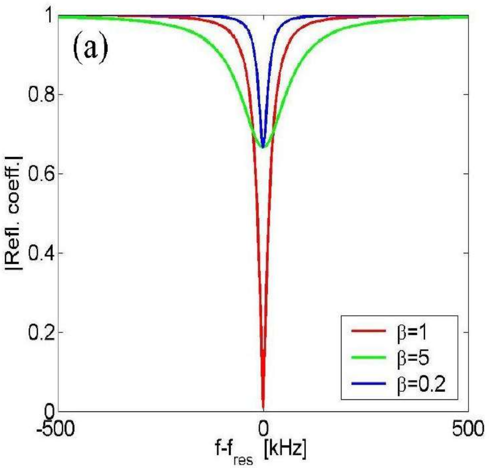
- loaded quality factor Q_L from the transmission line or S_{21}, where the external port1 is input and the external port2 output

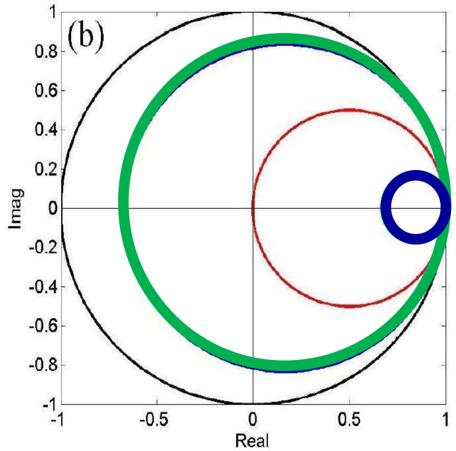
TM$_{010}$ mode Q$_0$ measurement at the frequency domain

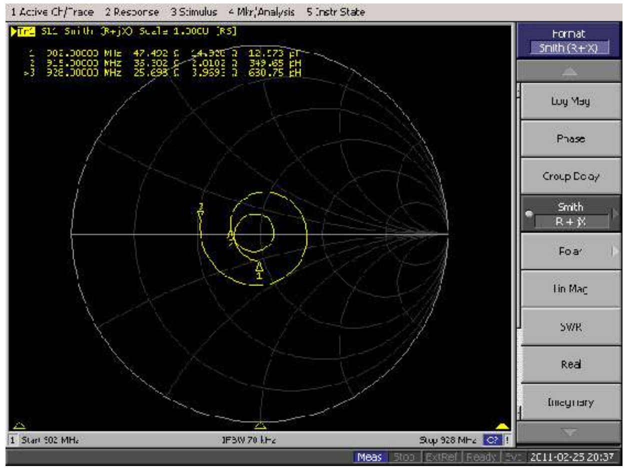
- $Q_0 = Q_L\left(1+\beta_{ext1}+\beta_{ext2}\right)$
- $\beta_{ext} \equiv \dfrac{Q_0}{Q_{ext}} = \begin{cases} \dfrac{1+|\Gamma|}{1-|\Gamma|}, \text{if } \beta_{ext}>1 \text{ or overcoupled} \\ \dfrac{1-|\Gamma|}{1+|\Gamma|}, \text{if } \beta_{ext}<1 \text{ or undercoupled} \end{cases}$, where $|\Gamma|$: reflection coefficient
- reflection coeffients from S_{11} and $S_{22} \rightarrow \beta_{ext1}$ and β_{ext2} respectively



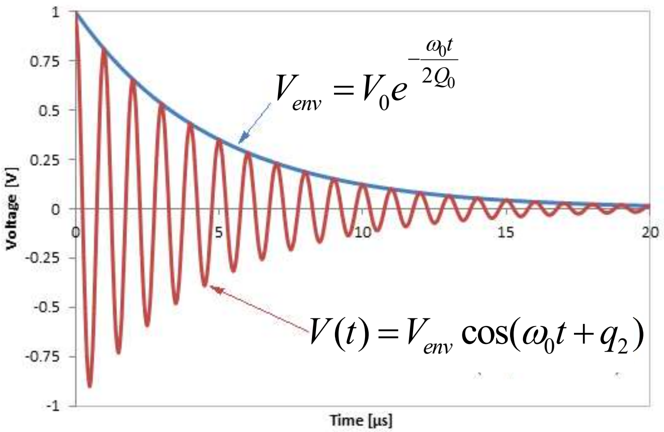
$|\Gamma|$





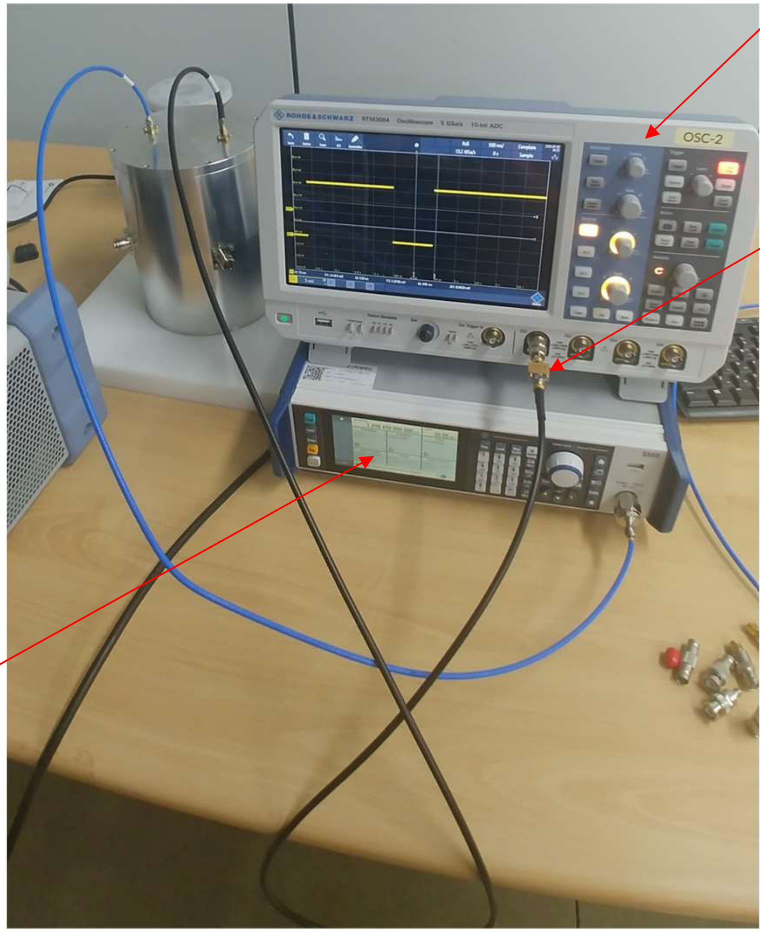


TM_{010} mode Q_0 measurement at the time domain

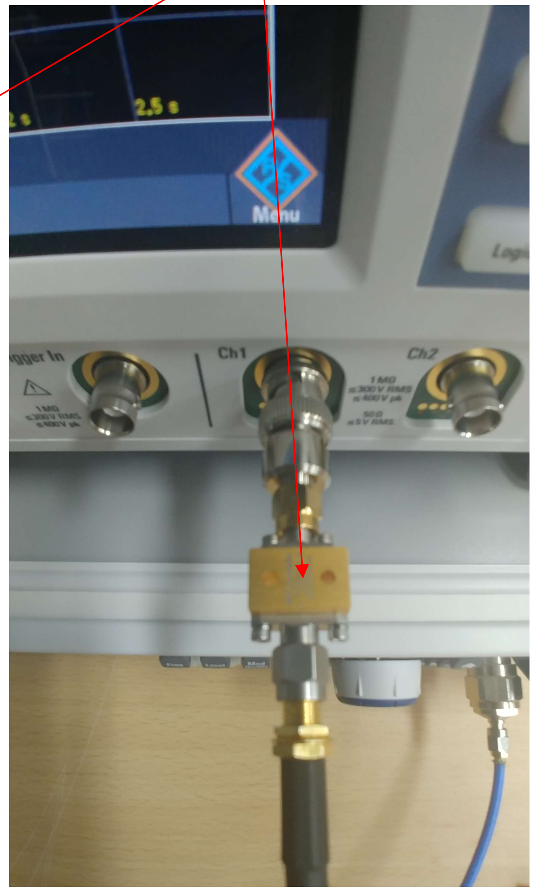


$$Q_L = \frac{\omega_0 (t_2 - t_1)}{2\ln\left(\frac{A_1}{A_2}\right)}$$

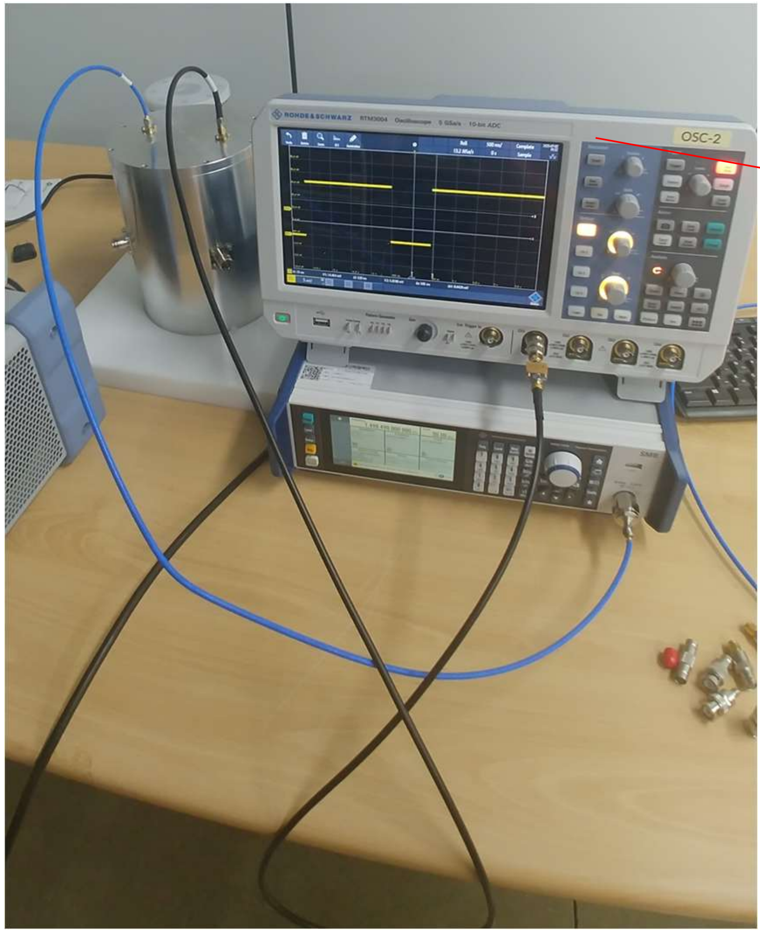
input power from an signal generator

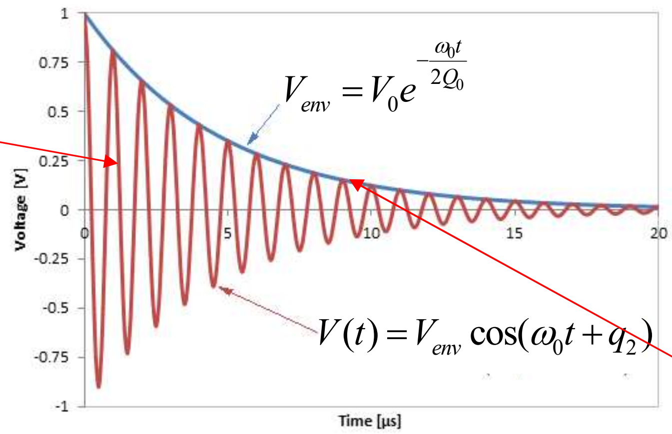


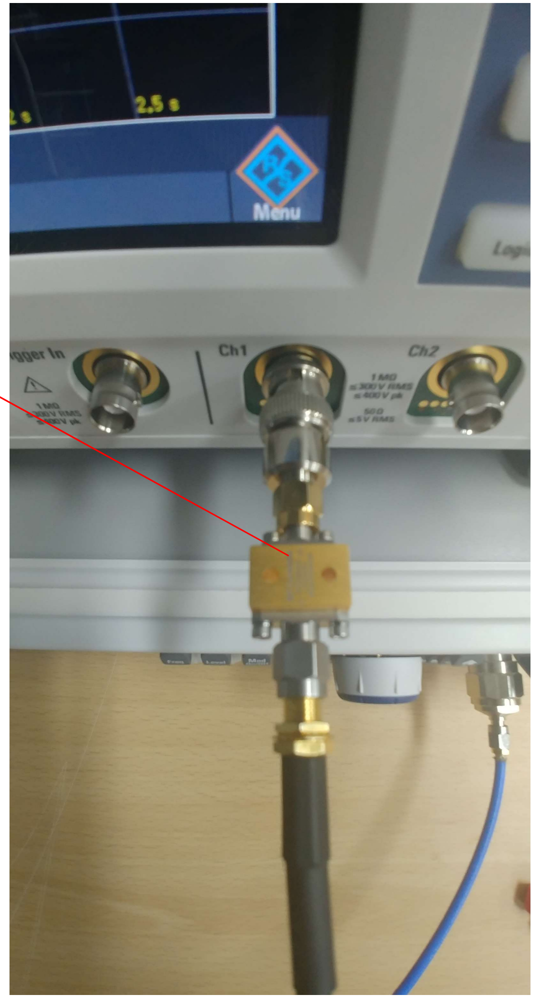
decay time and amplitude by an oscilloscope with help from an RF detector



TM_{010} mode Q_0 measurement at the time domain





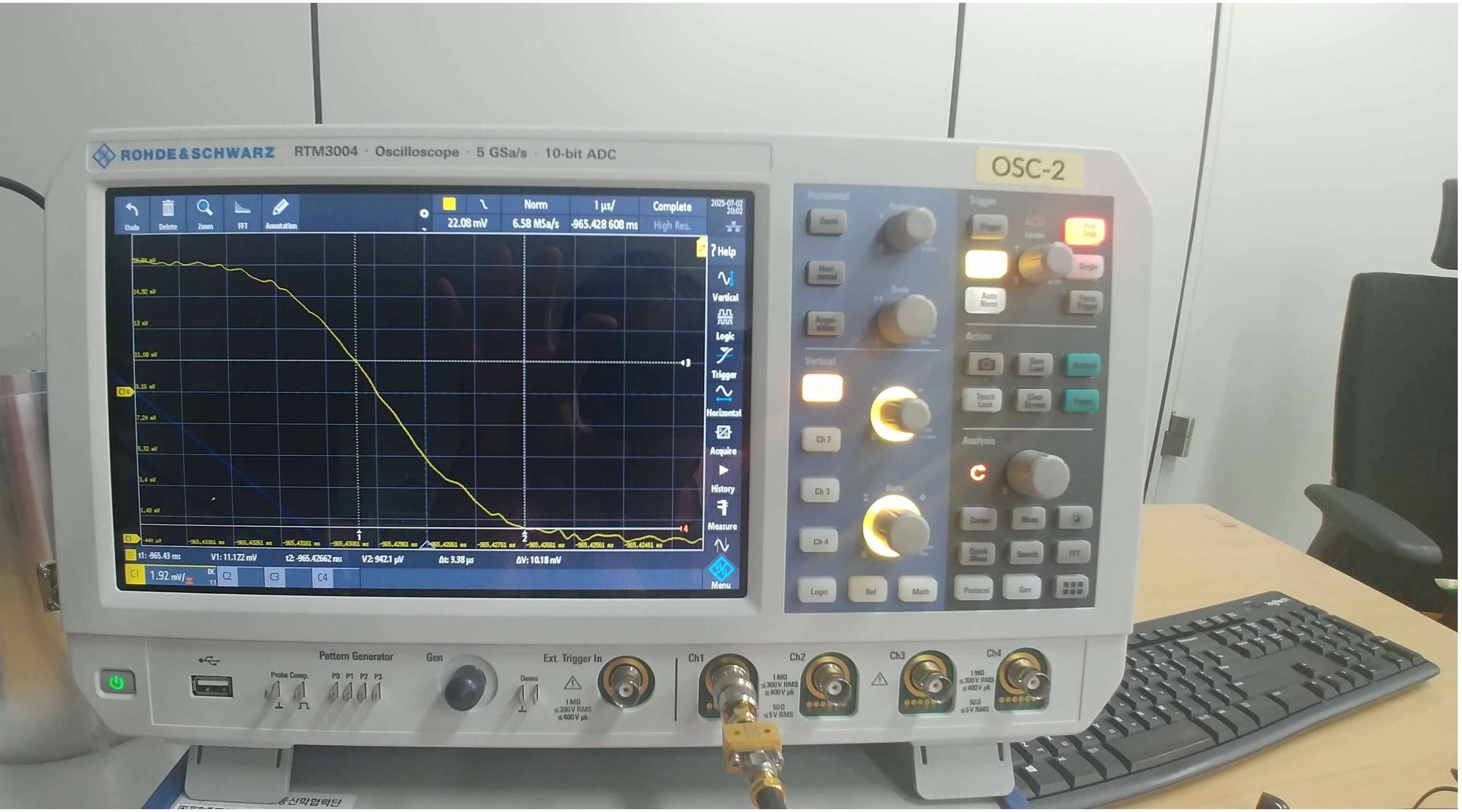


$$Q_L = \frac{\omega_0 (t_2 - t_1)}{2 \ln\left(\frac{A_1}{A_2}\right)}$$

oscilloscope up to 0.5 GHz

cavity mode frequency > 1 GHz

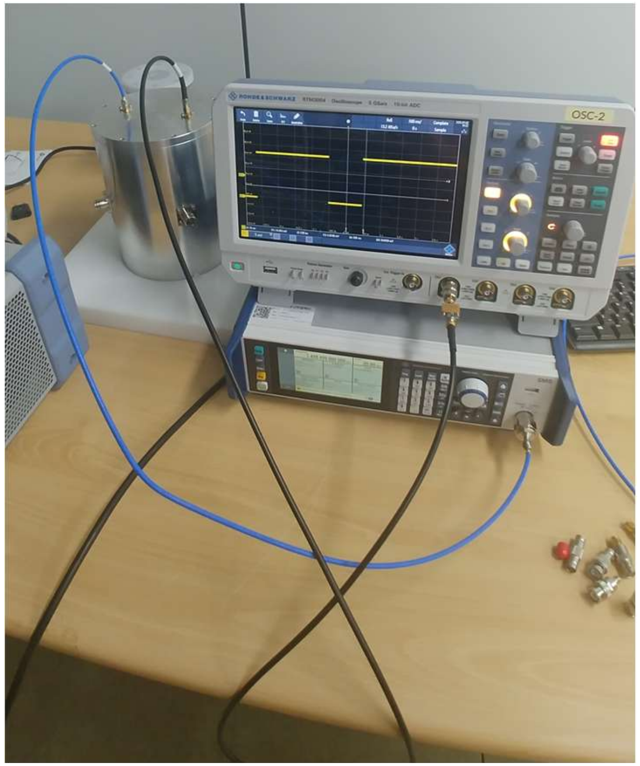
TM_{010} mode Q_0 measurement at the time domain

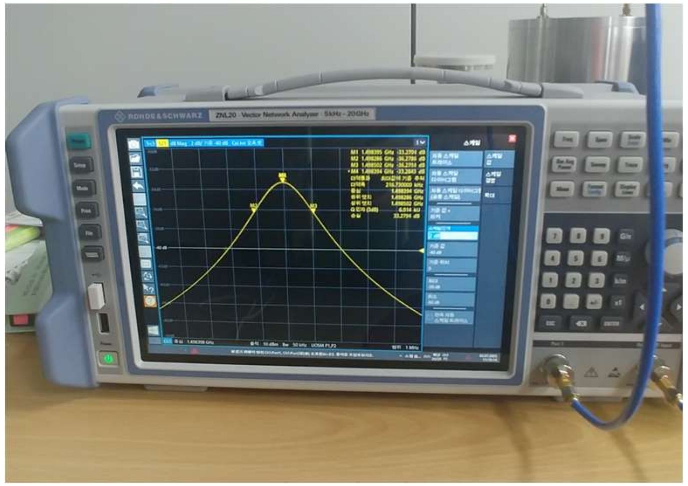


- $Q_L = \dfrac{\omega_0 (t_2 - t_1)}{2\ln\left(\dfrac{A_1}{A_2}\right)}$
- β_{ext1} and β_{ext2} from the frequency domain measurement
- $Q_0 = Q_L\left(1 + \beta_{ext1} + \beta_{ext2}\right)$

Equipment

- cavity
- network analyzer, signal generator, oscilloscope, and RF detector
- torque wrench
- coaxial cables





Notice