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# Beam dynamics in ring and linac

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EUN-SAN KIM (銀山 金)

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Department of Accelerator Science, Korea University

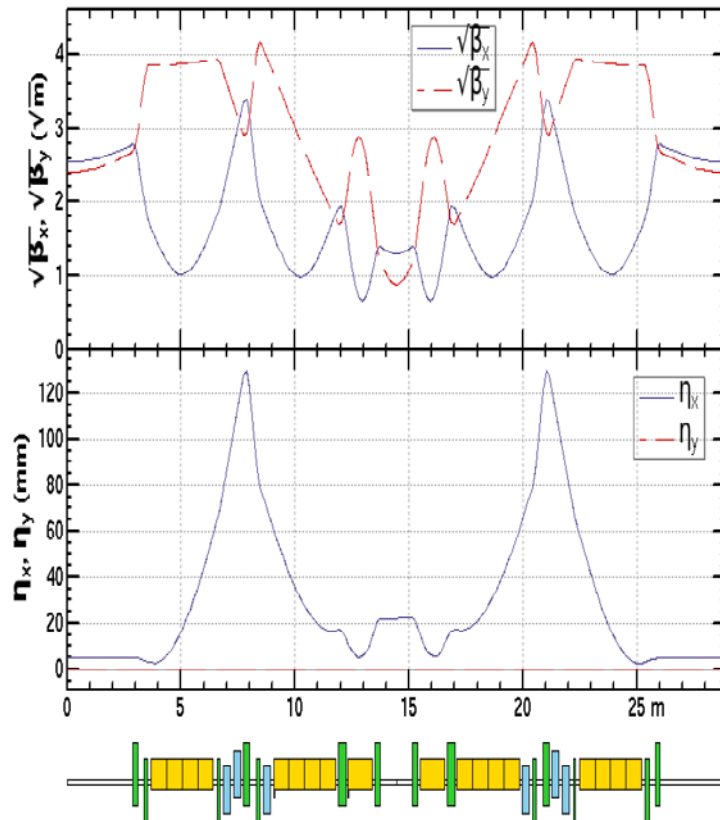
# Contents

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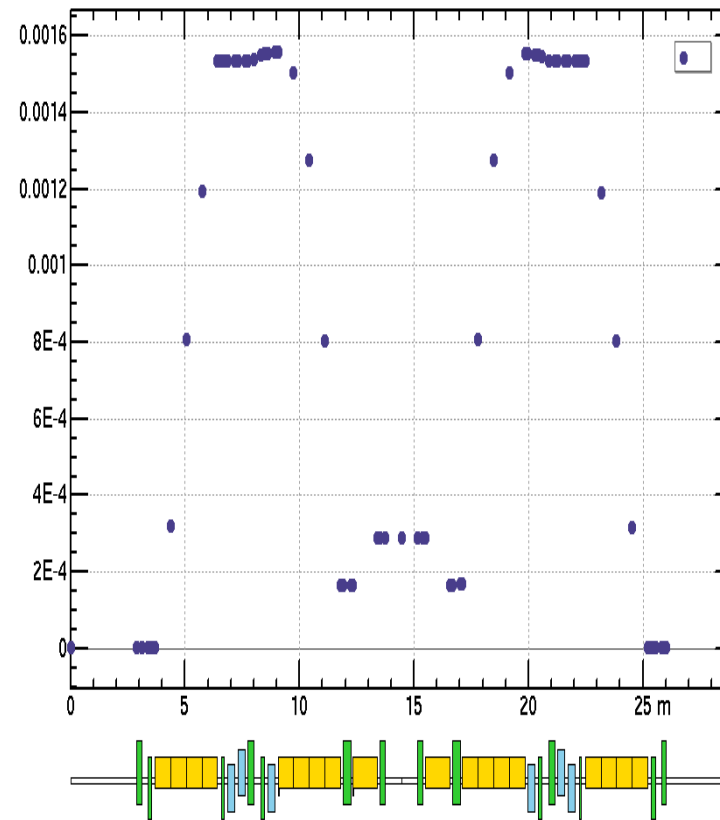
- ❑ Beam dynamics in 4 GeV light source
- ❑ Beam dynamics in ATF damping ring
- ❑ Beam dynamics in 2.5 GeV light source
- ❑ Beam dynamics in Energy Recovery Linac
- ❑ Beam dynamics in Free Electron Linac

# 1. Beam dynamics in 4 GeV light source

## Lattice design ( by code SAD)



HMBA



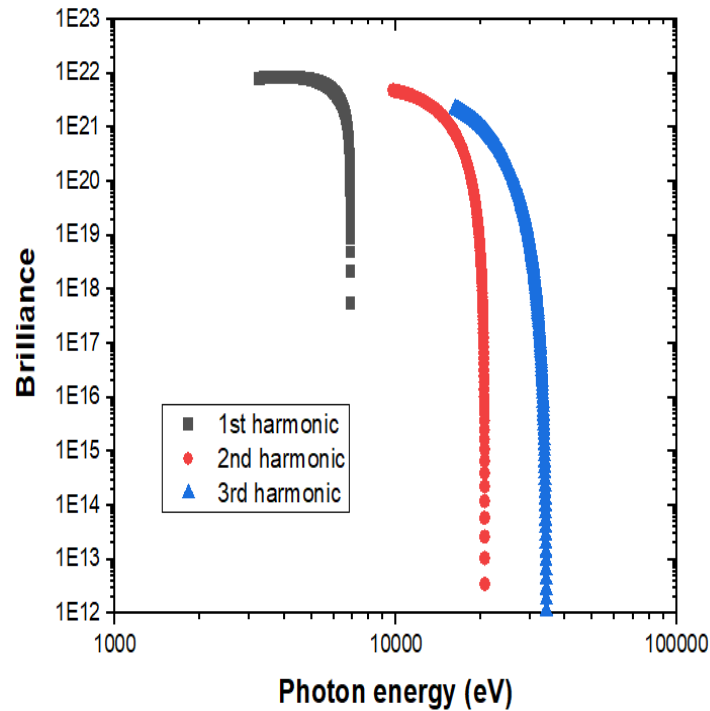
$$H_x = \gamma \eta_x^2 + 2\alpha_x \eta_x \eta_{px} + \beta_x \eta_{px}^2$$

Parameters	Value
Lattice type	HMBA
Beam energy (GeV)	4
Natural beam emittance (pm)	95
Number of cell	28
Circumference (m)	809.2
Magnetic field (T) in combined dipole	0.48
Gradient (T/m) in combined dipole	-26.3
RF frequency (MHz)	500
Harmonic number	1349
Number of cell	28
Rms energy spread	$6.5 \times 10^{-4}$
Synchrotron frequency (kHz)	1.59
Beam size(x,y) at long straight section ( $\mu\text{m}$ )	24/7.5
Momentum compaction factor	$1.4 \times 10^{-4}$
RF bucket height (%)	3.87
Energy loss per turn (MeV)	0.48
Touschek lifetime(hour) at 400 mA	3
Damping time (x/y/z)(ms)	29.7/44.4/29.4
Damping partition number (x/y/z)	1.49/1/1.5
RMS bunch length (mm)	2.68
RF voltage (MV)	2.5
Natural chromaticity(x/y)	-86.9/-76.4
Corrected chromaticity	0.1/0.1
Maximum strength ( $T/m^2$ ) of sextupole	1750
Betatron tune (x/y)	64.29/22.85
Beta function (x/y)(m) in long straight section	6/6
Dispersion function(x)(cm) - in long/short straight section	0.6 /2.2

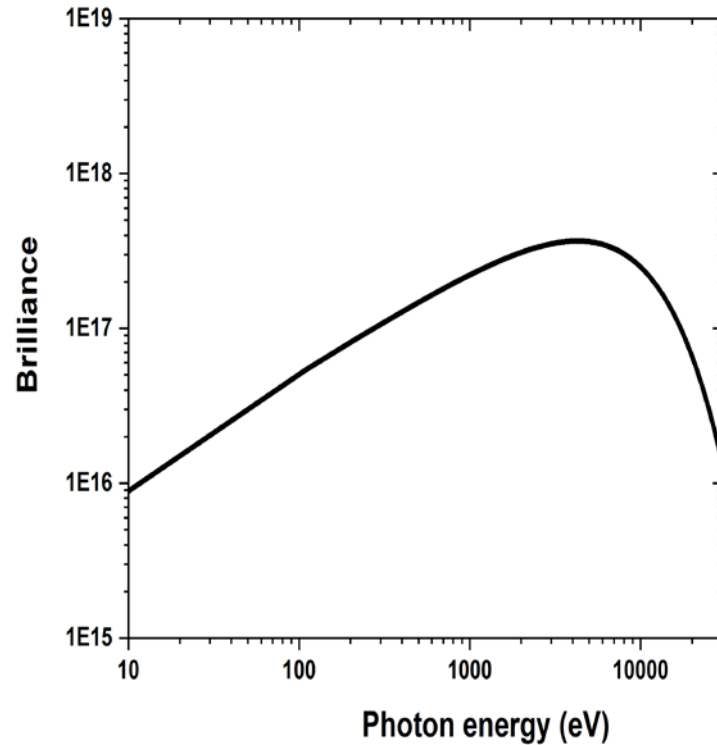
CEPC (APES) also uses code SAD for lattice design and beam simulations.

# 1. Beam dynamics in 4 GeV light source

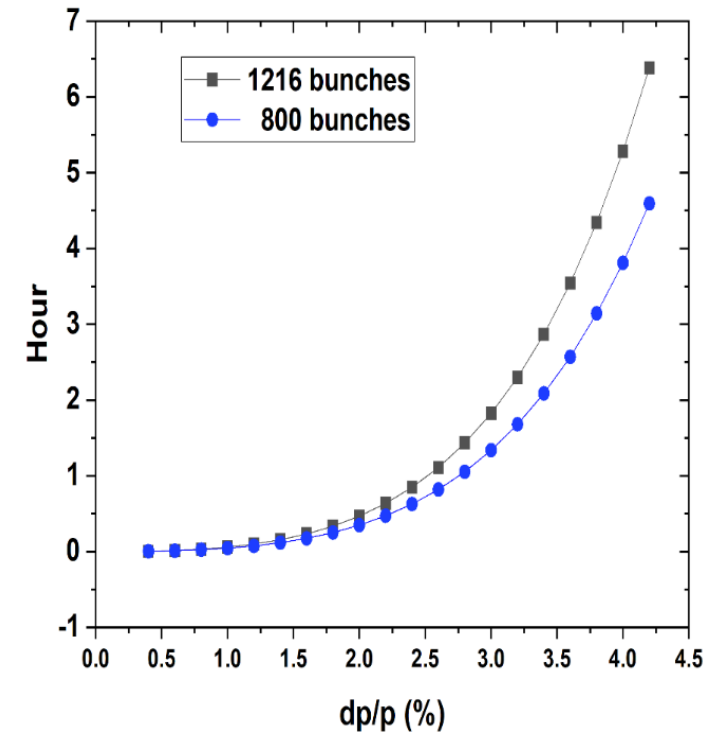
## Brilliance and beam lifetime



**In-vacuum undulator**  
(period 22mm, gap 8mm  
 $L=4\text{m}$   $K=1.52$ )



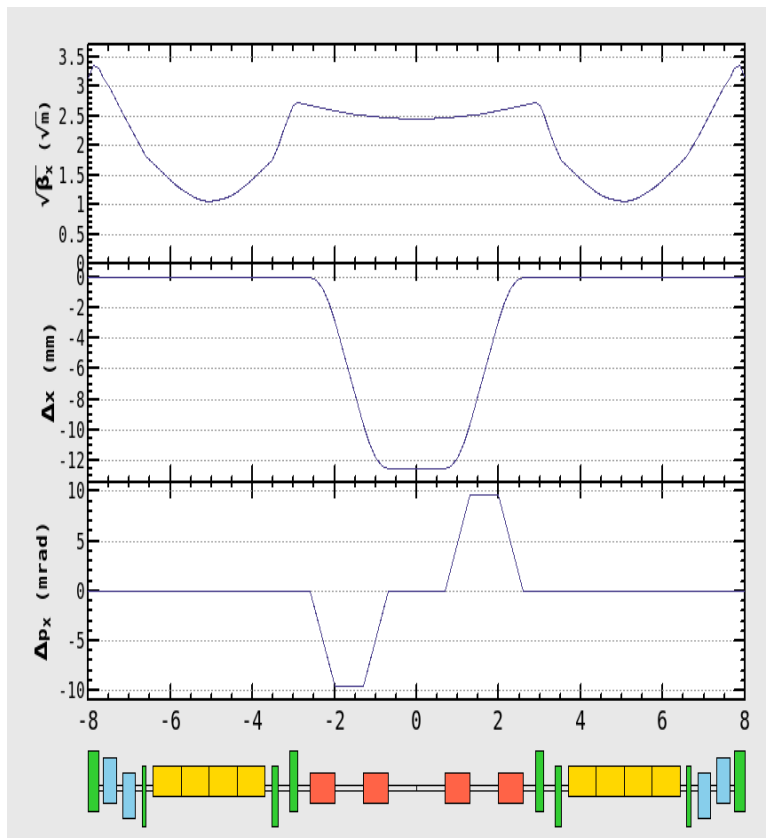
**dipole**



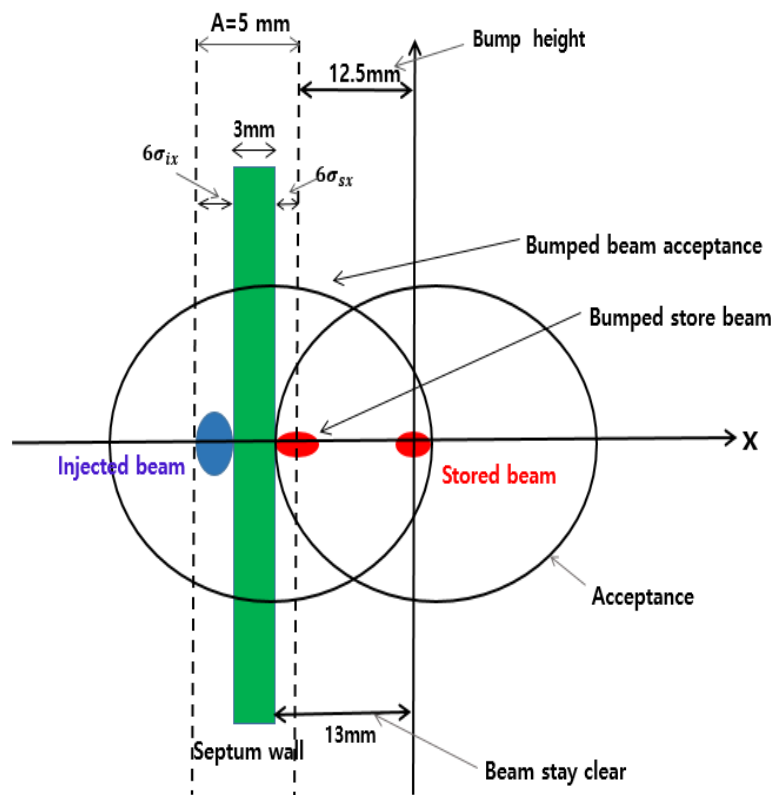
**beam lifetime**

# 1. Beam dynamics in 4 GeV light source

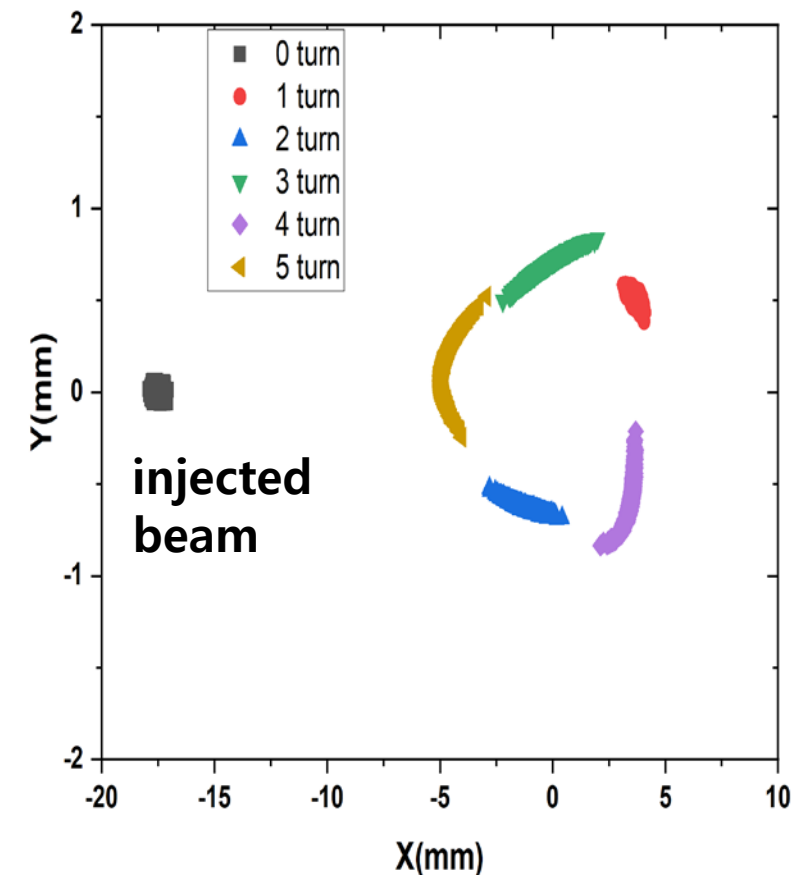
## Beam injection by 4 kickers



Beta, bump height, kick angle



Horizontal space in injection scheme



Horizontal beam distribution after for 5 turns after injection

# 1. Beam dynamics in 4 GeV light source

## Resistive wall Wake

### ■ Short range longitudinal wake function

$$W(s) = \frac{16}{a^2} \left[ \frac{e^{-s/s_0}}{3} \cos \frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi} \int_0^\infty \frac{dx x^2 e^{-x^2 s/s_0}}{x^6 + 8} \right]$$

### ■ Long range longitudinal wake function

$$W(s) = -\sqrt{c/\sigma} / (2\pi a s^{3/2}).$$

### ■ Short range transverse wake function

$$W(s) = \frac{8Z_0 c s_0}{\pi a^4} \left[ \frac{-1}{12} e^{-s/s_0} \cos \left( \sqrt{3} \frac{s}{s_0} \right) + \frac{1}{4\sqrt{3}} e^{-s/s_0} \sin \left( \sqrt{3} \frac{s}{s_0} \right) + \frac{\sqrt{2}}{\pi} \int_0^\infty dx \frac{e^{-x^2 s/s_0}}{x^6 + 8} \right]$$

### ■ Long range transverse wake function

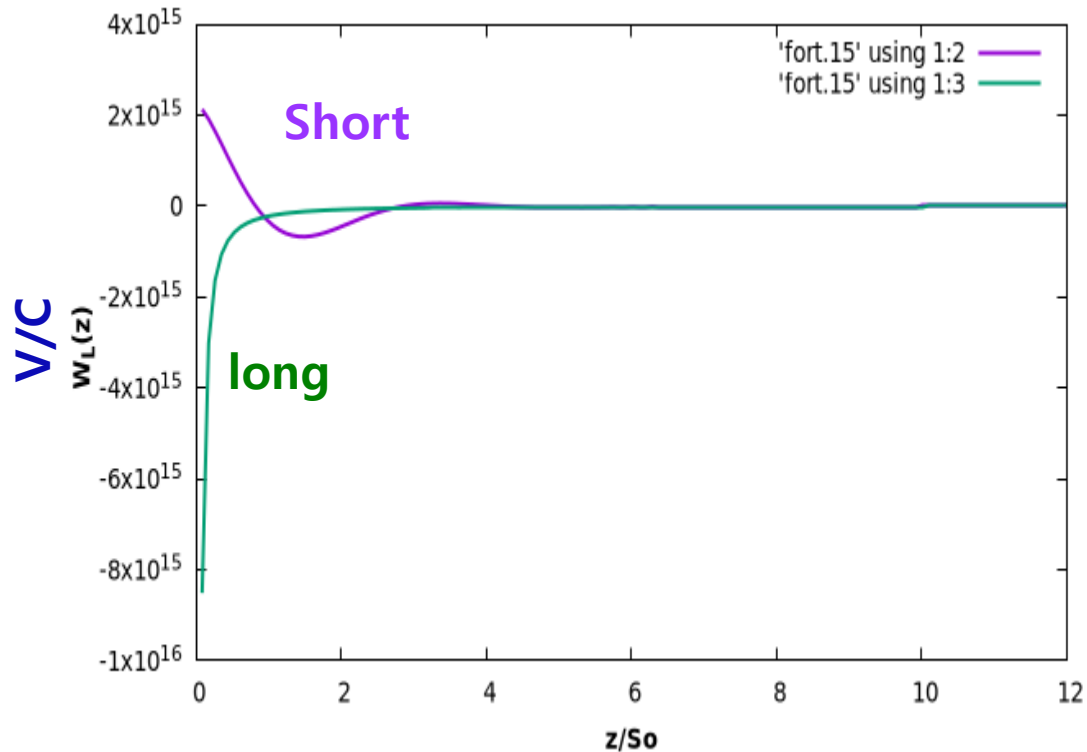
$$W(s) = \frac{c}{\pi^{3/2} a^3} \sqrt{\frac{Z_0}{\sigma s}}$$

characteristic distance :  $S_0 = (2a^2/Z_0\sigma)^{1/3}$

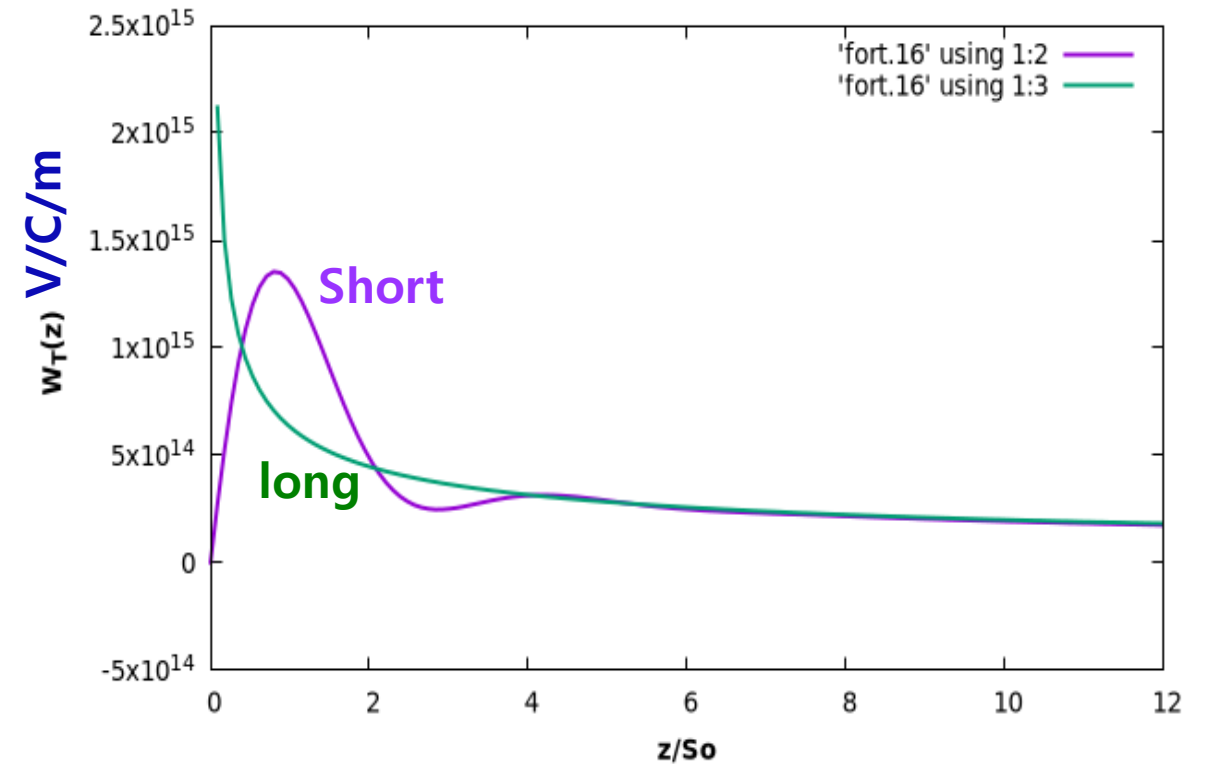
# 1. Beam dynamics in 4 GeV light source

## Resistive wall Wake

### Short and long range longitudinal wakes



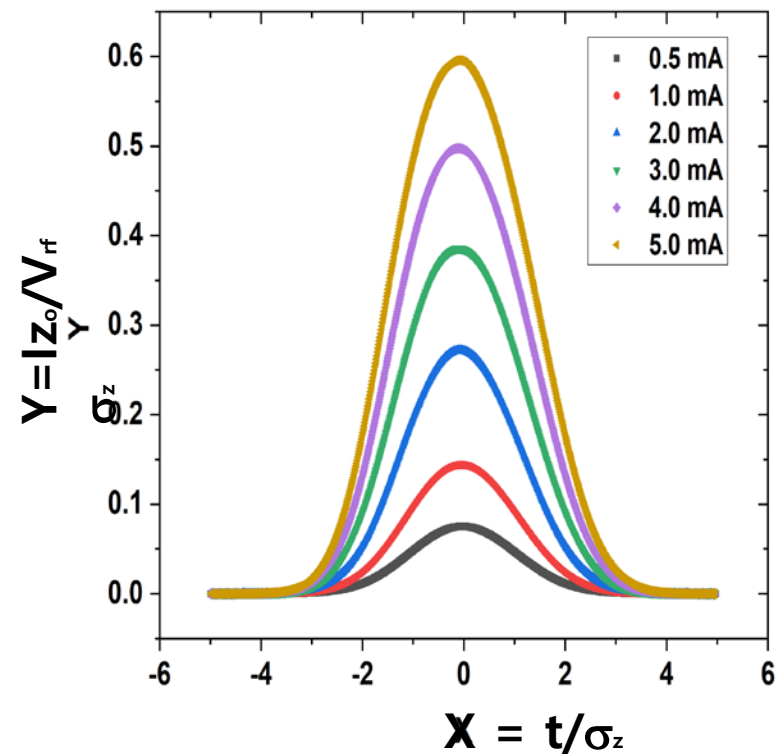
### Short and long range transverse wakes



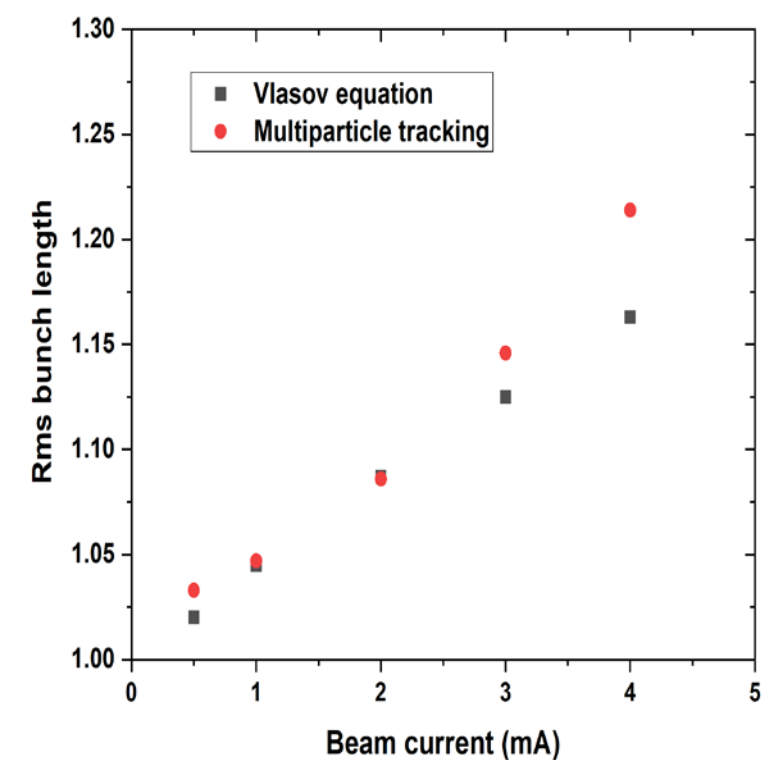
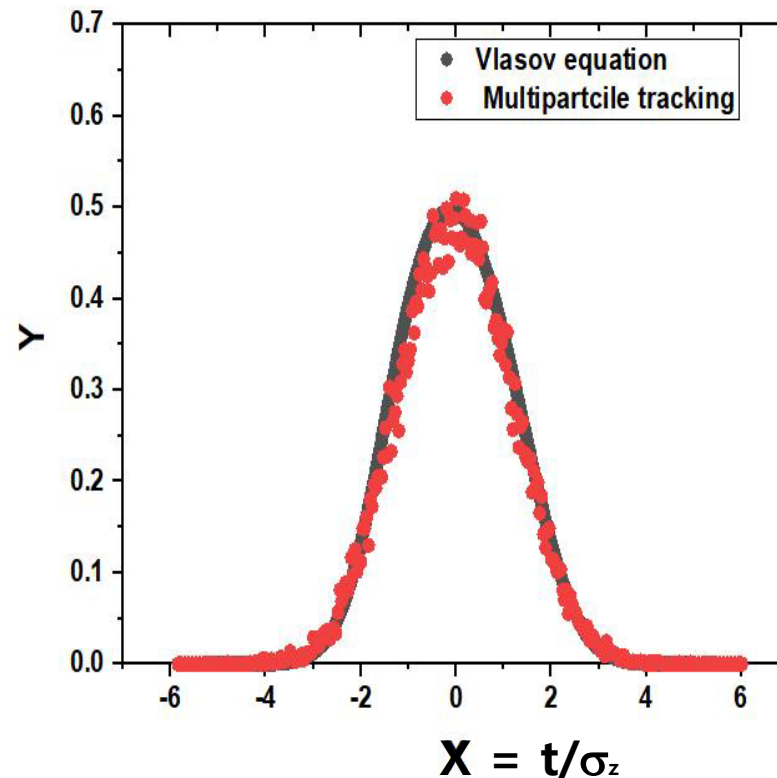
$$S_0 = (2a^2/Z_0s)^{1/3}$$

# 1. Beam dynamics in 4 GeV light source

## Potential well distortions due to Resistive wall Wake



Calculated longitudinal beam distributions



Calculated bunch length for the different beam currents



# 1. Beam dynamics in 4 GeV light source

## Resistive wall Wake

### Muiparticle tracking (binning method)

Longitudinal wake ( for  $i^{\text{th}}$  macroparticle)

$$W_o(z_i(n)) = -\frac{eN_b}{N_p} \sum_j^{z^i(n) < z^j(n)} N_j W'_o(z^i(n) - z^j(n))$$

wake between different bins ( $z^i$  and  $z^j$ )

$$-\frac{eN_b}{N_p} \sum_j^{z_i(n) < z_j(n)} W'_o(z_i(n) - z_j(n))$$

wake between macroparticle in a same bin

Transverse wake ( for  $i^{\text{th}}$  macroparticle)

$$W_i^x(z_i(n)) = -\frac{eN_b}{N_p} \sum_j^{z^i(n) < z^j(n)} \bar{x}^j(n) N_j W_T(z^i(n) - z^j(n))$$

$N_b$  : bunch population,  $N_p$  : number of macroparticles

$\bar{x}^j$  : average horizontal or vertical displacements of the preceeding particles

# 1. Beam dynamics in 4 GeV light source

## Muilparticle tracking simulation (x,x',y,y',z,ε)

$$\epsilon_i(n) = \epsilon_i(n-1) - \frac{2T_o}{\tau_d} \epsilon_i(n-1) + 2\sigma_{\epsilon o}$$

$$\sqrt{\frac{T_o}{\tau_d}} r_{1i}(n) + V'_{rf} z_i(n-1) + W(z_i)(n),$$

$$z_i(n) = z_i(n-1) + \frac{\alpha c T_o}{E_o} \epsilon_i(n),$$

$$V'_{rf} = 2\pi\nu_{rf} \hat{V}_{rf} [1 - (U_o/\hat{V}_{rf})^2]^{1/2}.$$

$$x_i(n) = M_{11}[\epsilon_i(n)]x_i(n-1) + M_{12}[\epsilon_i(n)]x'_i(n-1)(1 - \frac{T_o}{\tau_x}) \\ + \sqrt{\frac{2\epsilon_z\beta_x T_o}{\tau_x}} r_{1i}(n) + M_{12}[\epsilon_i(n)] \frac{W_i^x(n-1)}{E_o},$$

$$x'_i(n) = M_{21}[\epsilon_i(n)]x_i(n-1) + M_{22}[\epsilon_i(n)]x'_i(n-1)(1 - \frac{T_o}{\tau_x}) \\ + \sqrt{\frac{2\epsilon_z T_o}{\beta_x \tau_x}} r_{2i}(n) + M_{22}[\epsilon_i(n)] \frac{W_i^x(n-1)}{E_o},$$

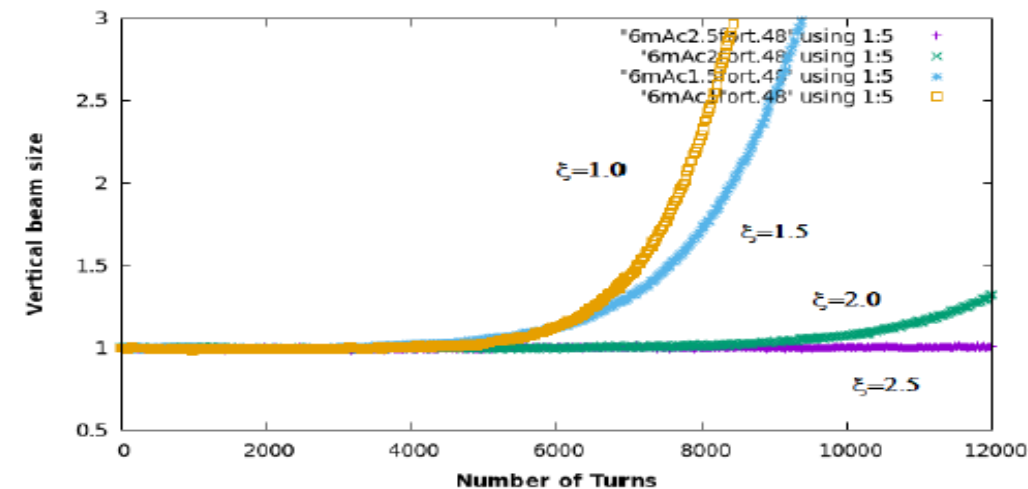
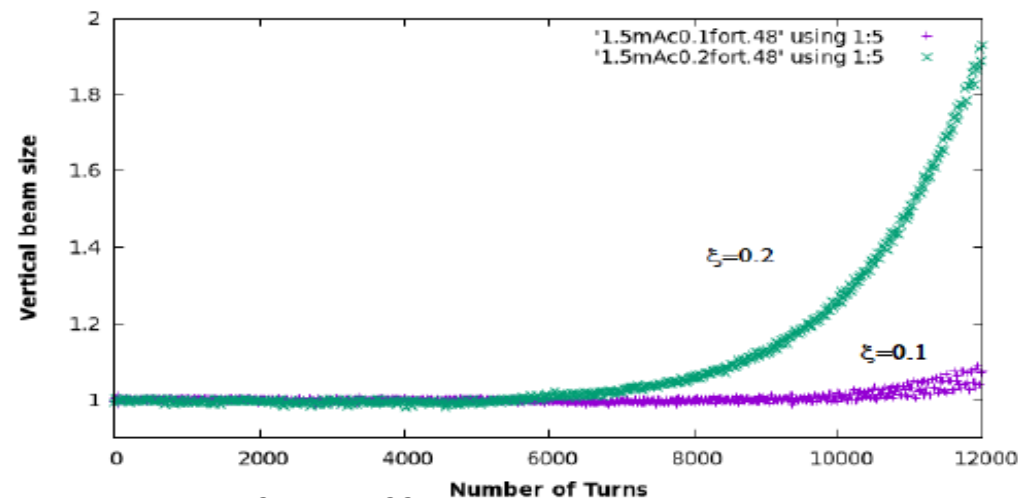
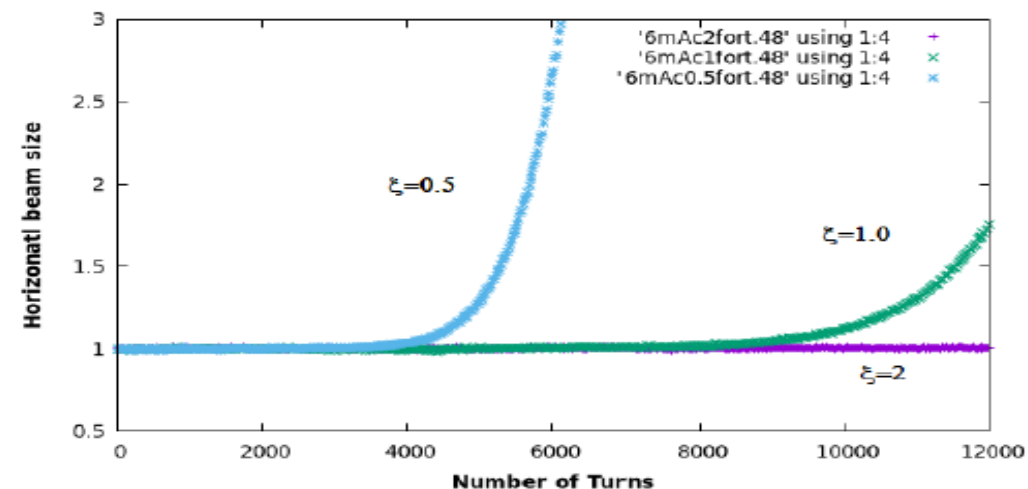
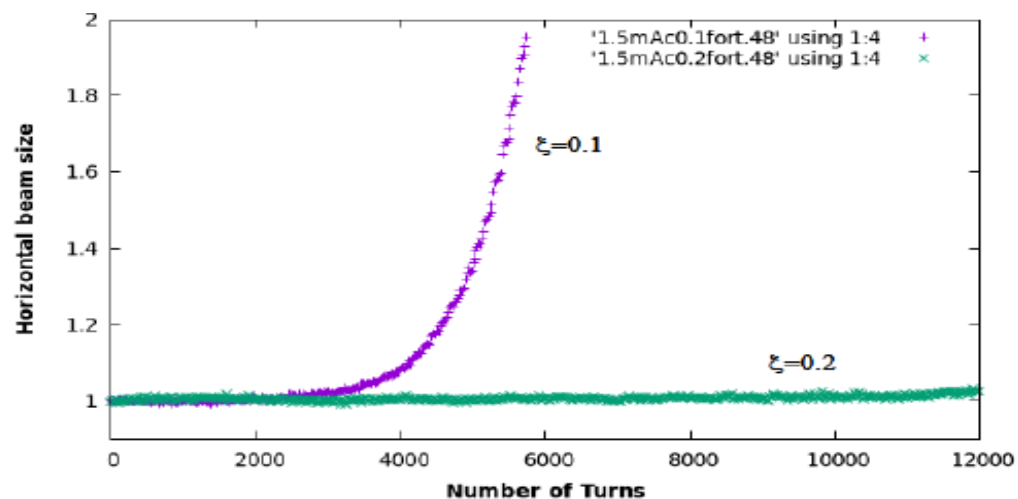
$$M(\epsilon) = \begin{bmatrix} \cos 2\pi Q_x(\epsilon) & \beta_x \sin 2\pi Q_x(\epsilon) \\ -1/\beta_x \sin 2\pi Q_x(\epsilon) & \cos 2\pi Q_x(\epsilon) \end{bmatrix},$$

$$Q_x(\epsilon) = Q_x(1 + \epsilon\xi/E_o) \quad (\xi : \text{chromaticity})$$

# 1. Beam dynamics in 4 GeV light source

## Resistive wall Wake

## Beam sizes due to chromaticity



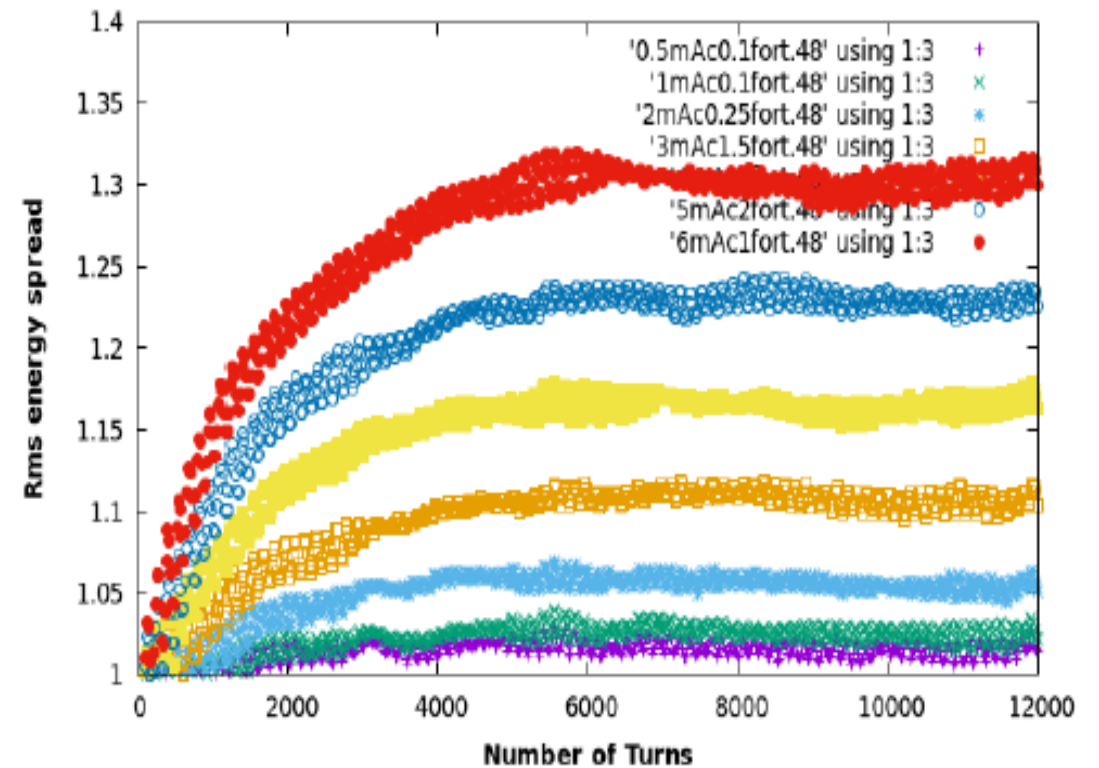
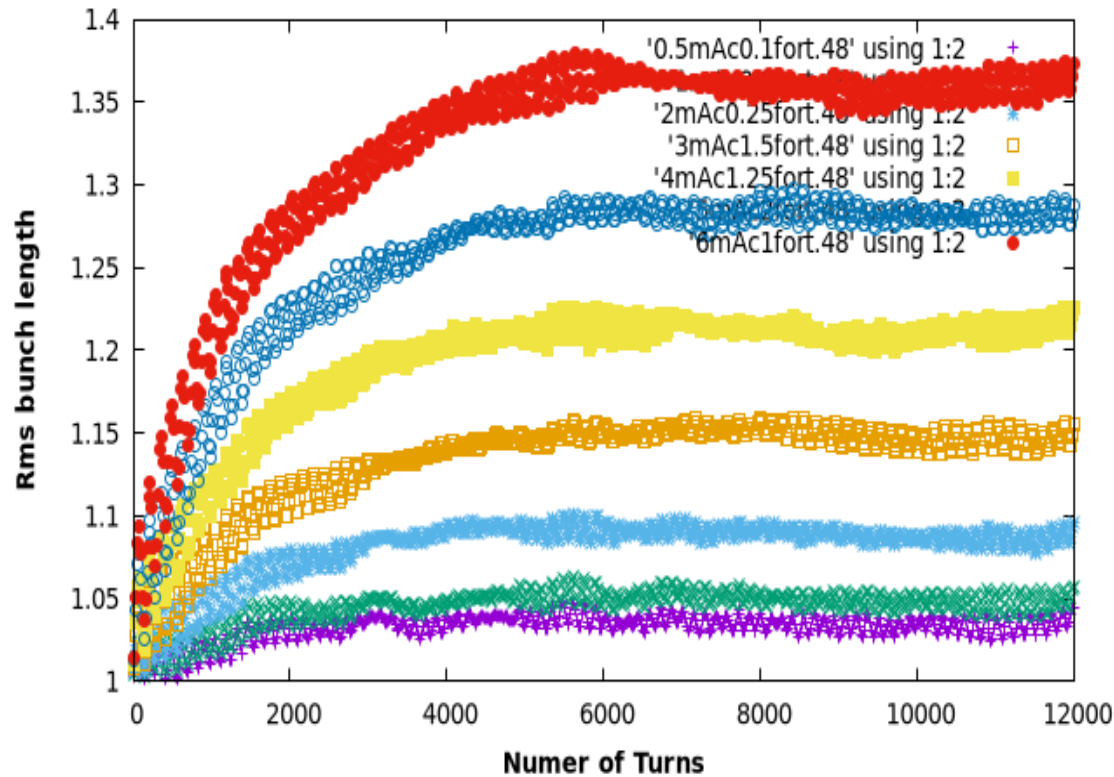
beam sizes for different chromaticities in 1.5 mA

beam sizes for different chromaticities at 6 in mA

# 1. Beam dynamics in 4 GeV light source

## Resistive wall Wake

### Bunch length and energy spread vs beam current

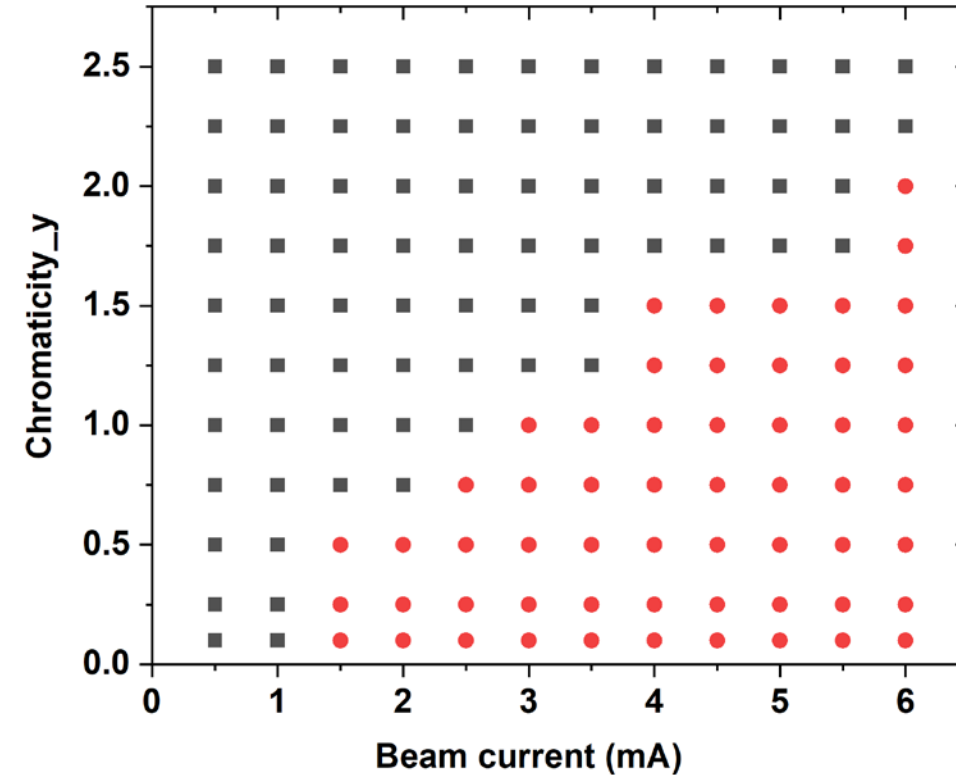
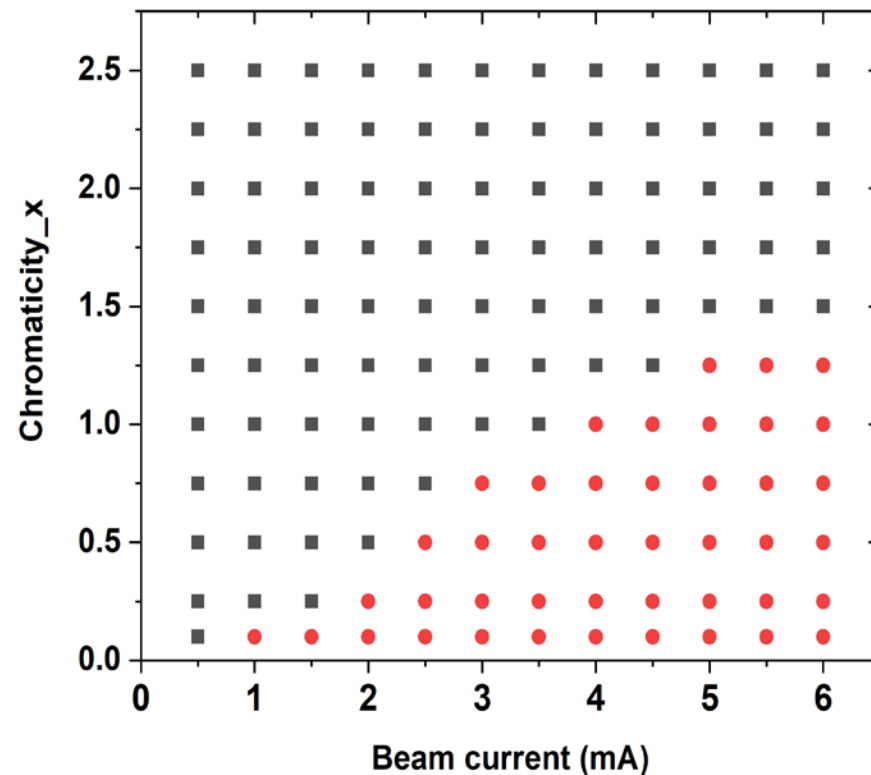


Bunch length and energy spread for the different beam currents ( 6mA to 0.5mA)

# 1. Beam dynamics in 4 GeV light source

## Resistive wall Wake

### Transverse instability at chromaticities for different beam currents

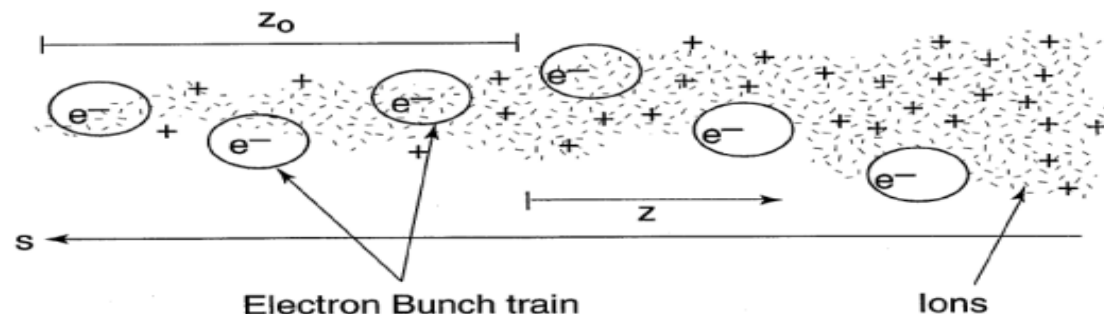


Black and red dots indicate that the beam size is not increased and increased due to the wake, respectively.

# 1. Beam dynamics in 4 GeV light source

## Fast-ion instability

- In electron storage rings, residual gas molecules can be ionized by the beam. The **resulting positive ions are trapped in the electric field of the beam**, and accumulate to high density. → **The fields of the ions can then drive beam instabilities.**
- While electrons move rapidly on the time scale of a single bunch passage, ions move relatively slowly. The dynamical behaviour is then somewhat different.
- If a storage ring is uniformly filled with electron bunches, then ions accumulate over many turns. This leads to the well-known phenomenon of ion trapping, which is usually solved by including "gaps" in the fill pattern.
- Under certain conditions, sufficient ions can accumulate in the passage of a small number of bunches to drive an instability, known as the "fast ion instability".



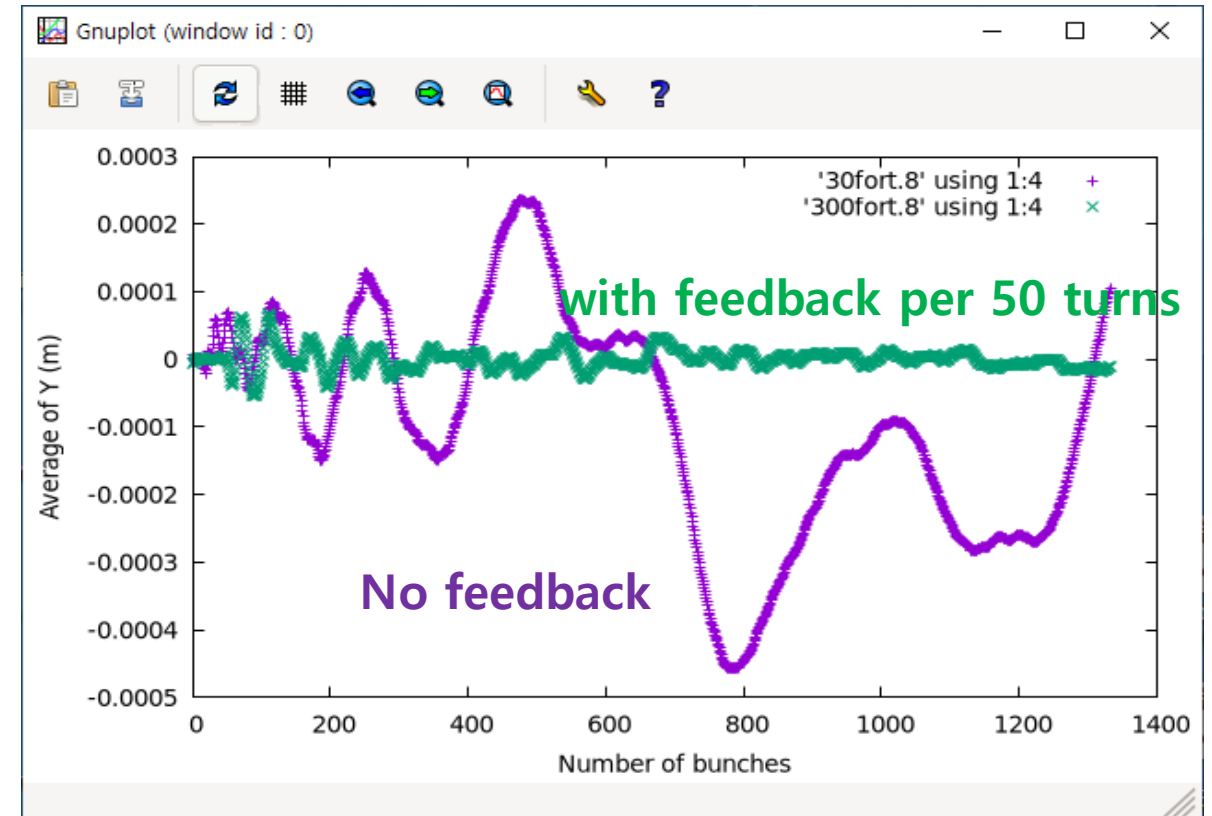
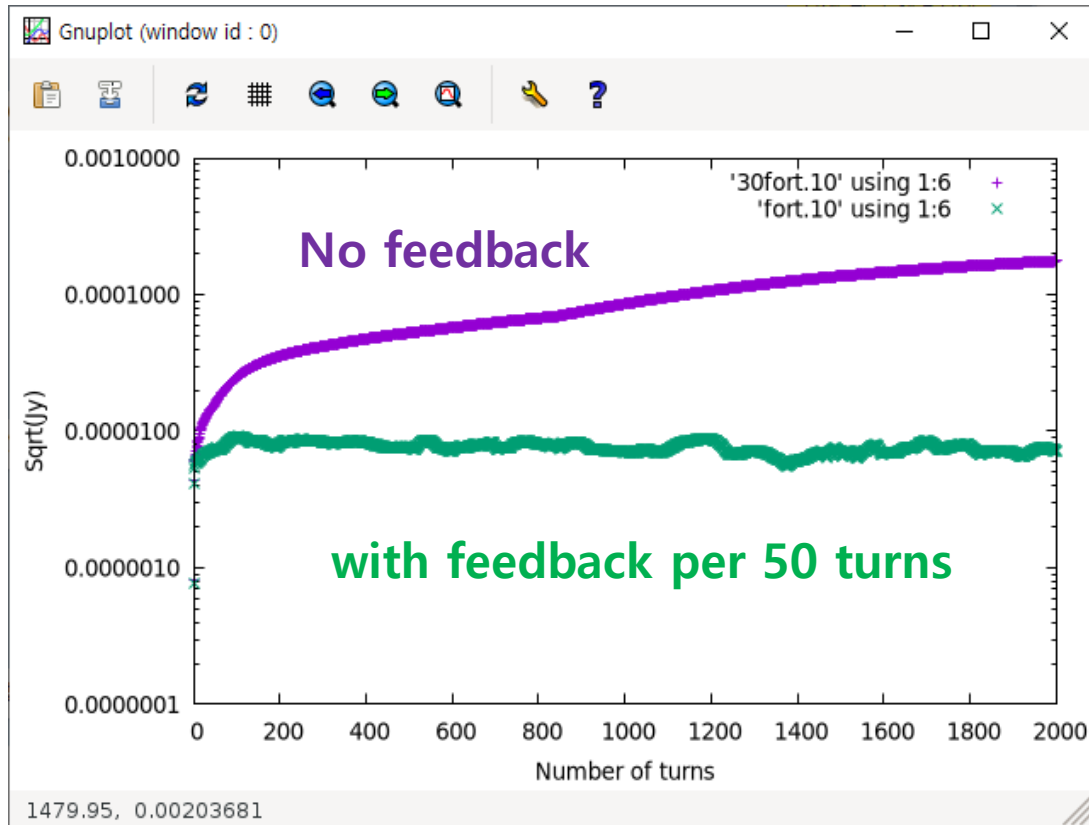
# 1. Beam dynamics in 4 GeV light source

## Fast-ion instability

1332 bunches

400 mA

0.1 nT



Vertical oscillation amplitude of the bunch centroid is half of the Courant–Synder invariant

$$J_y = [\gamma y^2 + 2\alpha y y' + \beta y'^2]/2$$

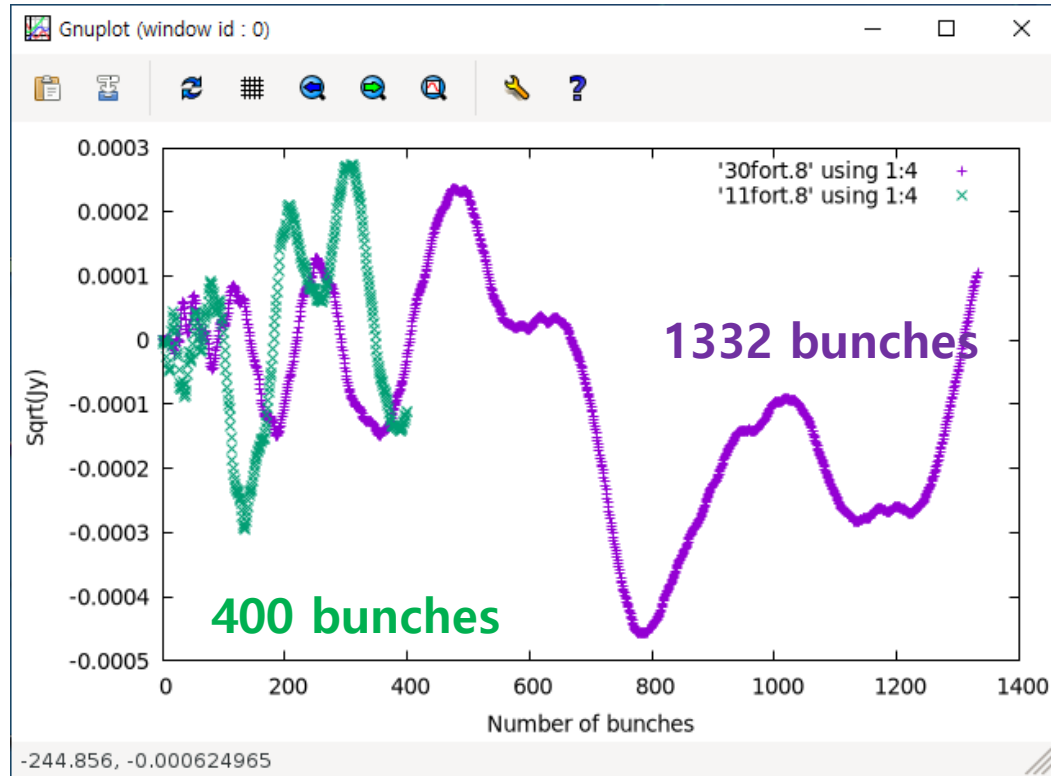


# 1. Beam dynamics in 4 GeV light source

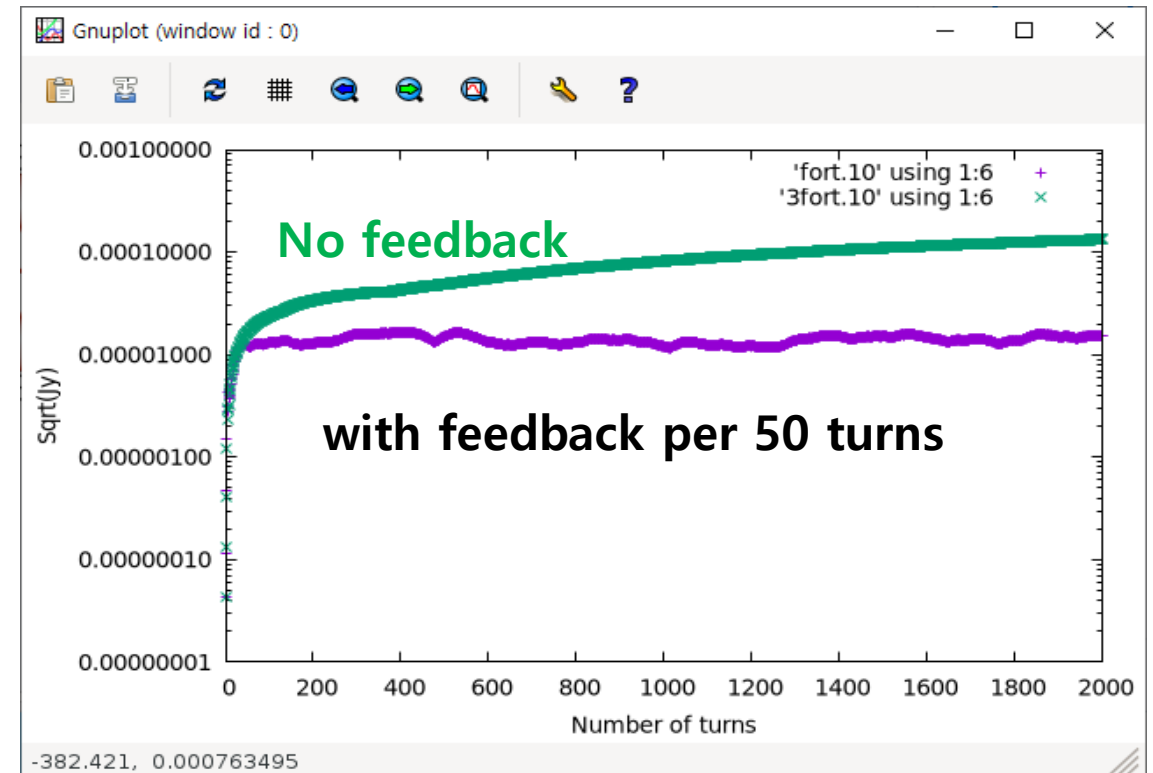
## Fast-ion instability

400 mA and 0.1 nT

1 train case of 1332 and 400 bunches



400 bunches with 4 trains  
(233 buckets are empty between trains)

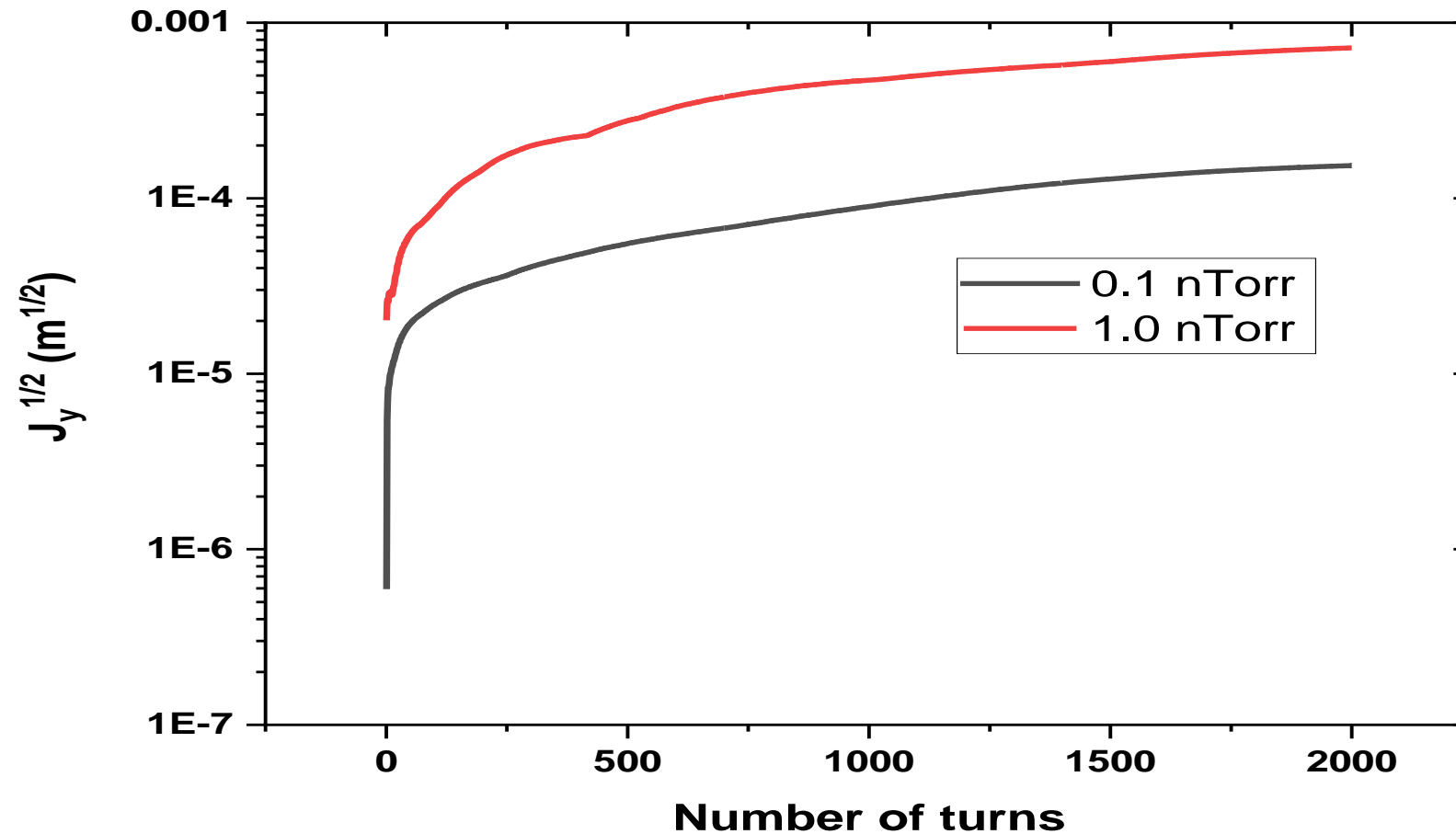




# 1. Beam dynamics in 4 GeV light source

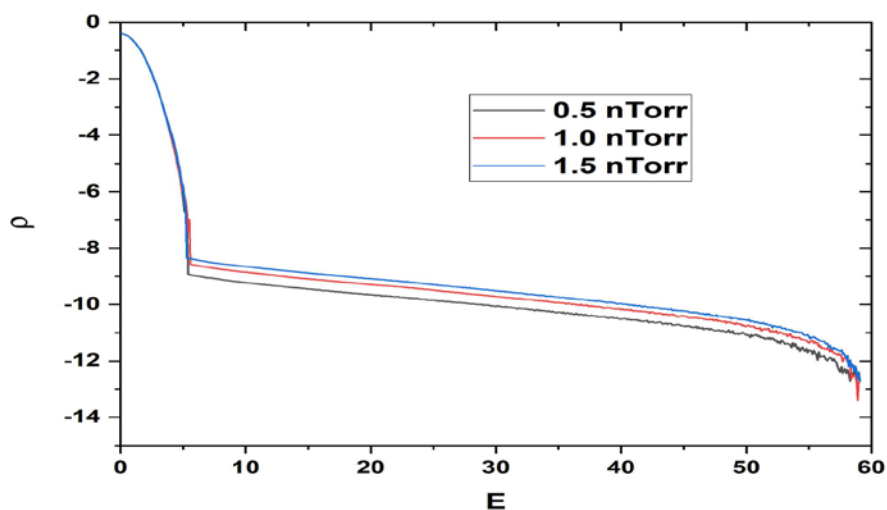
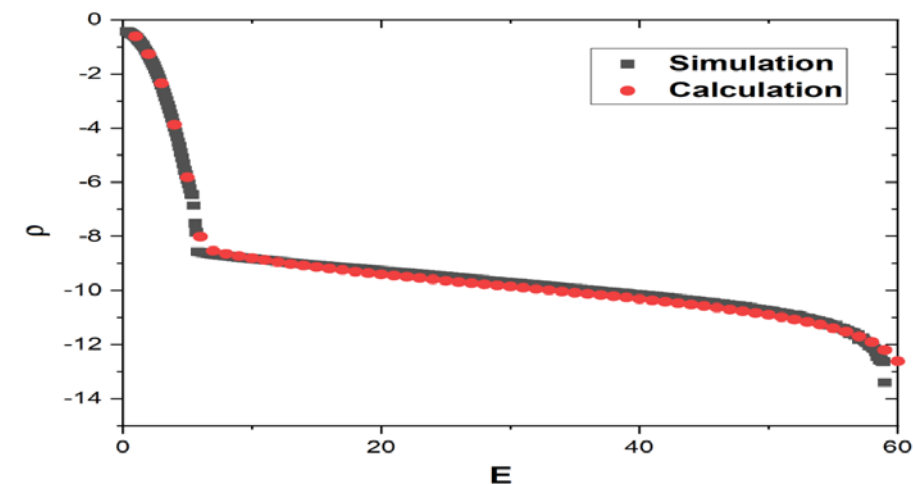
## Fast-ion instability

400 mA



# 1. Beam dynamics in 4 GeV light source

## Beam tail distribution due to beam-gas Bremsstrahlung



$$\rho(E) = \frac{1}{\pi} \int dK \cos(K E) \exp \left[ -\frac{K^2}{2} + \frac{N \tau_{\epsilon}}{2} \hat{f} \left( \frac{K}{E_0 \sigma_{\epsilon}^0} \right) \right]$$

### Simulation result

Energy aperture	1 nTorr	1.5 nTorr
2.5 %	41.1 h	28.1 h
3.0 %	44.6 h	29.3 h
3.87%	48.5 h	33.1 h

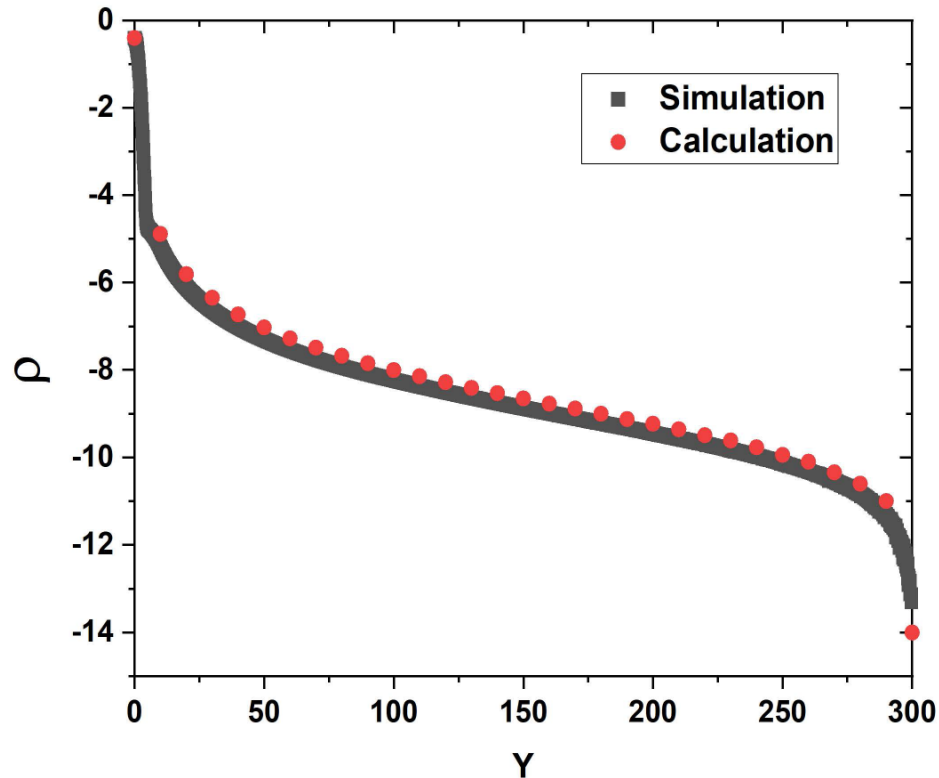
### Calculation result

Energy aperture	1 nTorr	1.5 nTorr
2.5 %	43.7 h	29.1 h
3.0 %	45.7 h	30.5 h
3.87%	49.4 h	32.9 h

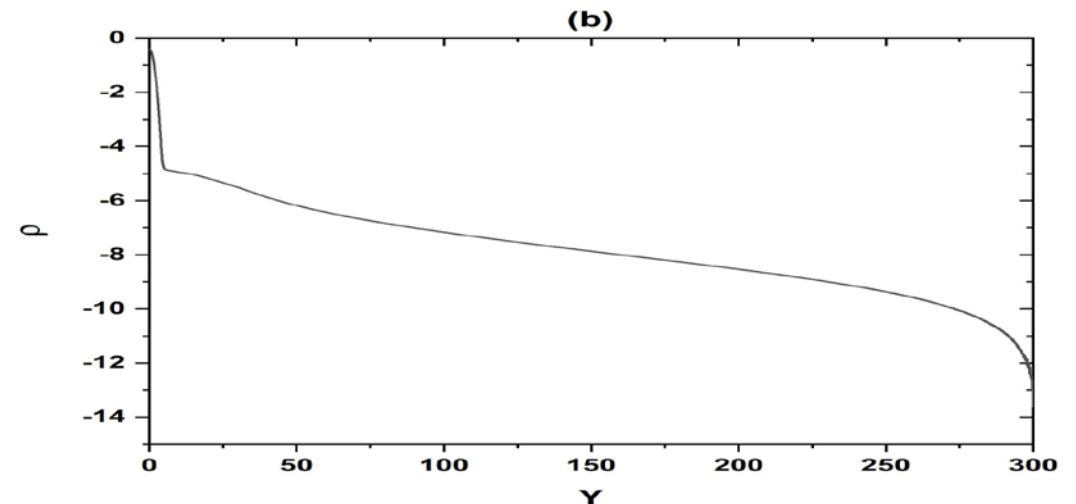
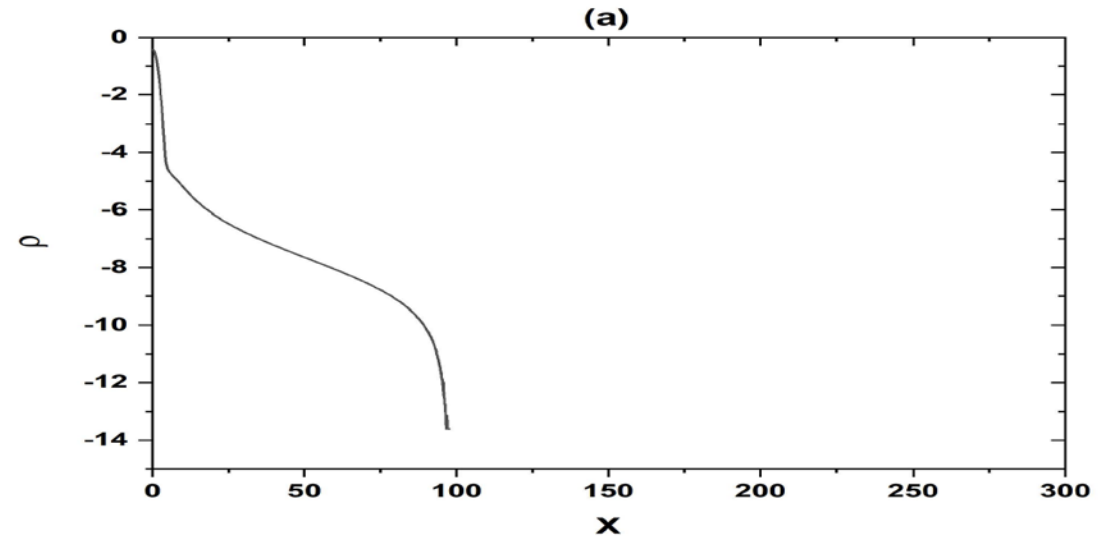
Both simulation and calculation results agree well.

# 1. Beam dynamics in 4 GeV light source

## Beam tail distribution due to beam-gas scattering

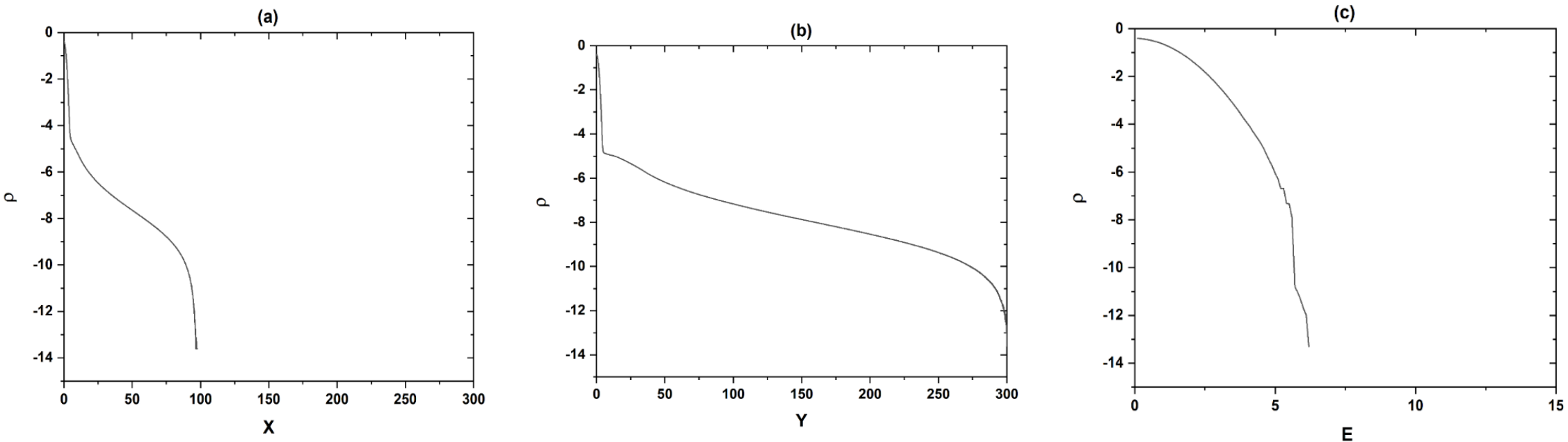


$$\rho(X) = \frac{1}{\pi} \int dK \cos(KX) \exp \left[ -\frac{K^2}{2} + N\tau \hat{f} \left( \frac{K}{\sigma'_x} \right) \right]$$



# 1. Beam dynamics in 4 GeV light source

## Beam tail distribution due to intra-beam scattering



Simulation result

Vertical aperture	300 $\sigma_y$	400 $\sigma_y$	500 $\sigma_y$
Residual gas scattering	21.2 h	37.5 h	58.8 h
Intra-beam scattering	1.6 h	2.5 h	3.6 h

calculation result

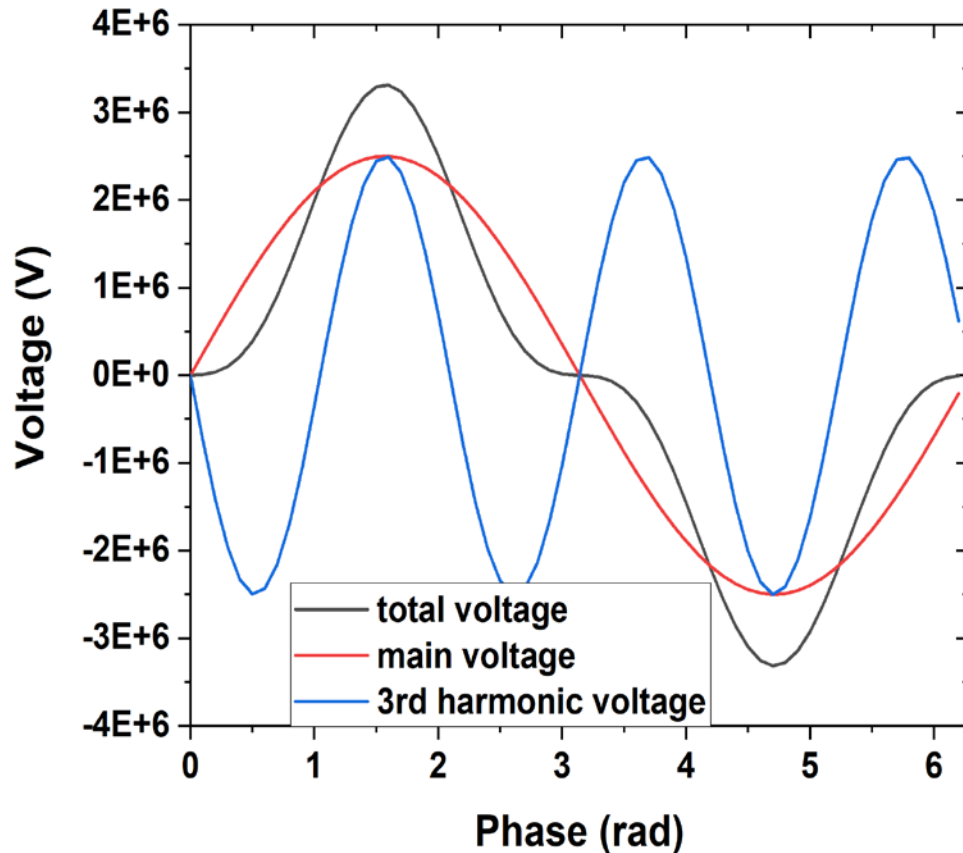
Vertical aperture	300 $\sigma_y$	400 $\sigma_y$	500 $\sigma_y$
Residual gas scattering	21.7 h	38.7 h	60.5 h
Intra-beam scattering	1.67 h	2.87 h	4.3 h

Both simulation and calculation results agree well.

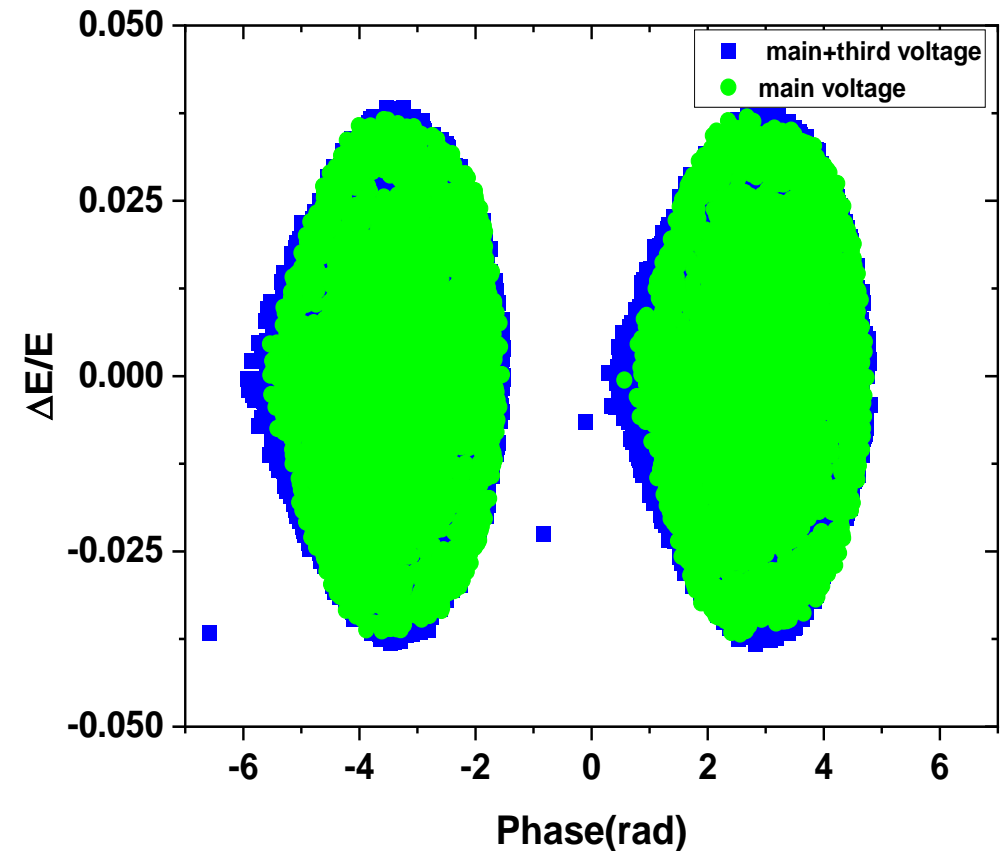
Total beam lifetime : 3.2 hour for 400 mA and 1 nTorr

# 1. Beam dynamics in 4 GeV light source

## Bunch-lengthening due to 3<sup>rd</sup> harmonic cavity (1.5 GHz)



Rf voltages seen by a bunch for bunch lengthening



RF buckets with and without third-harmonic voltage for bunch lengthening

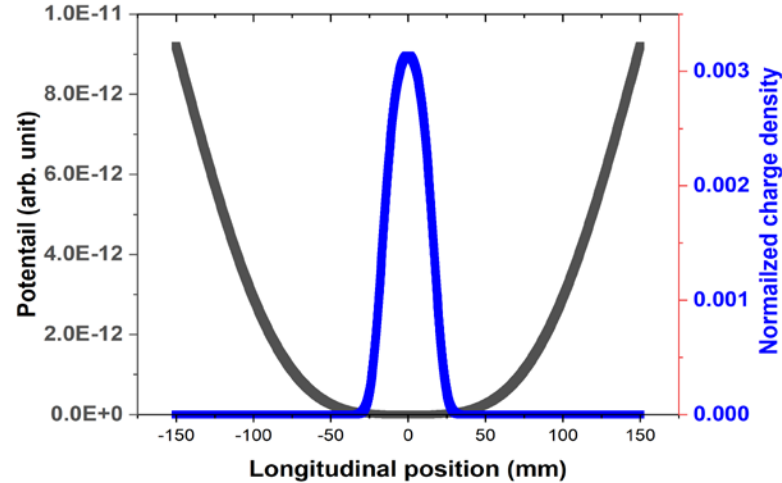
# 1. Beam dynamics in 4 GeV light source

## Bunch-lengthening due to 3<sup>rd</sup> harmonic cavity

$$V_{3rd\ rf} / V_{M\ rf} = 0.32$$

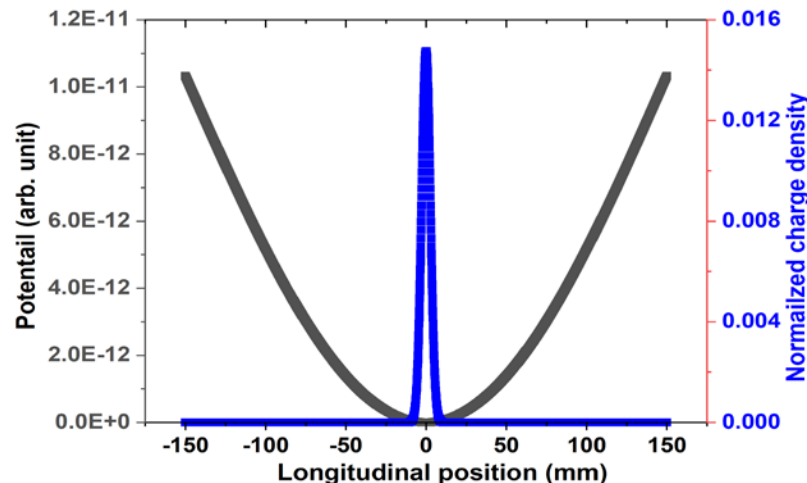
$$\phi_{3rd} = 1.047\ rad$$

with  
harmonic  
cavity

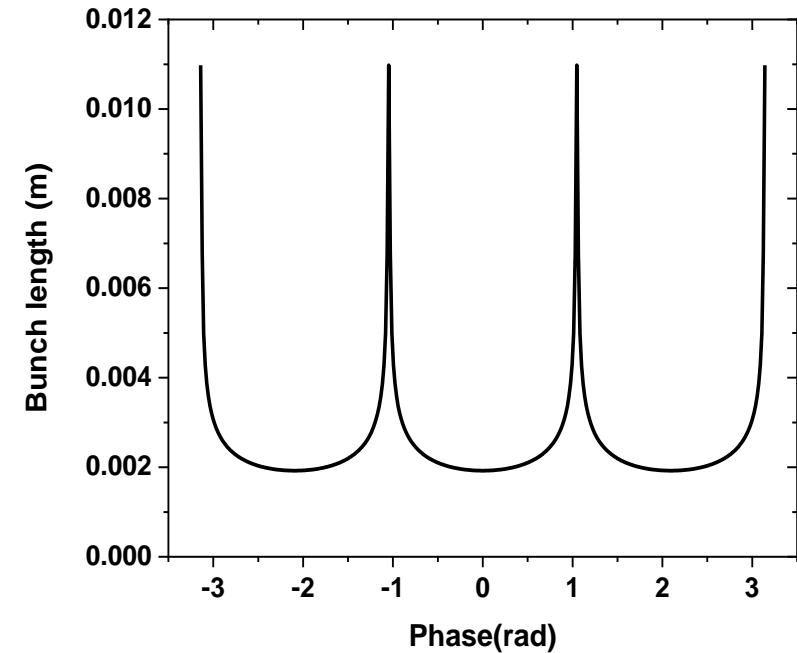


Potential-Well and  
longitudinal beam  
distributions

without  
harmonic  
cavity



bunch-lengthening : factor of 3.5



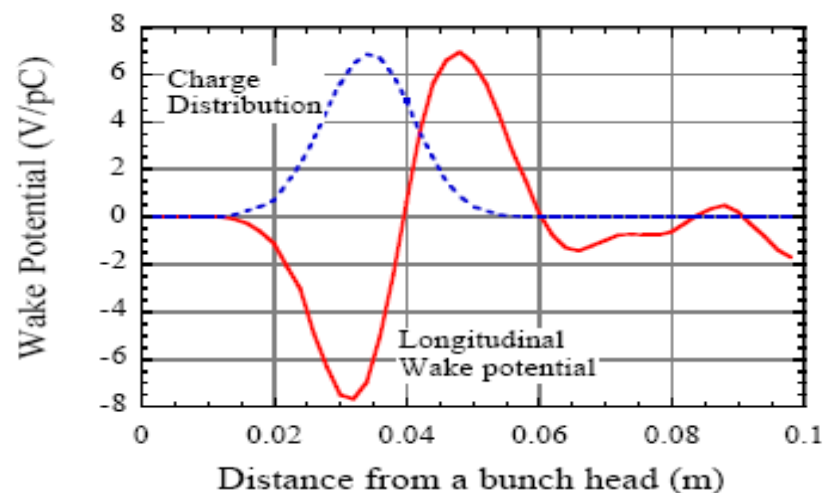
Bunch length as a function of phase angle  
in 3<sup>rd</sup> harmonic cavity

## 2. Beam dynamics in ATF damping ring

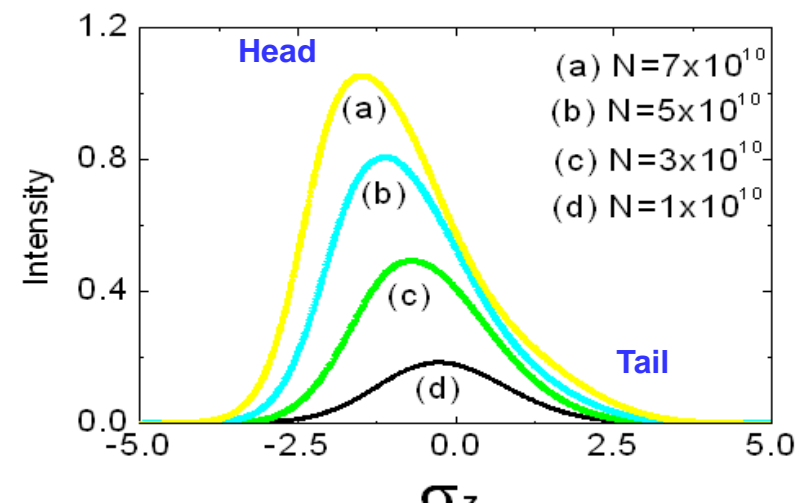
### Impedance estimation of damping ring

Main contributions to the longitudinal impedance

Components	Type	Number	$ \frac{Z_{  }(w)}{n} $ ( $\Omega$ )	L(nH)	Code used
BPM	Button-type	96	0.064	4.8	MAFIA
Bellows	Normal type with $\phi = 24$ mm	56	0.02	1.568	ABCI
	Elliptic type with $\phi = 24$ mm $\times$ 12 mm	8	0.0058	0.46	MAFIA
	External kicker with radius = 30 mm	1	0.01411		ABCI
	External kicker with radius = 15 mm	1	0.01017		ABCI
Masks	Wiggler	8	0.00096	0.0713	MASK30
	Straight section	8	0.047	3.544	MAFIA
Tapers	Length(= 120 mm), angle(= 10.3°)	2	0.0093	0.70	MAFIA
	Length(= 170 mm), angle(= 10.3°)	2	0.008	0.602	MAFIA
Septum		1	0.0082	0.6224	ABCI
Injection kicker	taper part	1	0.0015	0.1203	ABCI
	step part	1	0.00008	0.0063	ABCI
RF cavity	With tapers	2	0.03144	0.687	ABCI
RF absorber	With SiC with $\sigma = 80$ and $\varepsilon = 16$	4	0.0089	0.671	MAFIA
Total			0.23	13.9	



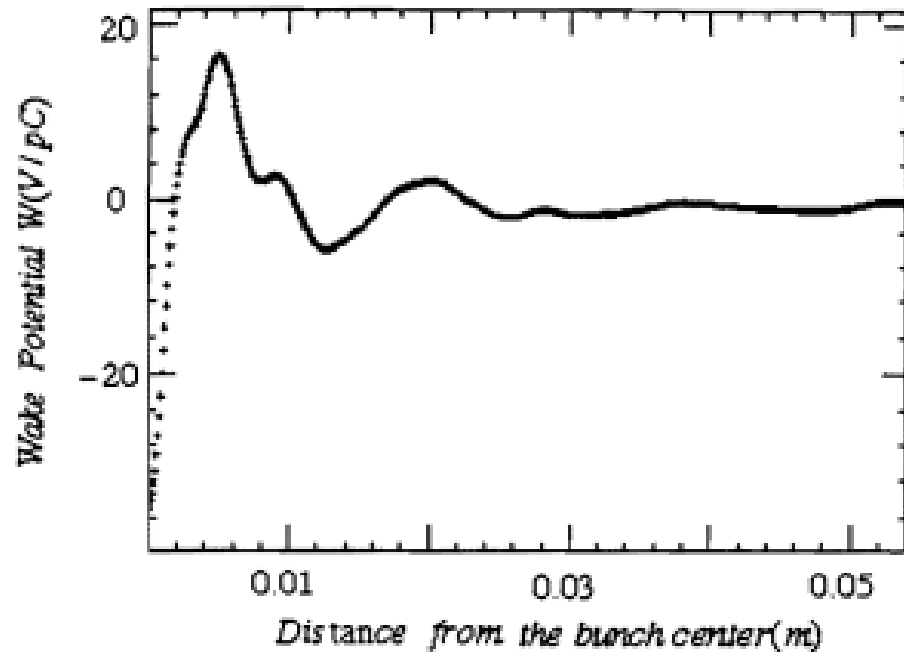
longitudinal wake potential of a 6.8 mm Gaussian bunch



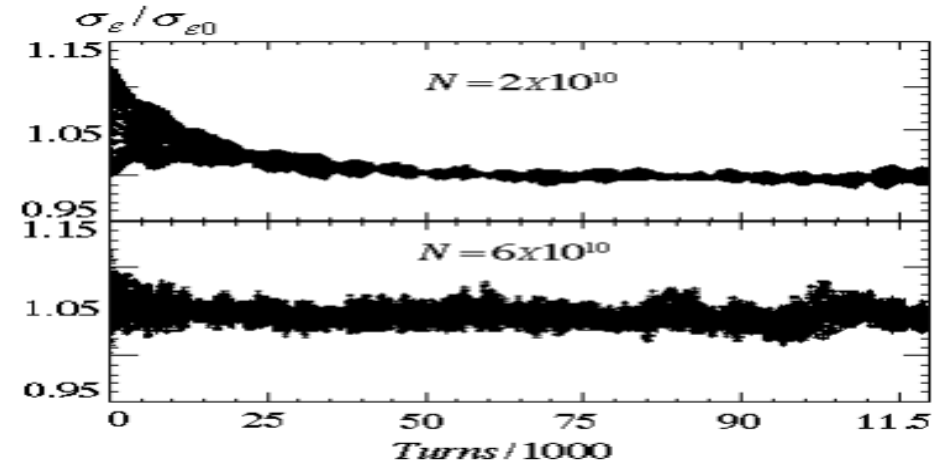
calculated longitudinal beam distributions due to potential well distortion

## 2. Beam dynamics in ATF damping ring

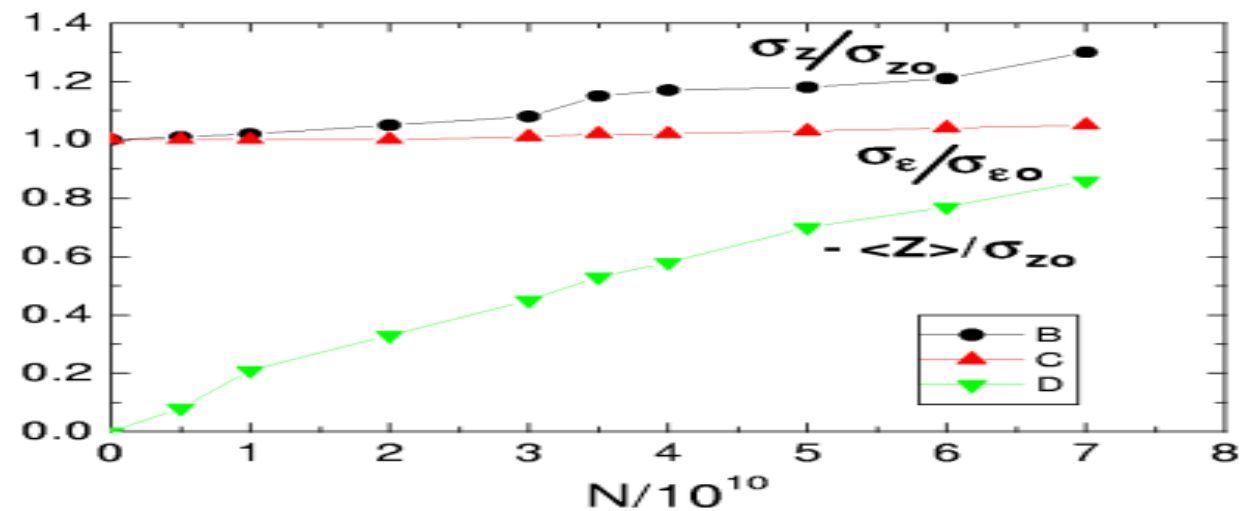
### longitudinal instability



Longitudinal wake potential for 2 rf cavities in damping ring



Rms energy spread vs turn number.



threshold instability :  $N = 3.3 \times 10^{10}$

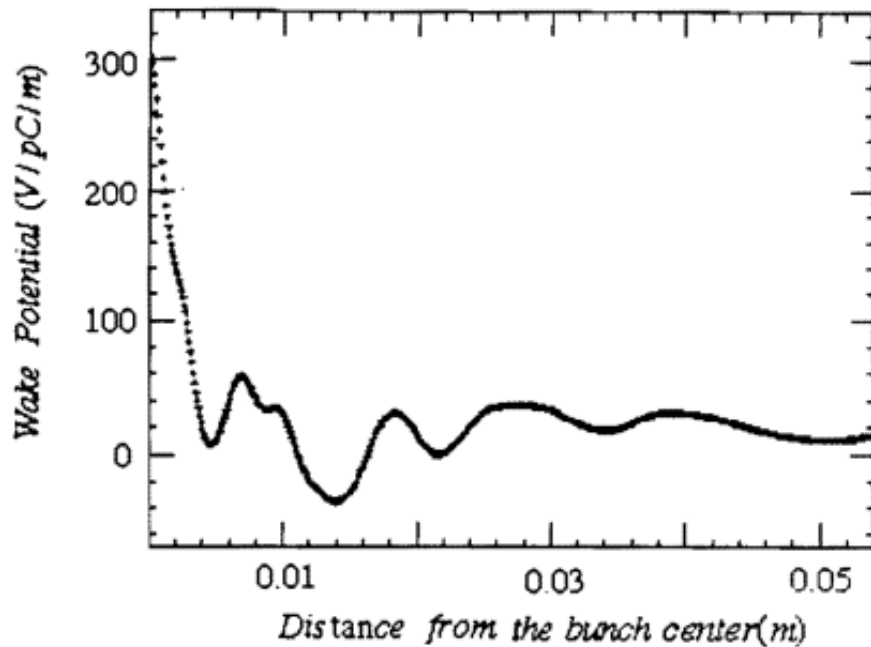


## 2. Beam dynamics in ATF damping ring

### Transverse instability

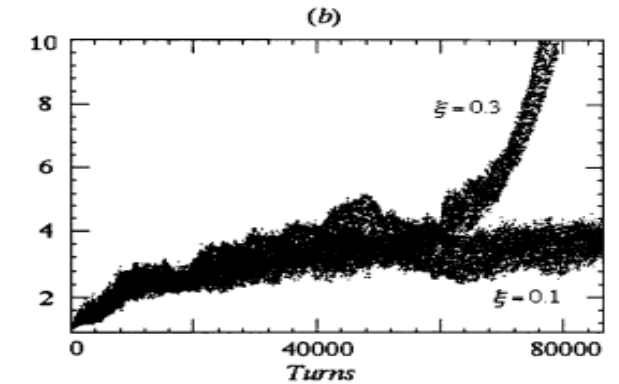
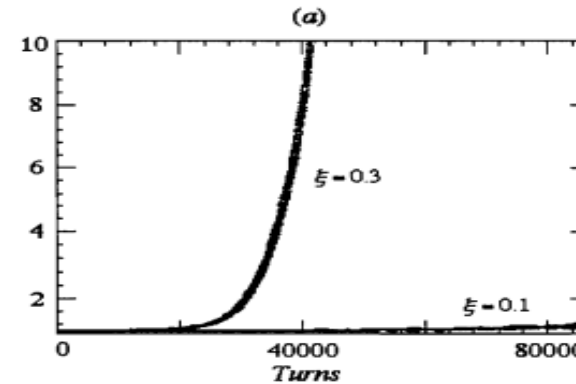
head-tail effect occuring at positive chromaticity

10.2 mA

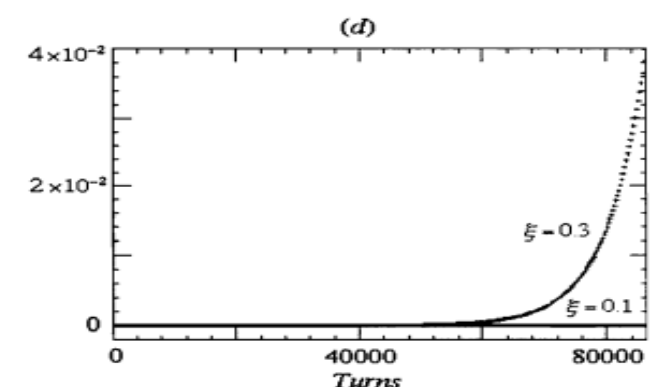
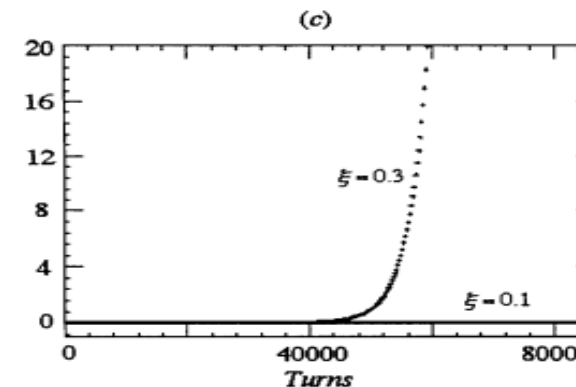


Transverse wake potential for two cavities

Charge bunch is a Gaussain with an rms of 1mm.  
Parts on front of bunch center ( $z < 0$ ) are discarded.



horizontal and vertical beam sizes



horizontal and vertical center-of-mass motion

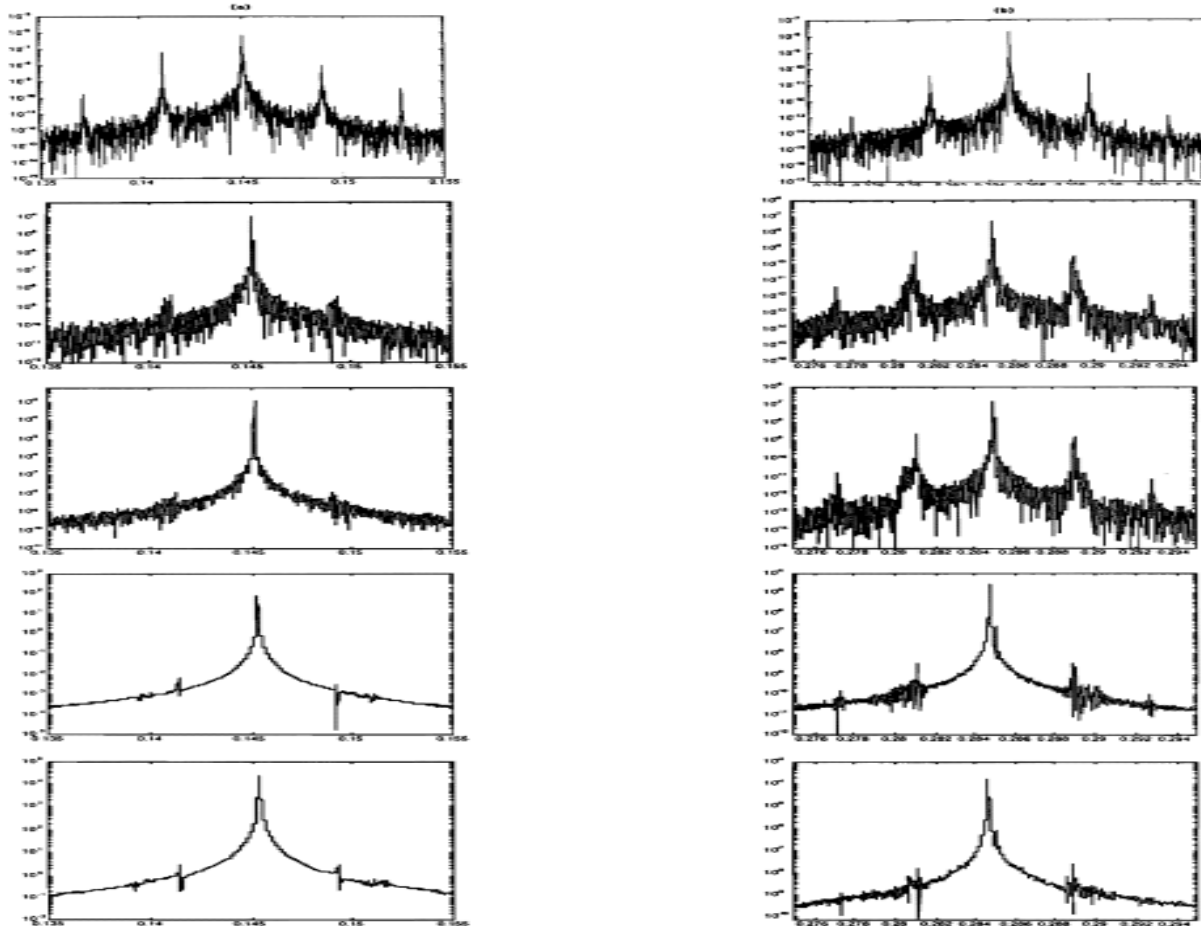
## 2. Beam dynamics in ATF damping ring

### Transverse instability

Fourier Transform of mean horizontal and vertical positions of the beam for different beam currents

$m=0$  mode has shifted to higher tunes as the beam currents increase.  
Mode  $m=-1$  and mode  $m=1$  are shown clearly.

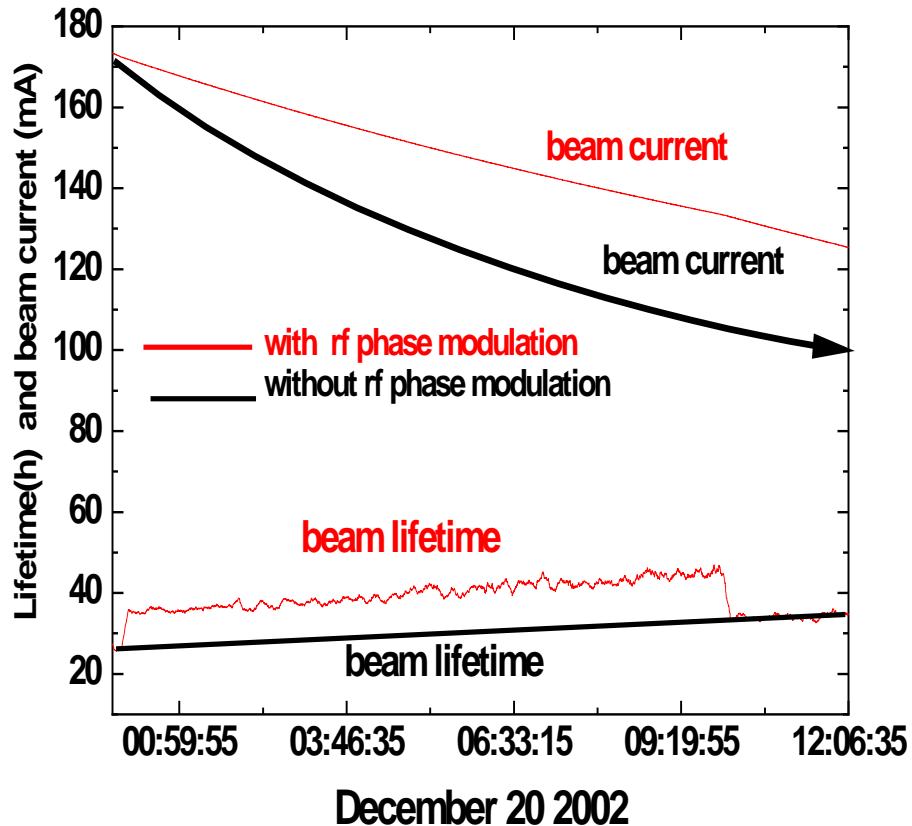
Chromaticity : 0.1  
 $Q_x=0.145$ ,  $Q_y=0.285$   
 $\omega_s = 1 \times 10^{-3}$ .



# 3. Beam dynamics in 2.5 GeV light source

## RF phase modulation

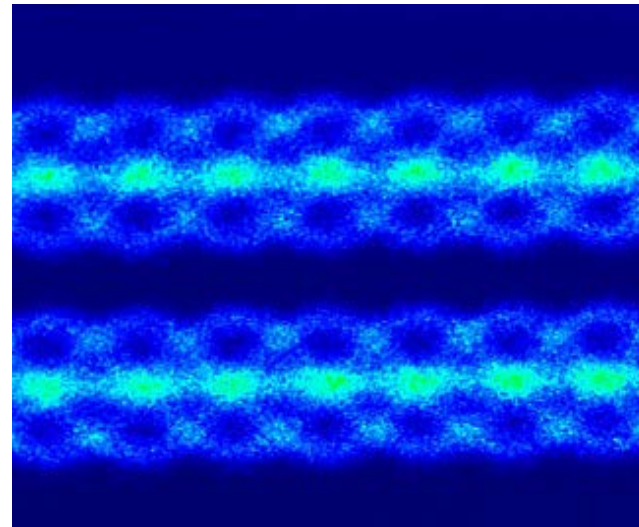
increased beam lifetime by a factor of 1.5  
by bunch-lengthening



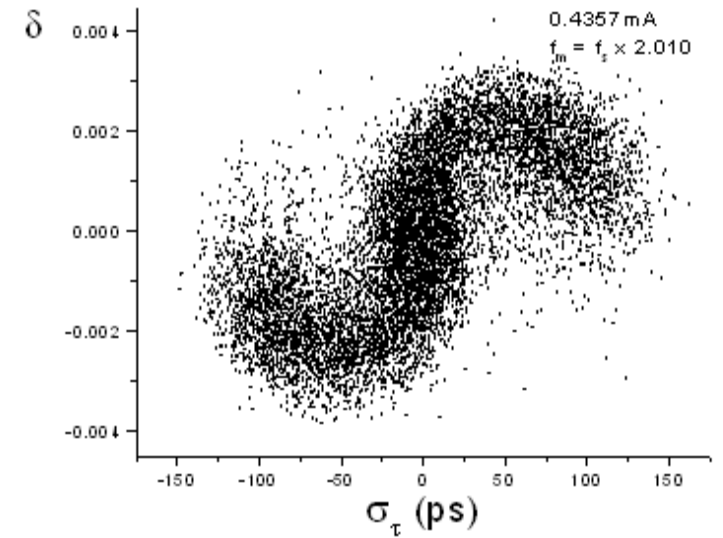
$$V_{rf} = \overline{V_o} \cos(\phi_o - \omega_{rf}\tau + \phi_m)$$

$$\phi_m = \phi_{mo} \cos(\omega_m T_o)$$

$$(\phi_{mo} = 5 \text{ deg.}, \omega_m = 2 \times 9.34 \text{ kHz})$$



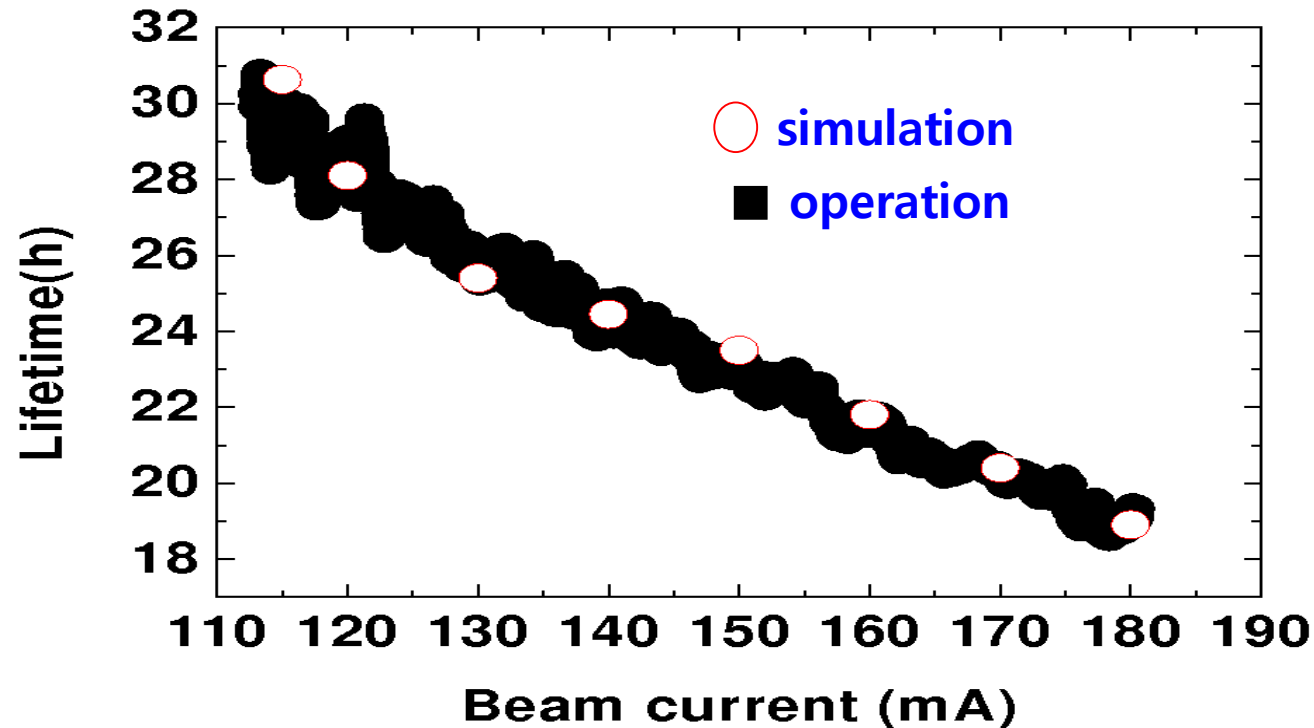
Streak camera image  
of the RF phase modulation



simulation result  
of the RF phase modulation

### 3. Beam dynamics in 2.5 GeV light source

#### Operation and simulation beam lifetimes



From the simulation results

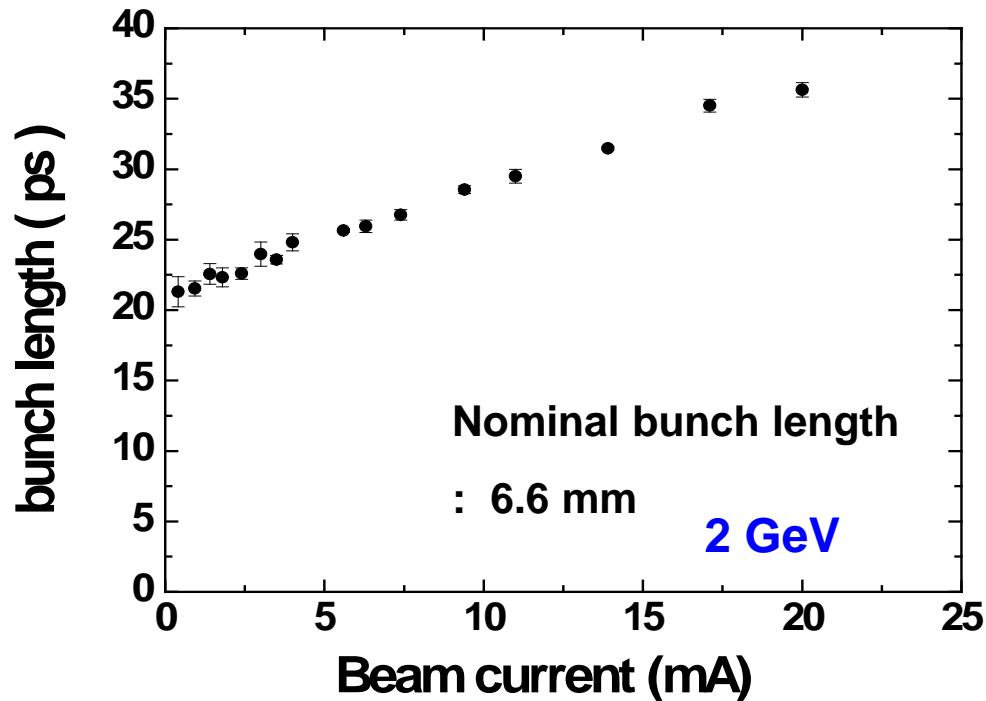
Beam-gas scattering lifetime : 290 h

Beam-gas Bremsstrahlung lifetime : 460 h

Touscheck lifetime : 18 h

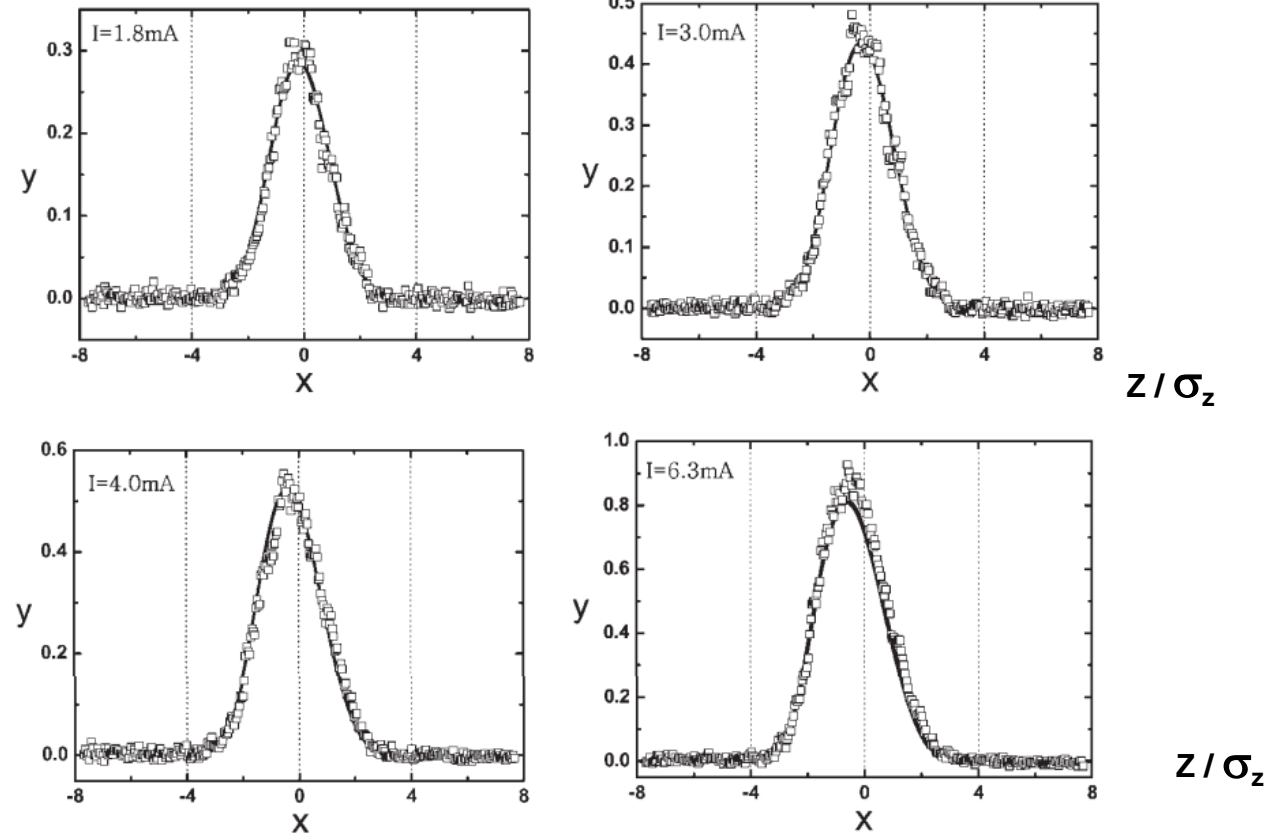
# 3. Beam dynamics in 2.5 GeV light source

## Estimation of ring impedance from bunch-lengthening



Resistive impedance :  $R = 743 \Omega \pm 84\Omega$

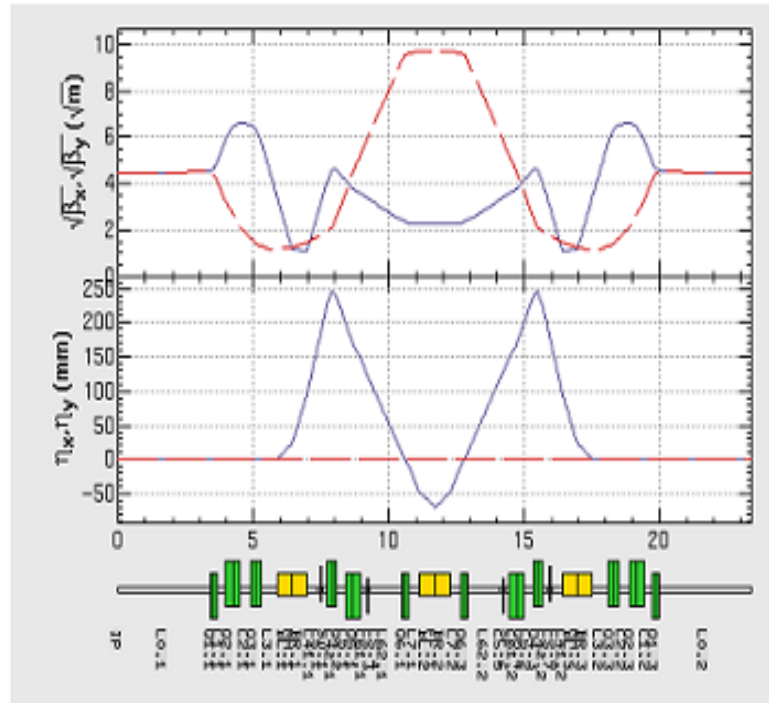
Inductive impedance :  $L = 13.7 \text{ nH} \pm 3.5 \text{ nH}$



Measured bunch shapes (squares) and their fits (black lines) to the Haissinski solution of a series R+L impedance

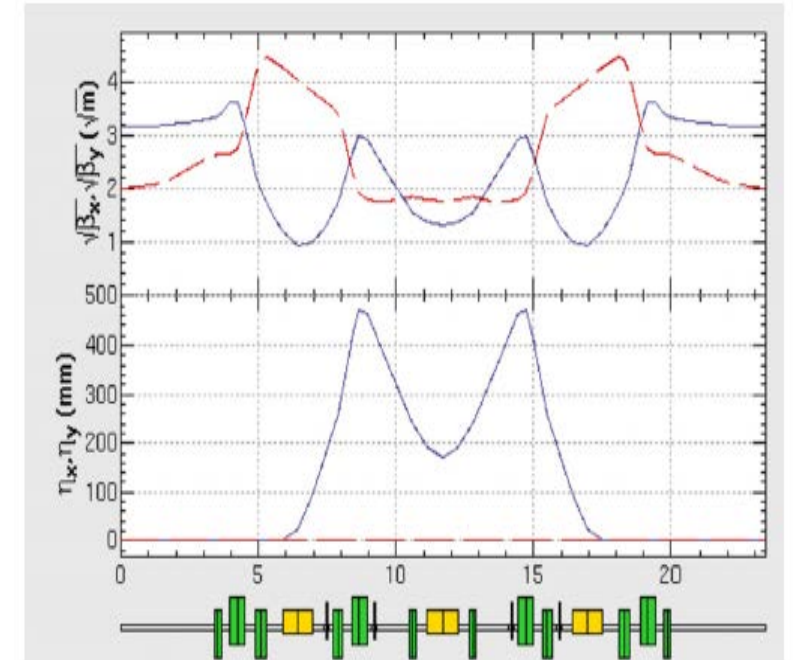
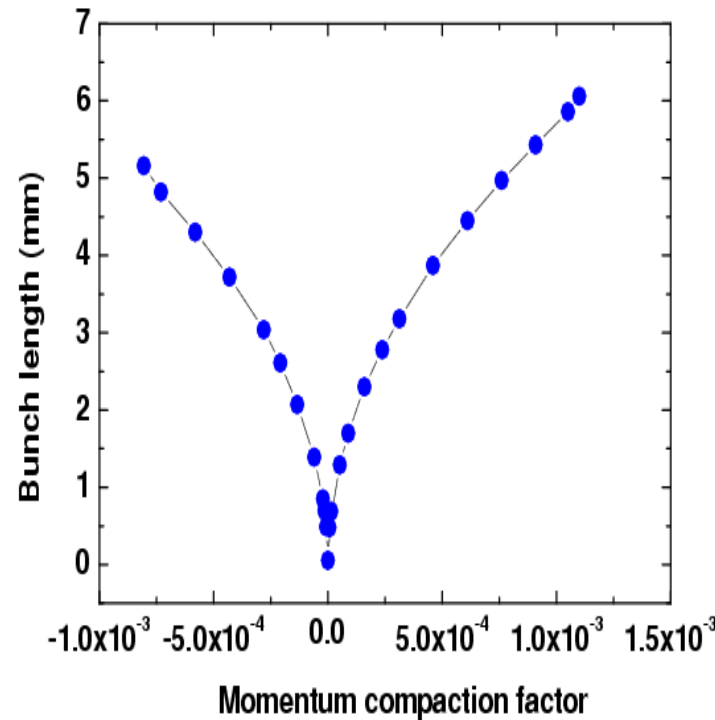
# 3. Beam dynamics in 2.5 GeV light source

## Lattice design for short-bunch length



Lattice for  
1 ps bunch length

## Bunch length vs momentum compaction factor

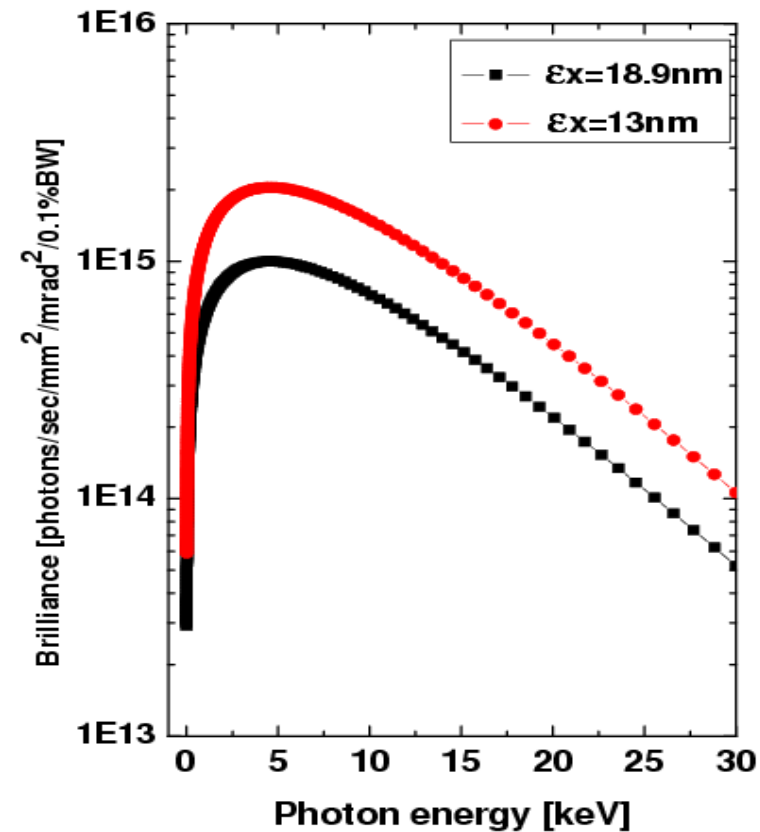
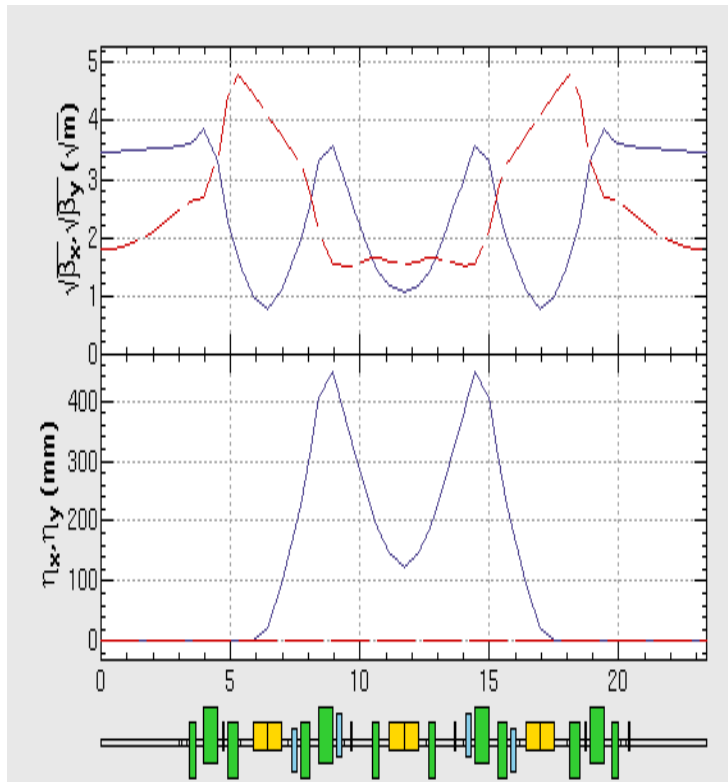


Lattice for  
28 ps nominal bunch length

# 3. Beam dynamics in 2.5 GeV light source

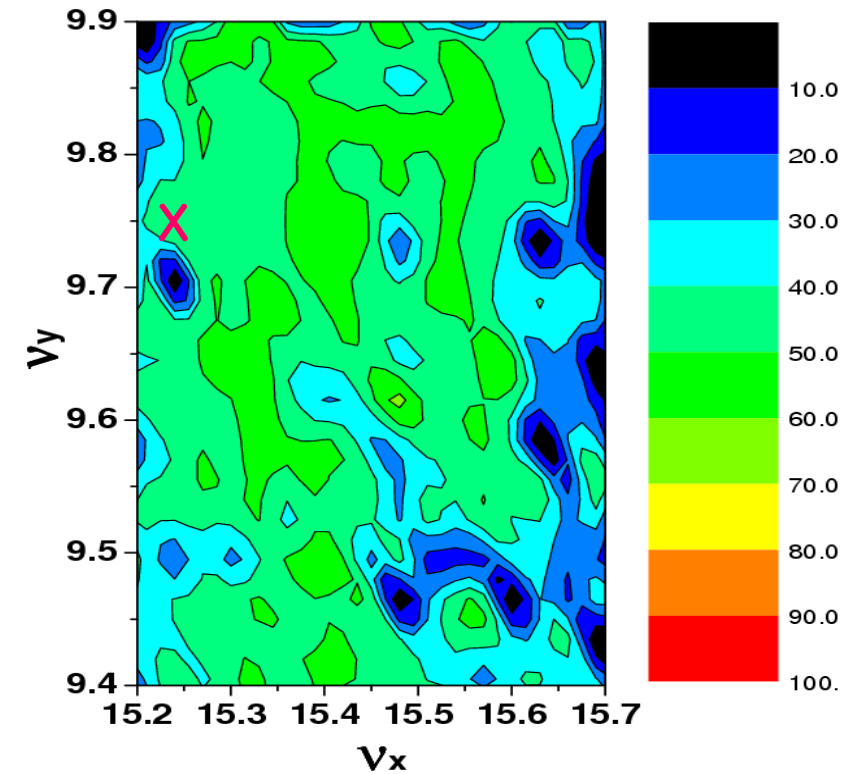
## Lattice design for a lower emittance

$\epsilon = 13 \text{ nm}$  (cf : 18.9 nm @ 2.5 GeV)



dipole

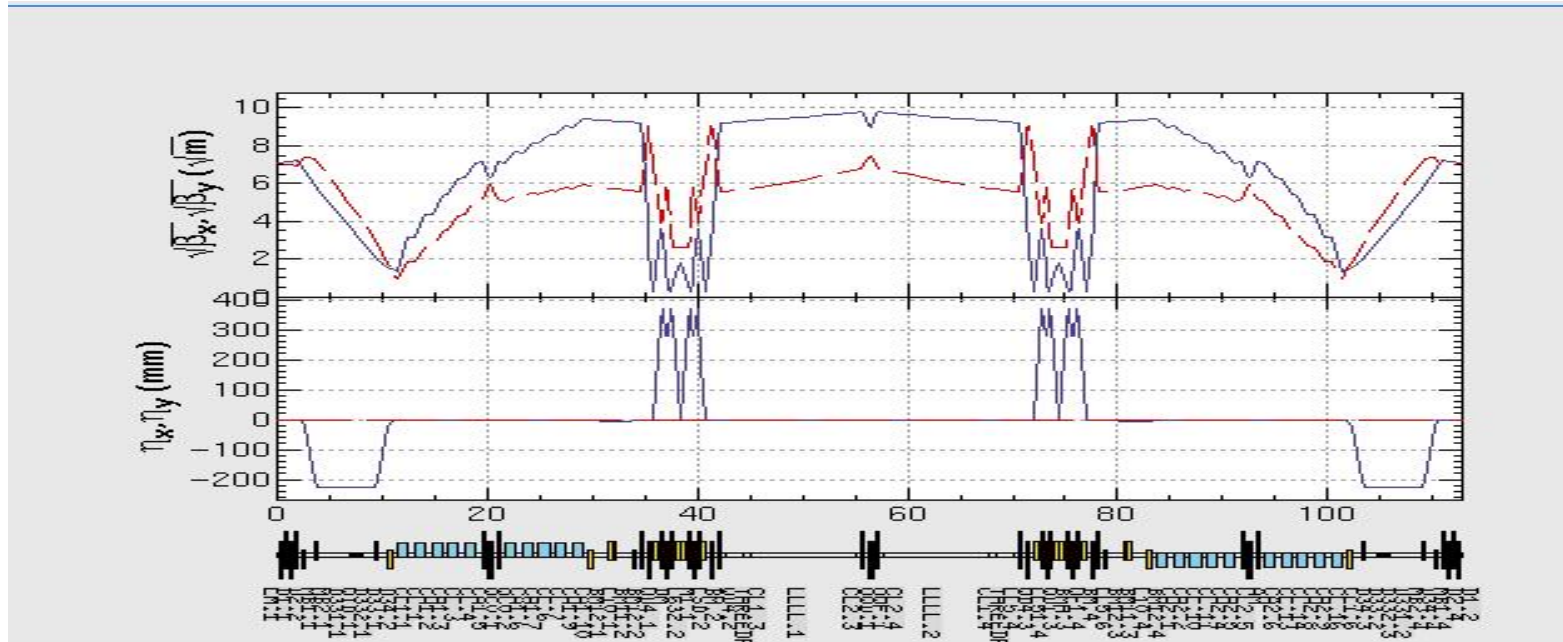
Tune survey for large dynamic aperture





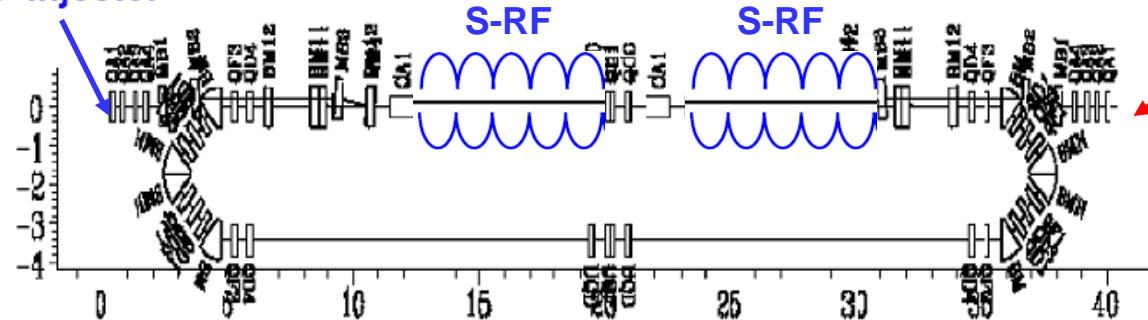
# 4. Beam dynamics in Energy Recovery Linac

## Lattice from merger to beam dump



5 MeV  $e^-$  Injector

Beam Dump 5 MeV





## 4. Beam dynamics in Energy Recovery Linac

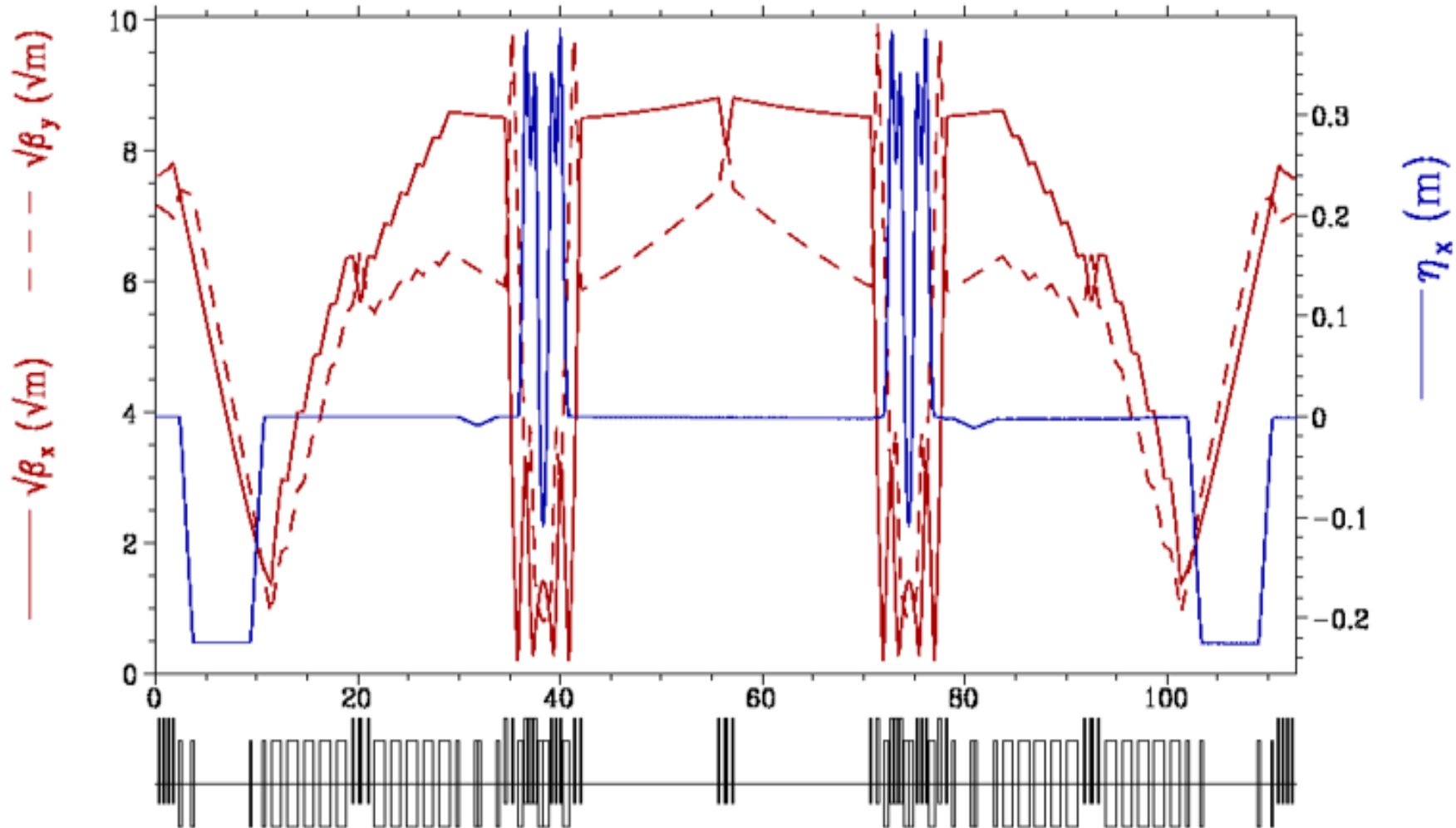
### Main parameters

Beam energy	: 205 MeV	Average beam current	: 100 mA
Bunch charge	: 77 pC	Injection beam energy	: 5 MeV
RF frequency	: 1.296 GHz	Harmonic number	: 320
Rms bunch length	: 1 ps	Rms energy spread	: $3 \times 10^{-4}$
Number of rf cavity units	: 10	RF cavity length	: 1 m
RF cavity gradient	: 20 MV /m	Momentum compaction	: $10^{-5}$
Normalized emittance (x/y) : 0.1 / 0.1 mm mrad			
Length of Bends	: 60 cm	Length of quadrupole	: 15cm/20cm

## 4. Beam dynamics in Energy Recovery Linac

Lattice design

Momentum compaction factor :  $10^{-5}$



## 4. Beam dynamics in Energy Recovery Linac

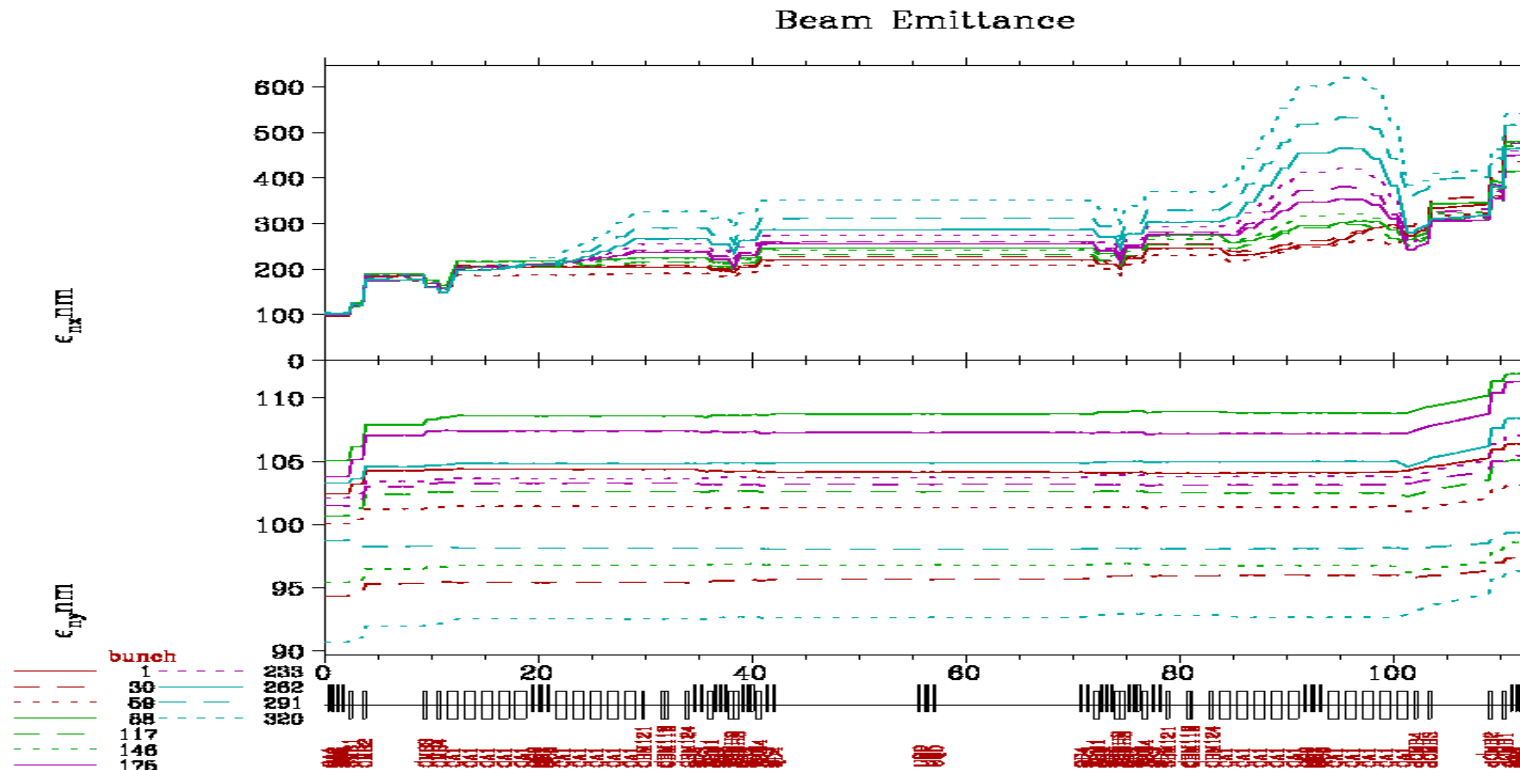
### Beam tracking for 320 bunches with cavity HOMs

$q=37\text{pC}$ ,  $\sigma_z = 1\text{ psec}$ ,  $\varepsilon_{nx}=0.1\text{ mm mrad}$

$R/Q=23.8 \times 10^4$ ,  $fr=2.57535 \times 10^9$ ,  $Q=5 \times 10^4$

$R/Q=8.69 \times 10^4$ ,  $fr=1.8722 \times 10^9$ ,  $Q=7 \times 10^4$

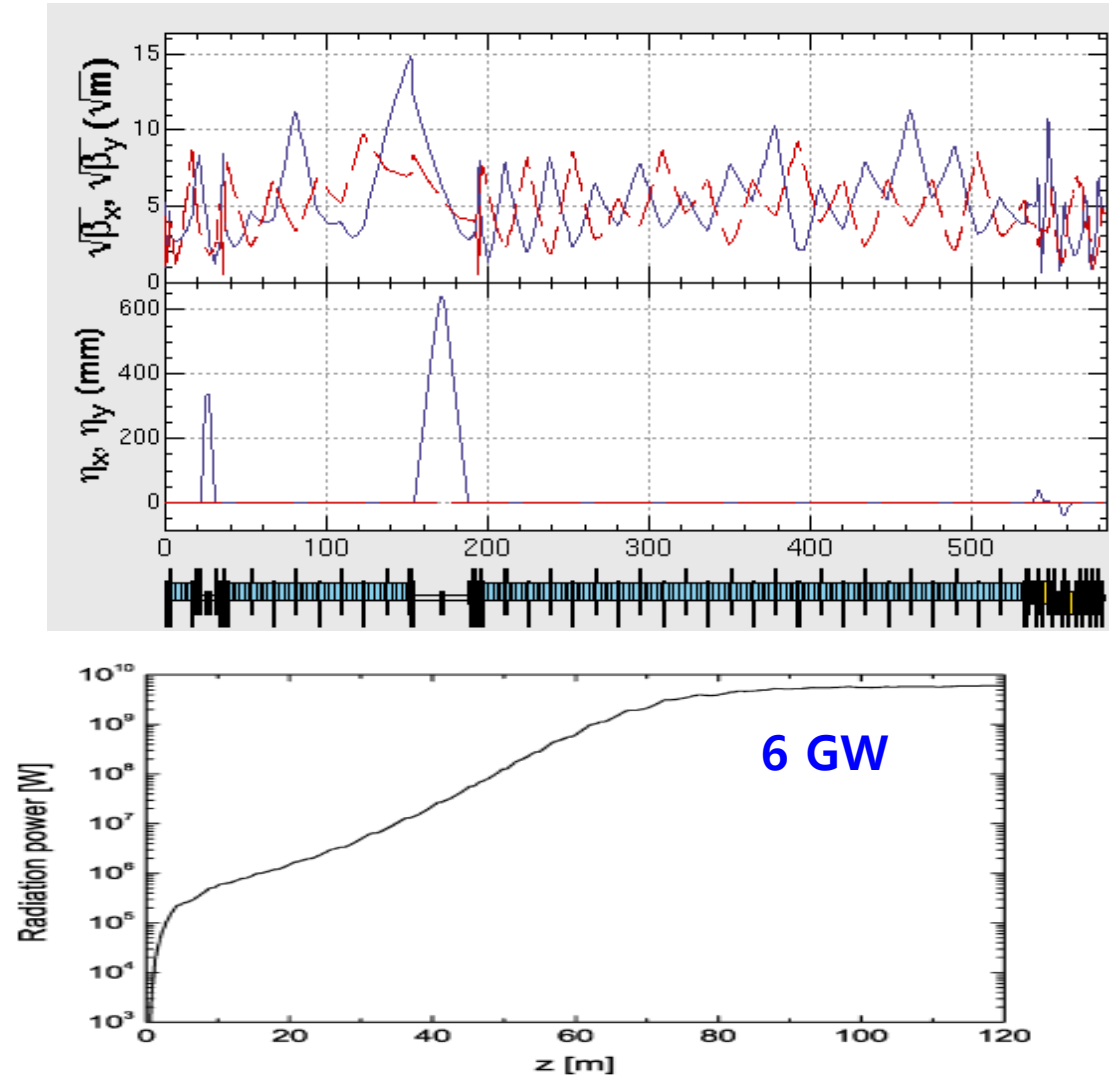
$R/Q=6.54 \times 10^4$ ,  $fr=1.8643 \times 10^9$ ,  $Q=5 \times 10^4$



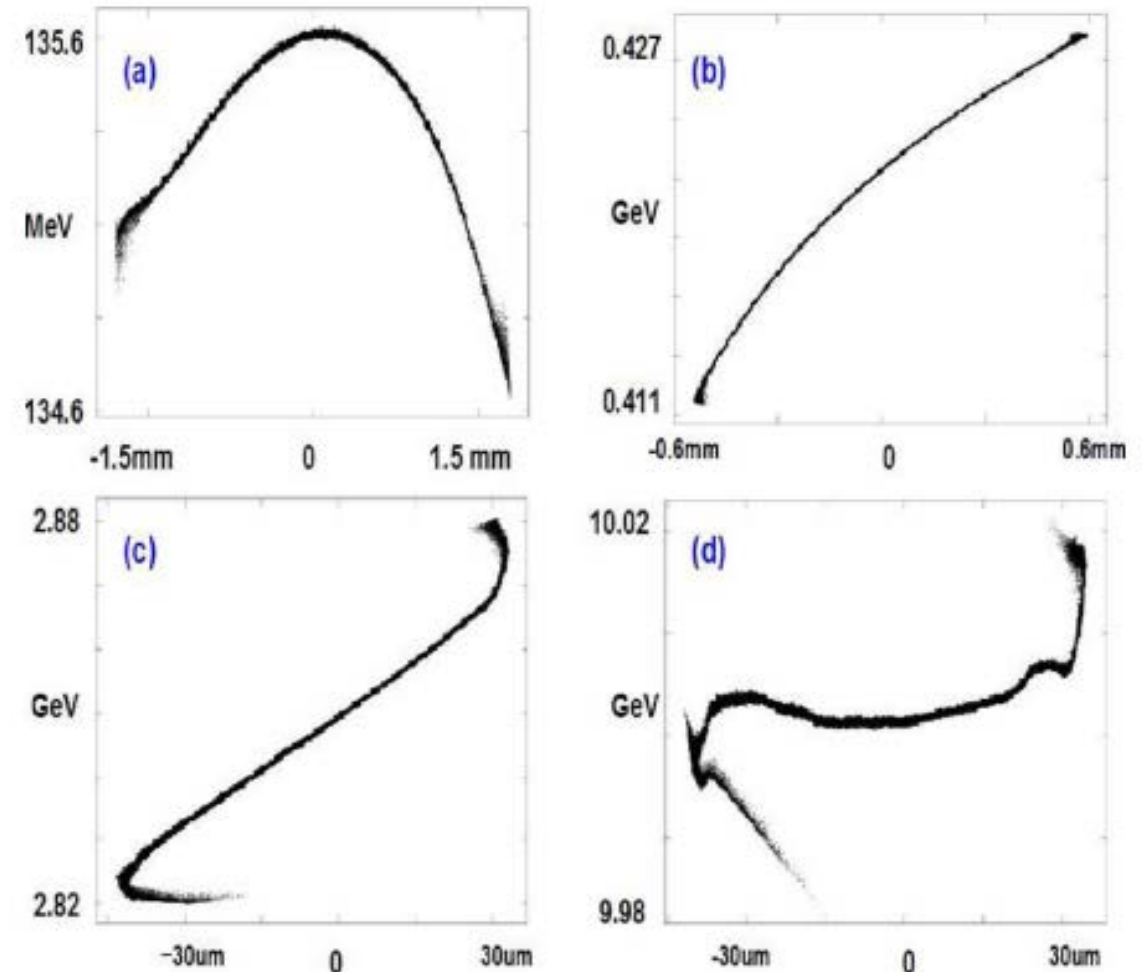
increased by factor of  $\sim 5$

# 5. Beam dynamics in Free Electron Linac

## Lattice for 10 GeV linac

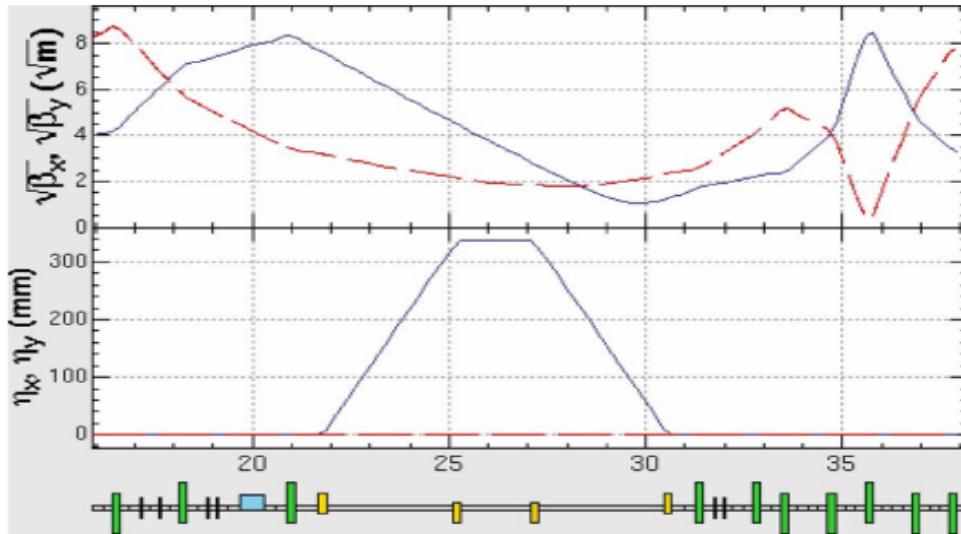


Radiation power as a function of the undulator length.

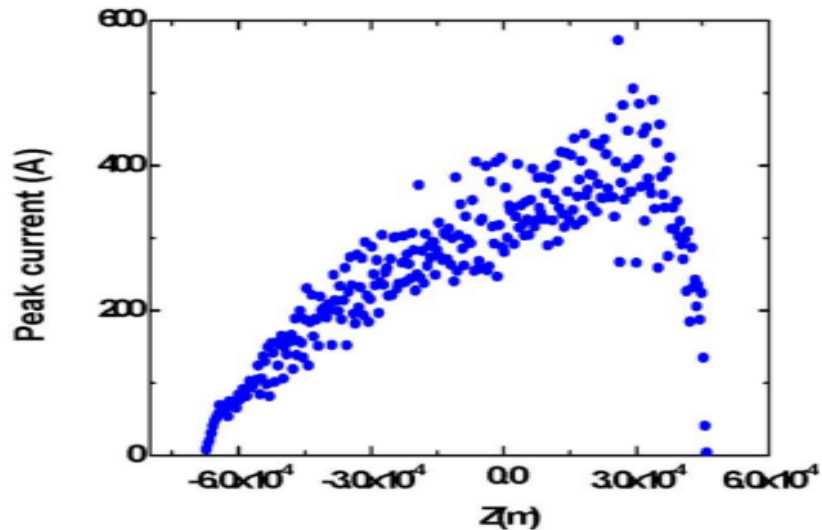


Longitudinal beam distributions (a) from the injector, (b) from the BC1, (c) from the BC2, (d) from the entrance of undulator.

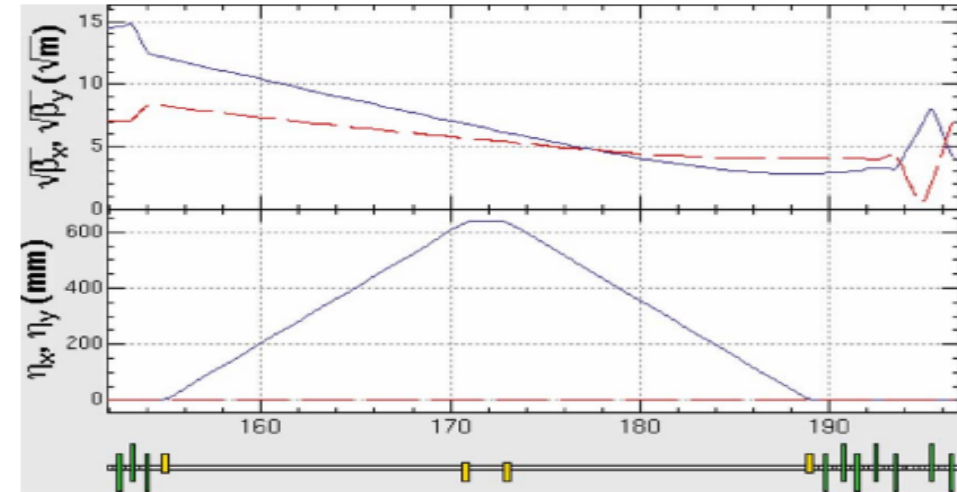
# 5. Beam dynamics in Free Electron Linac



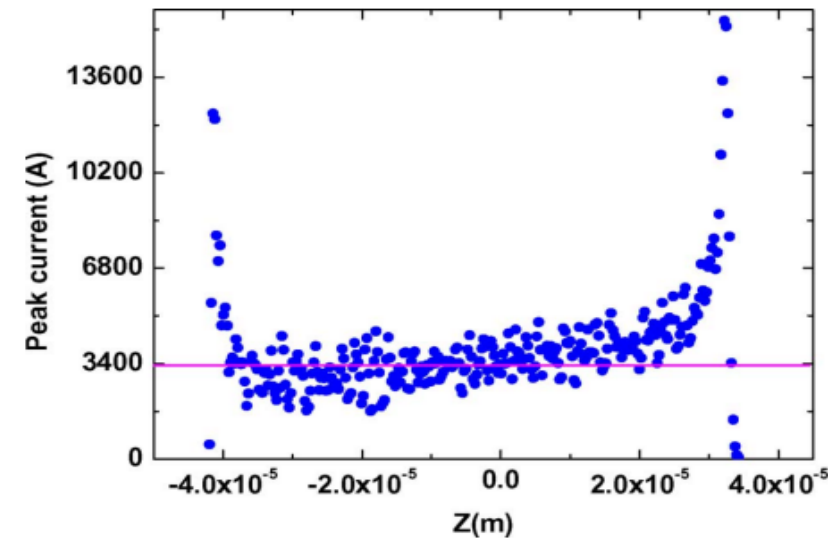
Optics for 1<sup>st</sup> bunch compressor



peak beam current in 1<sup>st</sup> bunch compressor.

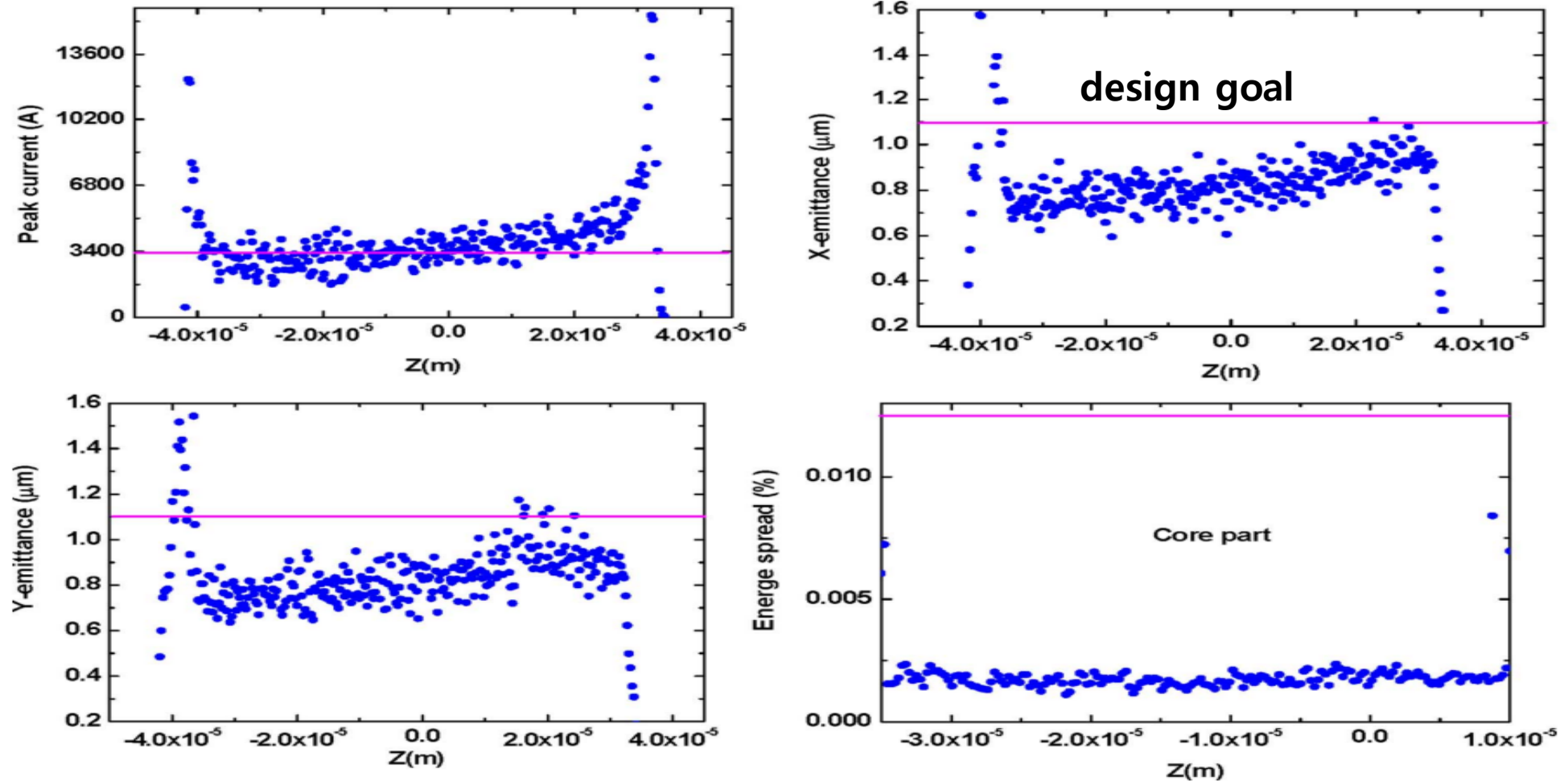


Optics for 2<sup>nd</sup> bunch compressor



peak beam current in 2<sup>nd</sup> bunch compressor.

# 5. Beam dynamics in Free Electron Linac



slice beams at the entrance of the undulator at 10 GeV

# Summary

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- We showed beam dynamics studies in ring and linac.
- Existing R&D items on beam dynamics and diagnostics with IHEP
  - Beam instabilities with Prof. Haisehng Xu
    - : invited (Feb. 24) to KU and visited (June. 24) to IHEP
  - are discussing with Prof. Yanfeng Sui on beam measurements for commissioning in HEPS.
  - We have an oral talk on “resistive wall instabilities in CEPC” in 2024 International workshop on High Energy CEPC (Oct. 22-24)
  - .....